Mass Estimating Relationships

- Mass Estimating Relationships (MERs)
- Sample vehicle design analysis
Design is an iterative process. The necessary number of iterations is one more than the number you have currently done. This is true at any point in time.
Vehicle-Level Preliminary Design - 1st Pass

- Single Stage to Orbit (SSTO) vehicle
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec
  - δ=0.08

\[ r = e^{\frac{-\Delta V}{gI_{sp}}} = 0.1127 \]
\[ \lambda = r - \delta = 0.0327 \]
\[ M_0 = \frac{M_L}{\lambda} = 153,000 \text{ kg} \]
\[ M_i = \delta M_0 = 12,240 \text{ kg} \]
\[ M_p = M_0 (1 - r) = 135,800 \text{ kg} \]
System-Level Estimation

- Start with propellant tanks (biggest part)
- LOX/LH2 engines generally run at mixture ratio of 6:1 (by weight)
  - LH2: 19,390 kg
  - LOX: 116,400 kg
- Propellant densities

\[
\rho_{\text{LOX}} = 1140 \frac{\text{kg}}{\text{m}^3} \quad \rho_{\text{LH}_2} = 112 \frac{\text{kg}}{\text{m}^3}
\]
LOX Tank MERs

- Mass of Tank
  \[ M_{\text{LOX Tank}}(kg) = 0.0152 M_{\text{LOX}}(kg) + 318 \]

- Mass of Insulation
  \[ M_{\text{LOX Insulation}}(kg) = 1.123 \frac{kg}{m^2} \]
LOX Tank Design

- Mass of LOX = 116,400 kg
  \[ M_{\text{LOX Tank}} (kg) = 0.0152(116,400) + 318 = 2087 \text{ kg} \]

- Need area to find LOX tank insulation mass
  - assume a sphere

  \[ V_{\text{LOX Tank}} = \frac{M_{\text{LOX}}}{\rho_{\text{LOX}}} = 102.1 \text{ m}^3 \]

  \[ r_{\text{LOX Tank}} = \left( \frac{V_{\text{LOX}}}{4\pi / 3} \right)^{\frac{1}{3}} = 2.90 \text{ m} \]

  \[ A_{\text{LOX Tank}} = 4\pi r_{\text{LOX}}^2 = 105.6 \text{ m}^2 \]

  \[ M_{\text{LOX Insulation}} (kg) = 1.123 \frac{kg}{m^2} \left( 105.6 \text{ m}^2 \right) = 119 \text{ kg} \]
LH2 Tank MERs

- Mass of Tank

\[ M_{LH_2 \text{ Tank}}(kg) = 0.0694 M_{LH_2}(kg) + 363 \]

- Mass of Insulation

\[ M_{LH_2 \text{ Insulation}}(kg) = 2.88 \frac{kg}{m^2} \]
LH2 Tank Design

- Mass of LH2 = 19,390 kg
  \[ M_{LH2\ Tank}(kg) = 0.0694(19,390) + 363 = 1709\text{ kg} \]
- Again, assume LH2 tank is spherical
  \[ V_{LH2\ Tank} = M_{LH2} / \rho_{LH2} = 346.3\ m^3 \]

\[ r_{LH2\ Tank} = \left( \frac{V_{LH2}}{4\pi / 3} \right)^{\frac{1}{3}} = 3.46\ m \]

\[ A_{LH2\ Tank} = 4\pi r_{LH2}^2 = 150.2\ m^2 \]

\[ M_{LH2\ Insulation}(kg) = 2.88\frac{kg}{m^2}(150.2\ m^2) = 433\ kg \]
**Current Design Sketch**

- **Masses**
  - LOX Tank 2089 kg
  - LOX Tank Insulation 1709 kg
  - LH2 Tank 119 kg
  - LH2 Tank Insulation 433 kg
Other Tankage MERs

- Storable Propellants (RP-1, N$_2$O$_4$, N$_2$H$_4$)
  \[ M_{\text{Storables Tank}}(kg) = 0.316[M_{\text{Storables}}(kg)]^6 \]

- Small tank (liquids)
  \[ M_{\text{Small Liquid Tank}}(kg) = 0.1M_{\text{contents}}(kg) \]

- Small tank (pressurized gases)
  \[ M_{\text{Small Gas Tank}}(kg) = 2M_{\text{contents}}(kg) \]
Boost Module Propellant Tanks

- Gross mass 23,000 kg
  - Inert mass 2300 kg
  - Propellant mass 20,700 kg
  - Mixture ratio $\text{N}_2\text{O}_4/\text{A50} = 1.8$ (by mass)

- $\text{N}_2\text{O}_4$ tank
  - Mass = 13,310 kg
  - Density = 1450 kg/m$^3$
  - Volume = 9.177 m$^3$ $\rightarrow r_{\text{sphere}} = 1.299$ m

- Aerozine 50 tank
  - Mass = 7390 kg
  - Density = 900 kg/m$^3$
  - Volume = 8.214 m$^3$ $\rightarrow r_{\text{sphere}} = 1.252$ m
N$_2$O$_4$ Tank Sizing

- Need total N$_2$O$_4$ volume = 9.177 m$^3$
- Single tank
  - Radius = 1.299 m
  - Mass = 94.2 kg
- Dual tanks
  - Radius = 1.031 m
  - Mass = 62.2 kg ($x_2 = 124.3$ kg)
- Triple tanks
  - Radius = 0.900 m
  - Mass = 48.7 kg ($x_3 = 146.2$ kg)
Other Structural MERs

- **Fairings and shrouds**
  \[ M_{Fairing}(kg) = 13.3 \frac{kg}{m^2} \]

- **Avionics**
  \[ M_{Avionics}(kg) = 10 \left[ M_0(kg) \right]^{0.361} \]

- **Wiring**
  \[ M_{Wiring}(kg) = 1.058 \sqrt{M_0(kg)} |^{0.25} \]
Fairing Analysis

- **Payload Shroud**
  - Area: 51.74 m²
  - Mass: 688 kg

- **Intertank Fairing**
  - Area: 126.1 m²
  - Mass: 1677 kg

- **Aft Fairing**
  - Area: 107.7 m²
  - Mass: 1433 kg
Avionics and Wiring Masses

- **Avionics**

\[ M_{\text{Avionics}}(\text{kg}) = 10 \times [153,000 \text{ kg}]^{0.361} = 744 \text{ kg} \]

- **Wiring**

\[ M_{\text{Wiring}}(\text{kg}) = 1.058 \times \sqrt{153,000} \times (16.95 \text{ m})^{0.25} = 840 \text{ kg} \]
Propulsion MERs

- Liquid Pump-Fed Rocket Engine Mass
  \[ M_{\text{Rocket Engine}}(kg) = 7.81 \times 10^{-4} T(N) + 3.37 \times 10^{-5} T(N) \sqrt{\frac{A_e}{A_t}} + 59 \]

- Solid Rocket Motor
  \[ M_{\text{Motor Casing}} = 0.135 M_{\text{propellants}} \]

- Thrust Structure Mass
  \[ M_{\text{Thrust Structure}}(kg) = 2.55 \times 10^{-4} T(N) \]
Mass Estimating Relationships

Propulsion MERs (continued)

• Gimbal Mass

\[ M_{Gimbals} \text{ (kg)} = 237.8 \left[ \frac{T(N)}{P_0(Pa)} \right]^{.9375} \]

• Gimbal Torque

\[ \tau_{Gimbals} \text{ (N \cdot m)} = 990,000 \left[ \frac{T(N)}{P_0(Pa)} \right]^{1.25} \]
Propulsion System Assumptions

- Initial T/mg ratio = 1.3
  - Keeps final acceleration low with reasonable throttling
- Number of engines = 6
  - Positive acceleration worst-case after engine out
- Chamber pressure = 1000 psi = 6897 kN
  - Typical for high-performance LOX/LH2 engines
- Expansion ratio $A_e/A_t = 30$
  - Compromise ratio with good vacuum performance
Propulsion Mass Estimates

- Rocket Engine Thrust (each)
  \[ T(N) = \frac{m_0 g (T/W)_0}{n_{\text{engines}}} = 324,900 \text{ N} \]

- Rocket Engine Mass (each)
  \[ M_{\text{Rocket Engine}}(kg) = 7.81 \times 10^{-4} (324,900) + 3.37 \times 10^{-5} (324,900)^{\sqrt{30}} + 59 = 373 \text{ kg} \]

- Thrust Structure Mass
  \[ M_{\text{Thrust Structure}}(kg) = 2.55 \times 10^{-4} (324,900) = 497 \text{ kg} \]
First Pass Vehicle Configuration
## Mass Summary - First Pass

- **Initial Inert Mass Estimate**: 12,240 kg
- **LOX Tank**: 2087 kg
- **LH2 Tank**: 1709 kg
- **LOX Insulation**: 119 kg
- **LH2 Insulation**: 433 kg
- **Payload Fairing**: 688 kg
- **Intertank Fairing**: 1677 kg
- **Aft Fairing**: 1433 kg
- **Engines**: 2236 kg
- **Thrust Structure**: 497 kg
- **Gimbals**: 81 kg
- **Avionics**: 744 kg
- **Wiring**: 840 kg
- **Reserve**: -
- **Total Inert Mass**: 12,543 kg
- **Design Margin**: -2.4 %
Modifications for Second Pass

- Keep all initial vehicle sizing parameters constant
- Pick vehicle diameter and make tanks cylindrical to fit
- Redo MER analysis
Effect of Vehicle Diameter on Mass Margin

Vehicle Diameter (m)

Mass Estimating Relationships
Launch And Entry Vehicle Design
Effect of Mass-Optimal Diameter Choice

- Vehicle has L/D of 25.2 - severe complications from structural dynamics
- Mass margin goes from -2.4% to +18.3%
- Decreased volume for rocket engines in aft fairing
- Infeasible configuration
Effect of Diameter on Vehicle L/D

Vehicle Diameter (m)

Vehicle Length/Diameter

0.0
20.0
40.0
60.0
80.0
100.0
120.0
Second Pass Vehicle Configuration
### Mass Summary - Second Pass

- **Initial Inert Mass Estimate**: 12,240 kg
- **LOX Tank**: 2087 kg
- **LH2 Tank**: 1709 kg
- **LOX Insulation**: 119 kg
- **LH2 Insulation**: 433 kg
- **Payload Fairing**: 688 kg
- **Intertank Fairing**: 1677 kg
- **Aft Fairing**: 1433 kg
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- **Thrust Structure**: 497 kg
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### Design Margin

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### Design Margin

- **LOX Tank**: 2087 kg
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- **Thrust Structure**: 497 kg
- **Gimbals**: 81 kg
- **Avionics**: 744 kg
- **Wiring**: 840 kg
- **Reserve**: -
- **Total Inert Mass**: 12,543 kg
- **Design Margin**: -2.4 %
Modifications for Iteration 3

- Keep 4 m tank diameter
- Change initial assumption of $\delta$ iteratively, with resulting changes in $m_0$ and $m_i$, to reach 30% mass margin
Vehicle-Level Preliminary Design - 3rd Pass

- Single Stage to Orbit (SSTO) vehicle
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec
  - $\delta=0.08655$

\[ r = e^{\frac{-\Delta V}{gI_{sp}}} = 0.1127 \]
\[ \lambda = r - \delta = 0.0261 \]
\[ M_0 = \frac{M_L}{\lambda} = 191,300 \text{ kg} \]
\[ M_i = \delta M_0 = 16,560 \text{ kg} \]
\[ M_p = M_0(1 - r) = 169,800 \text{ kg} \]
Third Pass Vehicle Configuration
## Mass Summary - Third Pass

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<thead>
<tr>
<th>Component</th>
<th>Mass Estimate 1</th>
<th>Mass Estimate 2</th>
<th>Mass Estimate 3</th>
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<tbody>
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<td>12,240 kg</td>
<td>16,560 kg</td>
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<tr>
<td>LOX Tank</td>
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<td>LH2 Tank</td>
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<td>Design Margin</td>
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## Mass Budgeting

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