Course Overview/Orbital Mechanics

• Course Overview
  - Problems with launch and entry
  - Course goals
  - Web-based Content
  - Syllabus
  - Policies
  - Project Content

• An overview of orbital mechanics at "point five past lightspeed"
Space Launch - The Physics

• Minimum orbital altitude is ~200 km

\[
\text{Potential Energy} \quad \frac{\text{kg in orbit}}{\text{J}} = gh = 1.96 \times 10^6 \frac{J}{kg}
\]

• Circular orbital velocity there is 7784 m/sec

\[
\text{Kinetic Energy} \quad \frac{\text{kg in orbit}}{\text{J}} = \frac{1}{2} v^2 = 30 \times 10^6 \frac{J}{kg}
\]

• Total energy per kg in orbit

\[
\text{Total Energy} \quad \frac{\text{kg in orbit}}{\text{J}} = PE + KE = 32 \times 10^6 \frac{J}{kg}
\]
Theoretical Cost to Orbit

- Convert to usual energy units

\[
\frac{\text{Total Energy}}{\text{kg in orbit}} = 32 \times 10^6 \frac{J}{kg} = 8.888 \frac{kWhrs}{kg}
\]

- Domestic energy costs are \(~$0.05/kWhr\)

▷ Theoretical cost to orbit \$0.44/kg\)
Actual Cost to Orbit

- Delta IV Heavy
  - 23,000 kg to LEO
  - $150 M per flight
- $6500/kg of payload
- Factor of 15,000x higher than theoretical energy costs!
What About Airplanes?

• For an aircraft in level flight,

\[
\frac{\text{Weight}}{\text{Thrust}} = \frac{\text{Lift}}{\text{Drag}}, \text{ or } \frac{mg}{T} = \frac{L}{D}
\]

• Energy = force x distance, so

\[
\frac{\text{Total Energy}}{\text{kg}} = \frac{\text{Thrust} \times \text{Distance}}{\text{mass}} = \frac{Td}{m} = \frac{gd}{L/D}
\]

• For an airliner (L/D=25) to equal orbital energy, d=81,000 km (2 roundtrips NY-Sydney)
Equivalent Airline Costs?

- Average economy ticket NY-Sydney round-round-trip (Travelocity 1/28/04) ~$1300
- Average passenger (+ luggage) ~100 kg
- Two round trips = $26/kg
  - Factor of 60x electrical energy costs
  - Factor of 250x less than current launch costs
- But...
  you get to refuel at each stop!
Equivalence to Air Transport

- 81,000 kg ~ twice around the world
- Voyager - only aircraft to ever circle the world non-stop, non-refueled - once!
Orbital Entry - The Physics

- 32 MJ/kg dissipated by friction with atmosphere over ~8 min = 66kW/kg
- Pure graphite (carbon) high-temperature material: $c_p = 709 \text{ J/kg} \cdot \text{K}$
- Orbital energy would cause temperature gain of 45,000°K!
“Once you make it to low Earth orbit, you’re halfway to anywhere!”
- Robert A. Heinlein
How To Approach Teaching This Course

• As a system user
• As a system provider
• As a venture capitalist
• As a system designer
Goals of ENAE 791

• Learn the underlying physics (orbital mechanics, flight mechanics, aerothermodynamics) which constrain and define launch and entry vehicles
• Develop the tools for preliminary design synthesis, including the fundamentals of systems analysis
• Provide an introduction to engineering economics, with a focus on the parameters affecting cost of launch and entry vehicles, such as reusability
• Examine specific challenges in the underlying design disciplines, such as thermal protection and structural dynamics
Contact Information

Dr. Dave Akin
Space Systems Laboratory
Neutral Buoyancy Research Facility/Room 2100D
301-405-1138
dakin@ssl.umd.edu
http://spacecraft.ssl.umd.edu
Web-based Course Content

- Data web site at spacecraft.ssl.umd.edu
  - Course information
  - Syllabus
  - Lecture notes
  - Problems and solutions
- Interactive web site at www.ajconline.umd.edu
  - Communications for team projects
  - Surveys for course feedback
Syllabus Overview (1)

• Fundamentals of Launch and Entry Design
  - Orbital mechanics
  - Basic rocket performance

• Entry flight mechanics
  - Ballistic entry
  - Lifting entry

• Aerothermodynamics

• Thermal Protection System (TPS) analysis

• Entry, Descent, and Landing (EDL) systems
Syllabus Overview (2)

• Launch flight mechanics
  - Gravity turn
  - Targeted trajectories
  - Optimal trajectories
  - Airbreathing trajectories

• Launch vehicle systems
  - Propulsion systems
  - Structures and structural dynamics analysis
  - Payload accommodations
Syllabus Overview (3)

• Systems Analysis
  - Cost estimation
  - Engineering economics
  - Reliability issues
  - Safety design concerns
  - Fleet resiliency

• Case studies

• Design project
Policies

• Grade Distribution
  - 20% Problems
  - 25% Midterm Exam
  - 25% Term Project*
  - 30% Final Exam

• Late Policy
  - On time: Full credit
  - Before solutions: 70% credit
  - After solutions: 20% credit

* Team Grades
Term Project

- Teams of ~4 people (you pick)
- Given a mission model (mass per year), design a launch system which minimizes cost/kg of payload to LEO over the life of the project
- Detailed requirements forthcoming
- Design process should proceed throughout the term
- Formal design presentations at end of term
Orbital Mechanics: 500 years in 40 min.

• Newton’s Law of Universal Gravitation

\[ F = \frac{Gm_1 m_2}{r^2} \]

• Newton’s First Law meets vector algebra

\[ \vec{F} = m\vec{a} \]
Relative Motion Between Two Bodies

\[ m_1 \frac{d^2 \vec{r}_1}{dt^2} = G \frac{m_1 m_2}{|\vec{r}_{21}|^2} \frac{\vec{r}_{21}}{|\vec{r}_{21}|} \]

\[ = G \frac{m_1 m_2}{|\vec{r}_{21}|^3} \vec{r}_{21} = G \frac{m_1 m_2}{|\vec{r}_{21}|^3} (\vec{r}_2 - \vec{r}_1) \]

\[ m_2 \frac{d^2 \vec{r}_2}{dt^2} = G \frac{m_1 m_2}{|\vec{r}_{12}|^3} (\vec{r}_1 - \vec{r}_2) \]

\( F_{12} = \text{force due to body 1 on body 2} \)
Gravitational Motion

\[
\frac{d^2 \vec{r}}{dt^2} = \frac{G}{r^3} \left[ m_2 (-\vec{r}) - m_1 (\vec{r}) \right] = \frac{-G}{r^3} (m_1 + m_2) \vec{r}
\]

Let \( r = |\vec{r}_{12}| = |\vec{r}_{21}| \)

Let \( \vec{r} = \vec{r}_1 - \vec{r}_2 \)

\[
\frac{d^2 \vec{r}}{dt^2} + \mu \frac{\vec{r}}{r^3} = \vec{0}
\]

"Equation of Orbit" - Orbital motion is simple harmonic motion
**Orbital Angular Momentum**

\[
\vec{v} = \frac{d\vec{r}}{dt}
\]

\[
\frac{d\vec{v}}{dt} + \mu \frac{\vec{r}}{r^3} = \vec{0}
\]

\[
\vec{r} \times \frac{d\vec{v}}{dt} + \frac{\mu}{r^3} (\vec{r} \times \vec{r}) = \vec{0}
\]

\[
\frac{d}{dt} (\vec{r} \times \vec{v}) = \frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt}
\]

\[
= \vec{v} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt} = \vec{r} \times \frac{d\vec{v}}{dt} = \vec{0}
\]

\[
\frac{d}{dt} (\vec{r} \times \vec{v}) = \vec{0}
\]

\[
\vec{r} \times \vec{v} = constant
\]

\[
\vec{r} \times \vec{v} = \vec{h}
\]

\(\vec{h}\) is angular momentum vector - \(\vec{r}\) and \(\vec{v}\) planar
Fun and Games with Algebra

\[ \frac{dv}{dt} + \mu \frac{r}{r^3} = 0 \]

\[ \frac{dv}{dt} \times \vec{h} + \frac{\mu}{r^3} (\vec{r} \times \vec{h}) = 0 \]

\[ \frac{d}{dt} (\vec{v} \times \vec{h}) = \frac{dv}{dt} \times \vec{h} + \vec{v} \times \frac{d\vec{h}}{dt} \]

\[ \frac{d}{dt} (\vec{v} \times \vec{h}) = -\frac{\mu}{r^3} (\vec{r} \times \vec{h}) = -\frac{\mu}{r^3} (\vec{r} \times \vec{r} \times \vec{v}) \]

\[ \frac{d}{dt} (\vec{v} \times \vec{h}) = -\frac{\mu}{r^3} [ (\vec{r} \cdot \vec{v}) \vec{r} - (\vec{r} \cdot \vec{r}) \vec{v} ] \]

\[ \vec{r} \cdot \vec{v} = rv \cos \gamma = r \frac{dr}{dt} \]
\[\frac{d}{dt} (\vec{v} \times \vec{h}) = -\frac{\mu}{r^3} \left[ r \frac{dr}{dt} \vec{r} - r^2 \frac{d\vec{r}}{dt} \right]\]

\[\frac{d}{dt} \left( \frac{\vec{r}}{r} \right) = \frac{\left( r \frac{d\vec{r}}{dt} - \vec{r} \frac{dr}{dt} \right)}{r^2} = \left( \frac{1}{r} \frac{d\vec{r}}{dt} - \frac{\vec{r}}{r^2} \frac{dr}{dt} \right)\]

\[\frac{d}{dt} (\vec{v} \times \vec{h}) = -\mu \left( \frac{1}{r^2} \frac{dr}{dt} \vec{r} - \frac{1}{r} \frac{d\vec{r}}{dt} \right) = \mu \frac{d}{dt} \left( \frac{\vec{r}}{r} \right)\]

\[\frac{d}{dt} \left( \vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} \right) = 0\]
Orientation of the Orbit

\[ \vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \text{constant} \]
\[ \vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \mu \vec{e} \]

\[ \vec{e} \equiv \text{eccentricity vector, in orbital plane} \]

\( \vec{e} \) points in the direction of periapsis

\[ \vec{r} \cdot \vec{v} \times \vec{h} - \vec{r} \cdot \mu \frac{\vec{r}}{r} = \mu (\vec{r} \cdot \vec{e}) \]

\[ \vec{r} \times \vec{v} \cdot \vec{h} - \mu \frac{\vec{r} \cdot \vec{r}}{r} = \mu r e \cos \theta \]

\[ \vec{h} \cdot \vec{h} - \mu \frac{r^2}{r} = \mu r e \cos \theta \]
Position in Orbit

\[ h^2 - \mu r = \mu re \cos \theta \]

\[ r = \frac{h^2/\mu}{1 + e \cos \theta} \]

\( \theta = \text{true anomaly: angular travel from perigee passage} \)

at \( \theta = \pm \frac{\pi}{2}; \cos \theta = 0; r = p \equiv \frac{h^2}{\mu} \)
\[ \mu \vec{e} = \vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} \]

\[ \mu \vec{e} \cdot \mu \vec{e} = \vec{v} \times \vec{h} \cdot \vec{v} \times \vec{h} - 2\mu \left( \vec{v} \times \vec{h} \right) \cdot \frac{\vec{r}}{r} + \mu^2 \left( \frac{\vec{r}}{r} \cdot \frac{\vec{r}}{r} \right) \]

\[ \mu^2 e^2 = v^2 h^2 - 2\mu \frac{h^2}{r} + \mu^2 \]

\[ e^2 = \frac{v^2}{\mu} p - 2\frac{p}{r} + 1 \]

\[ p \equiv a(1 - e^2) = \frac{1 - e^2}{2} \frac{v^2}{\mu} \frac{1}{r} \]
Vis-Viva Equation

\[ v^2 = \mu \left( \frac{2}{r} - \frac{1}{a} \right) \]

\[-\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r} \]
Energy in Orbit

- **Kinetic Energy**

  \[
  K.E. = \frac{1}{2}mv^2 \Rightarrow \frac{K.E.}{m} = \frac{v^2}{2}
  \]

- **Potential Energy**

  \[
  P.E. = -\frac{mu}{r} \Rightarrow \frac{P.E.}{m} = -\frac{\mu}{r}
  \]

- **Total Energy**

  \[
  Const. = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \quad \text{ <-- Vis-Viva Equation}
  \]
Implications of Vis-Viva

- Circular orbit \((r=a)\)
  \[
  v_{\text{circular}} = \sqrt{\frac{\mu}{r}}
  \]

- Parabolic escape orbit \((a \to \infty)\)
  \[
  v_{\text{escape}} = \sqrt{\frac{2\mu}{r}}
  \]

- Relationship between circular and parabolic orbits
  \[
  v_{\text{escape}} = \sqrt{2}v_{\text{circular}}
  \]
Some Useful Constants

• Gravitation constant $\mu = GM$
  - Earth: 398,604 km$^3$/sec$^2$
  - Moon: 4667.9 km$^3$/sec$^2$
  - Mars: 42,970 km$^3$/sec$^2$
  - Sun: $1.327 \times 10^{11}$ km$^3$/sec$^2$

• Planetary radii
  - $r_{\text{Earth}} = 6378$ km
  - $r_{\text{Moon}} = 1738$ km
  - $r_{\text{Mars}} = 3393$ km
Classical Parameters of Elliptical Orbits
Basic Orbital Parameters

- Semi-latus rectum (or parameter)
  \[ p = a(1 - e^2) \]

- Radial distance as function of orbital position
  \[ r = \frac{p}{1 + e \cos \theta} \]

- Periapse and apoapse distances
  \[ r_p = a(1 - e) \quad r_a = a(1 + e) \]

- Angular momentum
  \[ h = r \times v \quad h = \sqrt{\mu p} \]
The Classical Orbital Elements

The Hohmann Transfer

\[ V_{perigee} \] \[ V_{apogee} \]

\[ r_1 \] \[ r_2 \]
First Maneuver Velocities

- Initial vehicle velocity
  \[ v_1 = \sqrt{\frac{\mu}{r_1}} \]

- Needed final velocity
  \[ v_{\text{perigee}} = \sqrt{\frac{\mu}{r_1}} \sqrt{\frac{2r_2}{r_1 + r_2}} \]

- Delta-V
  \[ \Delta v_1 = \sqrt{\frac{\mu}{r_1}} \left( \sqrt{\frac{2r_2}{r_1 + r_2}} - 1 \right) \]
Second Maneuver Velocities

- Initial vehicle velocity
  \[ v_{\text{apogee}} = \sqrt{\frac{\mu}{r_2}} \sqrt{\frac{2r_1}{r_1 + r_2}} \]

- Needed final velocity
  \[ v_2 = \sqrt{\frac{\mu}{r_2}} \]

- Delta-V
  \[ \Delta v_2 = \sqrt{\frac{\mu}{r_2}} \left( 1 - \sqrt{\frac{2r_1}{r_1 + r_2}} \right) \]
Limitations on Launch Inclinations
Differences in Inclination
Choosing the Wrong Line of Apsides
Simple Plane Change

\[ v_{\text{perigee}} \quad v_1 \quad v_{\text{apogee}} \quad \Delta v_2 \quad v_2 \]
Optimal Plane Change

\[ v_{\text{perigee}} \rightarrow v_1 \rightarrow \Delta v_1 \rightarrow v_{\text{apogee}} \rightarrow v_2 \]
First Maneuver with Plane Change $\Delta i_1$

- Initial vehicle velocity
  \[ v_1 = \sqrt{\frac{\mu}{r_1}} \]
- Needed final velocity
  \[ v_p = \sqrt{\frac{\mu}{r_1}} \sqrt{\frac{2r_2}{r_1 + r_2}} \]
- Delta-V
  \[ \Delta v_1 = \sqrt{v_1^2 + v_p^2 - 2v_1 v_p \cos(\Delta i_1)} \]
Second Maneuver with Plane Change $\Delta i_2$

- **Initial vehicle velocity**
  \[
v_a = \sqrt{\frac{\mu}{r_2}} \sqrt{\frac{2r_1}{r_1 + r_2}}
  \]

- **Needed final velocity**
  \[
v_2 = \sqrt{\frac{\mu}{r_2}}
  \]

- **Delta-V**
  \[
  \Delta v_2 = \sqrt{v_2^2 + v_a^2 - 2v_2 v_a \cos(\Delta i_2)}
  \]
Sample Plane Change Maneuver

![Graph showing Delta V (km/sec) vs. Initial Inclination Change (deg)]

Optimum initial plane change = 2.20°
Calculating Time in Orbit
Time in Orbit

• Period of an orbit

\[ P = 2\pi \sqrt{\frac{a^3}{\mu}} \]

• Mean motion (average angular velocity)

\[ n = \sqrt{\frac{\mu}{a^3}} \]

• Time since pericenter passage

\[ M = nt = E - e \sin E \]

⇒\( M \): mean anomaly
Dealing with the Eccentric Anomaly

• Relationship to orbit

\[ r = a(1 - e \cos E) \]

• Relationship to true anomaly

\[ \tan \left( \frac{\theta}{2} \right) = \sqrt{\frac{1 + e}{1 - e}} \tan \left( \frac{E}{2} \right) \]

• Calculating \( M \) from time interval: iterate

\[ E_{i+1} = nt + e \sin E_i \]

until it converges