Drag on a hemisphere

\[ dA = r^2 \sin \theta \, d\theta \, d\phi \]
\[ \alpha = \theta + \frac{\pi}{2} \]

Force on \( dA = \rho \, V^2 \, \sin^2 \alpha \, dA \)

\[ = \rho \, V^2 \, \sin^2 \left( \theta + \frac{\pi}{2} \right) \frac{r^2 \sin \theta}{\cos^2 \theta} \, d\theta \, d\phi \]
\[ = \rho \, V^2 \, r^2 \cos^2 \theta \sin \theta \, d\theta \, d\phi \]

Drag on \( dA = \) (Force on \( dA \)) \( \sin \alpha = \) (Force on \( dA \)) \( \cos \theta \)

\[ \text{Drag} = \rho \, V^2 \, r^2 \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \cos^2 \theta \sin \theta \, d\theta \, d\phi \]
\[ = 2\pi \rho \, V^2 \, r^2 \int_0^{\frac{\pi}{2}} \cos^2 \theta \sin \theta \, d\theta \]
\[ = 2\pi \rho \, V^2 \, r^2 \left[ \frac{\cos^4 \theta}{4} \right]_0^{\frac{\pi}{2}} \]
\[ = 2\pi \rho \, V^2 \, r^2 \left( \frac{1}{8} \right) \]
\[ D = 2\pi \rho \, V^2 \, r^2 \left( \frac{1}{8} \right) \]
\[ = \frac{1}{2} \pi \rho \, V^2 \, \frac{r^2}{A} \]
\[ = \frac{1}{2} \rho \, V^2 \, A \, C_D \quad \Rightarrow \quad C_D = 1 \]

Can solve this again for lift, but symmetry gives \( C_L = 0 \)
Newtonian Flow explains behavior of an ideal shock wave; past entry vehicles on Earth!

normal (detached shocks) - how do we model that?

Newtonian Flow: \( C_p = 2 \sin^2 \alpha \)

Modified Newtonian Flow: \( C_p = C_{pmax} \sin^2 \alpha \)

\[
C_{pmax} = \text{pressure coefficient behind normal shock}
\]

\[
C_{pmax} = \frac{2}{\gamma m^2} \left\{ \left[ \frac{(\gamma+1) m^2}{\gamma \delta m^2 - 2(\delta-1)} \right]^{\frac{\gamma}{\gamma-1}} \left[ \frac{1 - \delta + 2 \delta M^2}{\delta+1} \right] - 1 \right\}
\]

We can rewrite this as

\[
C_{pmax} = \left[ \frac{(\gamma+1)}{4\gamma - 2(\delta-1)} \right]^{\frac{\gamma}{\gamma-1}} \left[ \frac{2(1-\delta)}{\delta(\delta+1)m^2} + \frac{4}{\delta+1} \right]^{\frac{1}{\gamma}} - 1 \times \frac{2}{\gamma m^2}
\]

Now let \( M \to \infty \)

\[
C_{pmax} \lim_{M \to \infty} = \left[ \frac{(\delta+1)^2}{4\gamma} \right]^{\frac{\gamma}{\gamma-1}} \left( \frac{4}{\delta+1} \right)
\]

\[
= 1.839 \text{ for } \delta = 1.4
\]

\[
= 2.0 \text{ for } \delta = 1.0
\]
FIGURE 3.8
Surface pressure distribution over a paraboloid at $M_{\infty} = 8.0$; $p_0$ is the total pressure behind a normal shock wave at $M_{\infty} = 8.0$. 