HEATING RATES

DEFINITIONS

Prandtl Number

\[ Pr = \frac{\mu C_p}{K} \]

where \( C_p \) = specific heat @ constant pressure

\( K \) = thermal conductivity

\( \mu \) = viscosity

\[ Pr \propto \frac{\text{frictional dissipation}}{\text{thermal conduction}} \]

\( Pr \approx 0.715 \) for air at standard conditions

Sutherland's Law (empirical)

Viscosity depends on temp

\[ \frac{\mu}{\mu_{ref}} = \left( \frac{T}{T_{ref}} \right)^{\frac{3}{2}} \frac{T_{ref} + S}{T + S} \]

for air:

\[ \mu_{ref} = 1.789 \times 10^{-5} \text{ kg/m s} \]

\[ T_{ref} = 288 \text{ K} \]

\[ S = 110 \text{ K} \]

...good up to several thousand degrees

Stanton Number - applies to boundary layer prob

\[ St = \frac{\dot{q}_w}{\rho e V e (H_w - H_w)} \]

\( H \) = enthalpy

\[ = C_p T \] for perfect gas

\( H_w \) = enthalpy @ the wall

\( H_w \) = enthalpy @ the wall if it were adiabatic

\[ \Rightarrow \frac{\partial}{\partial \xi} \left( \frac{\Delta H}{\Delta \xi} \right) = 0 \]
Approximating Haw

\[ \text{Haw} = \text{H}_e + r \frac{u_w^2}{2} \]
\[ r \text{ = recovery factor} \]
\[ \text{ TOTAL ENTHALPY at edge of boundary layer } \]

\[ \text{H}_e = \text{H}_e + \frac{u_w^2}{2} \]

so...

\[ \text{Haw} = \text{H}_e + r (\text{H}_e - \text{H}_e) \]

for incompressible flow, \( r = \sqrt{\frac{\text{Pr}}{\text{Pr}}} \)
\( = 0.845 \) for std. air

\( r \) decreases only 2.4% from \( M = 0 \) to \( M = 16 \)

\( \rightarrow \) fairly constant!

Reynold's Analogy - pg 81 in Hankey

\[ \frac{\text{St}}{\text{C}_f} = \frac{1}{2} \text{Pr}^{-2/3} \]
\( \text{C}_f \) = skin friction coeff.

since \( \text{Pr} \approx 1 \)

\[ \text{St} \approx \frac{\text{C}_f}{2} \]

this is thru Reynold's Analogy

Empirical Correlation for \( \text{C}_f \):

\[ \frac{\text{C}_f}{2} = \frac{A}{\text{Re}^n} \]

<table>
<thead>
<tr>
<th>LAMINAR FLOW</th>
<th>TURBULENT FLOW</th>
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<tbody>
<tr>
<td></td>
<td>0.332 0.5</td>
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<tr>
<td>0.0294 0.2</td>
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</tbody>
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transition b/w laminar & turbulent @ ~ \( \text{Re} = 10^6 \)

\[ \text{St} \approx \frac{A}{\text{Re}^n \text{Pr}}^{-2/3} \]

STANTON NUMBER
FRACTION OF HEAT INTO WAKE

\[ q_e = \rho A V H_s \quad (\text{total entropy}) \]
\[ q_w = \dot{q}_B = S T P A V (H_w - H_w) \]
\[ y = \left[ k \frac{1}{\beta} \right] \text{wall} \]

usually, \( H_w >> H_w \)
\( H_w \approx H_s \)

\[ \frac{\dot{q}_B}{\dot{q}_A} = ST \approx 10^{-3} \]

\[ \Rightarrow 0.1\% \text{ of heating goes into vehicle!} \]
most of the energy goes into the atmosphere

FREE MOLECULAR FLOW

At equilibrium
- no heating

on a larger scale...

more accurately
(rough surface, molecule bounces a few times before leaving vehicle)
Newtownian Flow: \( q = 2 \sin^2 \theta = 2 \cos^2 \phi \)

\( \phi = \text{complement of } \theta \)

\[
\frac{P_e - P_{\infty}}{q_{\infty}} = 2 \cos^2 \phi
\]

\[
P_e = 2 q_{\infty} \cos^2 \phi + P_{\infty}
\]

\[
\frac{dP_e}{dx} = -4 q_{\infty} \cos \phi \sin \phi \frac{d\phi}{dx}
\]

\[
\frac{d u_e}{dx} = \frac{4 q_{\infty}}{\rho u_e} \cos \phi \sin \phi \frac{d\phi}{dx}
\]

everywhere on the body

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New Evaluate Heating Rate at STAGNATION POINT, \( S \)

\( u_e = (\frac{d u_e}{dx})_S \Delta x \)

\( 2 \text{ incompressible flow around } S \)

at \( S \):

\[
\cos \Delta \phi = 1
\]

\[
\sin \Delta \phi \approx \Delta \phi = \frac{\Delta x}{R}
\]

\[
\frac{\Delta \phi}{\Delta x} = \frac{1}{R}
\]

\( q_{\infty} = \frac{1}{2} (P_e - P_{\infty}) \)

\[
(\frac{d u_e}{dx})^2 = \frac{2 (P_e - P_{\infty})}{\rho u_e \Delta x} \left( \frac{\Delta x}{R} \right) \left( \frac{1}{R} \right) 
\]

\[
\frac{d u_e}{dx} = \frac{1}{R} \sqrt{\frac{2 (P_e - P_{\infty})}{\rho u_e}}
\]

---

\[q_{in} \propto \frac{1}{R}, \text{ Blunt Bodie Stay Cooler Than Sharp Ones}\]
APPROXIMATIONS FOR HEATING RATES
(these will get you within 10% or so)

Simple Set of Equations for HYPERSONIC CONTINUUM FLOW
\[ q_w = \rho_{\infty}^N V_{\infty}^M C \]
\[ q_w \text{ in } \frac{\text{W}}{\text{cm}^2} \]
\[ \rho_{\infty} \text{ in } \frac{\text{kg}}{\text{m}^3} \]
\[ V_{\infty} \text{ in } \frac{\text{m}}{\text{s}} \]
\[ R \text{ in } \text{m} \]

Stagnation Point:
- \( M = 3 \)
- \( N = 0.5 \)
- \( C = 1.83 \times 10^{-6} R^{-1/2} \left( 1 - \frac{H_w}{H_0} \right) \)

Laminar Flow on Flat Plate:
- \( M = 3.2 \)
- \( N = 0.5 \)
- \( C = 2.53 \times 10^{-9} (\cos \phi)^{1/2} (\sin \phi) R^{1/2} \left( 1 - \frac{H_w}{H_0} \right) \)

where \( R \) = distance along surface in \( \text{m} \)

Turbulent Flow on Flat Plate:
CASE I: \( V_{\infty} \leq 3962 \text{ m/s} \)
- \( M = 3.37 \)
- \( N = 0.8 \)
- \( C = 3.9 \times 10^{-9} (\cos \phi)^{1.38} (\sin \phi)^{1.6} R^{1/4} \left( \frac{T_w}{552} \right)^{1/4} \left( 1 - \frac{H_w}{H_0} \right) \)

CASE II: \( V_{\infty} > 3962 \text{ m/s} \)
- \( M = 3.7 \)
- \( N = 0.8 \)
- \( C = 3.2 \times 10^{-9} (\cos \phi)^{2.08} (\sin \phi)^{1.6} R^{1/4} \left( 1 - \frac{H_w}{H_0} \right) \)

where \( R \) = distance measured in the turbulent part of the boundary layer.