ENAE 483/788D

Fall, 2002

Midterm Exam

This is a take-home examination. You may use any notes or reference material you like. You may not consult other class members or work as a team on this exam! It is due in class on Tuesday, November 12. I estimate that this would be a challenging, but achievable in-class test that should take 75 minutes. You may take as long as you wish, but please time yourself as a calibration point.

1. (25 pts) You are at the International Space Station, in a 500 km altitude circular orbit at 51.2° inclination. An Ariane V has just launched a communications satellite into a GEO transfer orbit, which has a 500 km altitude perigee, an apogee radius of 42,240 km, and a 0° inclination. Some useful numbers: μ=398,604 km^3/sec^2; r_E=6378 km

    a. The satellite carries an AKM ("apogee kick motor") to circularize its orbit at GEO (radius 42,240 km). What ΔV would be required for this maneuver?

The initial circular orbital radius r_p is the altitude (500 km) plus the radius of the Earth (6378 km), or 6878 km. The velocity at apogee of the transfer orbit is $v_a = \sqrt{\frac{\mu}{r_a}}\sqrt{\frac{2r_p}{r_p + r_a}} = \sqrt{\frac{398604}{42240}}\sqrt{\frac{2\cdot 6878}{6878 + 42240}}$ **=1.626 km/sec. The necessary circular orbital velocity in geostationary orbit is** $v_c = \sqrt{\frac{\mu}{r_a}} = \sqrt{\frac{398604}{42240}}$ **=3.072 km/sec. Since the two velocities are collinear, the required ΔV is just the difference between the two, or 1.446 km/sec.**

    b. The AKM fails, leaving the spacecraft stranded in its initial parking orbit. You decide to take your experimental spaceplane (based at ISS) on a rescue mission. What ΔV would be required to change orbits from the ISS orbit to the GEO transfer orbit, by performing a single maneuver at the point where the two orbits are in contact? (In other words, what is the ΔV to go from the initial circular orbit to perigee of the GEO transfer orbit with a 51.2° plane change?)

The velocity at perigee of the transfer orbit is $v_p = \sqrt{\frac{\mu}{r_p}}\sqrt{\frac{2r_a}{r_p + r_a}}$ **=**

$\sqrt{\frac{398604}{6878}}\sqrt{\frac{2 \cdot 42240}{6878 + 42240}}$ **=9.934 km/sec. Your current circular orbital velocity in ISS orbit is** $v_{cp} = \sqrt{\frac{\mu}{r_p}} = \sqrt{\frac{398604}{6878}}$ **=7.613 km/sec. Since you also have to accomplish a 51.2° plane change, the law of cosines is used to find the required ΔV by**

$\Delta v = \sqrt{v_{cp}^2 + v_p^2 - 2v_{cp}v_p\cos(\Delta i)}$ **=7.898 km/sec.**

c. It turns out that someone else is using the spaceplane, so you give up on rescue and now have to deorbit the satellite to keep it from becoming orbital debris. What apogee ΔV will be required to lower the satellite's perigee to 100 km so it will enter Earth's atmosphere and burn up?

From (a), your apogee velocity in the transfer ellipse is 1.626 km/sec. You want to be in an ellipse with the same apogee, and a new perigee of 6378+100=6478 km. The apogee velocity in this ellipse is $v_{a2} = \sqrt{\frac{\mu}{r_a}}\sqrt{\frac{2r_{p2}}{r_{p2} + r_a}} = \sqrt{\frac{398604}{42240}}\sqrt{\frac{2 \cdot 6478}{6478 + 42240}}$ **=1.584 km/sec. Again, these velocities are collinear, and the required ΔV is the difference between the two, or 0.04152 km/sec.**

2. (10 pts) When Evel Knievel tried to jump the Snake River Canyon, he used a steam-powered rocket. Water was superheated to 1000°F at 500 psi, then expanded to ambient pressure (14.7 psi) to generate thrust. What specific impulse would you expect this rocket engine to have? (Assume a γ of 1.33)

The exhaust velocity equation is $V_e = \sqrt{\frac{2\gamma}{\gamma-1}\frac{\Re T_0}{\overline{M}}\left[1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$**. Plugging the data in gives** $V_e = \sqrt{\frac{2(1.33)}{0.33}\frac{8314.3(811)}{18}\left[1-\left(\frac{14.7}{500}\right)^{\frac{0.33}{1.33}}\right]}$**. (Note that I necessarily converted °F to °K, but since the pressures only appear in a ratio, I didn't bother.) Crunching the numbers gives an exhaust velocity of 1327 m/sec; dividing by 9.8 m/sec^2 gives a specific impulse of $\boxed{\text{135.4 sec}}$.**

3. (10 pts) A cubical microsat (15 cm on a side) is undergoing testing in a thermal vacuum chamber. Liquid nitrogen shrouds all around the vacuum chamber ensure that the spacecraft is radiating to a temperature of 100°K in all directions. Internal electronics in the microsat are generating 10W of power. All surfaces of the spacecraft have an absorptivity α of 0.2 and an emissivity ε of 0.8. What is the equilibrium temperature the spacecraft will reach?

Assuming the spacecraft is at an equilibrium temperature of T, the power radiated into a 100°K environment is $\varepsilon\sigma A\left(T^4-100^4\right)$**. The surface area A of the spacecraft is 6x0.15^2=0.135 m^2. Since no illumination source is specified, the heat into the spacecraft is just the 10W of the internal electronics. Setting these equal gives** $10 = 0.8\left(5.67\times10^{-8}\right)(0.135)\left(T^4-100^4\right)$**, which gives** $\boxed{T_{equil}=204°K}$**.**

4. (10 pts) The first flight of your new launch vehicle has just succeeded. The head of your marketing department wants to go ahead with an advertising campaign claiming a 97% reliability for the vehicle.

   a. What confidence value would correspond to this 97% reliability claim?

The definition of confidence is P+C=1. Since the claimed P=0.97, the confidence is $\boxed{C=0.03}$**.**

   b. Unfortunately, the second flight fails. The marketing manager graciously agrees to lower the reliability claim to 95%. What confidence value does this estimate have?

For this case, confidence is the likelihood that your results would have been worse than those observed and all better outcomes. This means that $P^2 + 2P(1-P) + C = 1$**.** **We can rearrange this to get** $C = 1 - \left[P^2 + 2P(1-P)\right] = 1 - 2 \cdot 0.95 + 0.95^2$ **or** $\boxed{\mathbf{C=0.0025}}$

5. (20 pts) A spacecraft on the surface of Mars requires 5000 m/sec ΔV to reach low Martian orbit. It has a payload mass of 20,000 kg and an inert mass of 10,000 kg, *excluding* the propellant tanks, which have a mass equal to 5% of the propellants they contain. It uses liquid oxygen and methane for propellants, with an I_{sp} of 330 sec.

   a. What is the required mass ratio for this spacecraft?

The rocket equation gives $r = e^{-\frac{\Delta V}{g \cdot I_{sp}}} = \exp\left(-\frac{5000}{9.8 \cdot 330}\right)$ $=\boxed{\mathbf{0.2131}}$**. Note that "g" in the rocket equation is used only to convert from mass to force units – it has nothing to do with where you are in the universe, because our systems of units is defined around Earth gravity, and g always equals 9.8 m/sec^2 in this equation!**

   b. What are the propellant mass, tank mass, and total initial mass for this spacecraft?

$r = \frac{m_{final}}{m_{initial}} = \frac{m_{inert} + m_{\text{tanks}} + m_{payload}}{m_{propellants} + m_{inert} + m_{\text{tanks}} + m_{payload}}$ **Using the supplied relationship between** m_{tanks} **and** $m_{propellants}$**, this becomes**

$r = \frac{m_{final}}{m_{initial}} = \frac{0.05 m_{propellants} + m_{inert} + m_{payload}}{1.05 m_{propellants} + m_{inert} + m_{payload}} = \frac{0.05 m_{propellants} + 30000}{1.05 m_{propellants} + 30000}$ **or, after a little algebra,** $m_{propellants} = \frac{30000(1-r)}{1.05r - 0.05}$**. Plugging in r=0.2131 from (a) gives** $\boxed{\mathbf{m_{propellants}=135{,}900\ kg}}$**. The tank mass is 5% of this, or** $\boxed{\mathbf{m_{tanks}=6794\ kg}}$**. Total initial mass is** $m_{inert} + m_{payload} + m_{tanks} + m_{propellant}$**, so** $\boxed{\mathbf{m_{initial}=172{,}700\ kg}}$**.**

   c. What is the inert mass fraction δ for this spacecraft?

The inert mass fraction is all vehicle mass remaining at burnout divided by initial mass, or $\delta = \frac{m_{inert} + m_{\text{tanks}}}{m_{initial}} = \frac{10000 + 6794}{172700}$ $=\boxed{\mathbf{0.0973}}$**.**

6. (10 pts) A spacecraft which requires a constant power of 5 kW is in a highly elliptical orbit around the moon. Due to the orientation of the orbit, it receives daylight for only 1 hr 15 min of its 6 hour orbital period. Batteries are used to supply power during the dark periods. Size the photovoltaic array (in kW) needed for this spacecraft, assuming perfect efficiency in battery charge/discharge cycles.

To supply a power of 5kW during 4.75 hours of darkness requires 4.75x5=23.75kW-hrs of energy storage. We have to gain this back during the 1h15m daylight pass, so the power generation for recharging will be 23.75kWhrs/1.25hrs = 19 kW. Adding in the 5 kW for daylight power usage gives the required photovoltaic power capacity of 24 kW. (Note that I didn't ask for mass, area, or battery parameters, just PV array power.)

7. (10 pts) A human lunar exploration program has been developed with will cost $1B for the first mission. Five missions will be flown over five years.

a. Mission costs are reduced for subsequent flights due to an 80% learning curve. What it the cost for each of the five missions?

Using the learning curve relation $C_n = C_1 n^p$ **and the definition of**

$p = \frac{\ln(LC\%)}{\ln(2)} = \frac{\ln(0.8)}{\ln(2)}$**=-0.3219, you get**

C_1=\$1000M, C_2=\$800M, C_3=\$702.1M, C_4=\$640M, C_5=\$595.6M.

b. What is the net present value of the total program cost (for all five missions), based on the year of the first mission, using a discount rate of 10%?

Since everything is referenced to the first year, the discounted cost C_{1d}=\$1000M. The next year is discounted by $C_{2d} = C_2(1+r)^{-1} = 800(1.1)^{-1}$ **or C_{2d}=\$727.3M. Similarly, C_{3d}=\$580.3M, C_{4d}=480.8M, and C_{5d}=\$406.8M. Adding these up gives the total discounted program cost $C_{tot,d}$=\$3.195B.**

8. (5 pts) How long did it take you to complete this exam?

Average: 2:12

Standard deviation: 42.6m

Longest: 4:00

Shortest: 1:10

<u>ENAE 483 Exam Results:</u>

Average: 84.1

Standard deviation: 14.9

Longest: 96

Shortest: 35

<u>ENAE 788D Exam Results:</u>

Average: 89.3

Standard deviation: 6.0

Longest: 98

Shortest: 81