Mass Estimating Relations

- Review of iterative design approach
- Mass Estimating Relations (MERs)
- Sample vehicle design analysis
Design is an iterative process. The necessary number of iterations is one more than the number you have currently done. This is true at any point in time.
Vehicle-Level Prelim Design - 1st Pass

- Single Stage to Orbit (SSTO) vehicle
- $\Delta V = 9200 \text{ m/sec}$
- 5000 kg payload
- LOX/LH2 propellants
  - $I_{sp} = 430 \text{ sec}$
  - $(V_e = 4214 \text{ m/sec})$
  - $\delta = 0.08$

\[ r = e^{-\frac{\Delta V}{V_e}} = 0.1127 \]
\[ \lambda = r - \delta = 0.0327 \]
\[ M_o = \frac{M_\ell}{\lambda} = 153,000 \text{ kg} \]
\[ M_i = \delta M_o = 12,240 \text{ kg} \]
\[ M_p = M_o(1 - r) = 135,800 \text{ kg} \]
System-Level Estimation

- Start with propellant tanks (biggest part)
- LOX/LH2 engines generally run at mixture ratio of 6:1 (by weight)
  - LH2: 19,390 kg
  - LOX: 116,400 kg
- Propellant densities

\[
\rho_{LOX} = 1140 \, \frac{kg}{m^3} \quad \rho_{LH2} = 71 \, \frac{kg}{m^3}
\]
Propellant Tank Regression Data

\[ y = 9.0911x \quad R^2 = 0.9896 \]

\[ y = 12.158x \quad R^2 = 0.9328 \]

- LH2 Tanks
- LOX Tanks
- RP-1 Tanks

- Linear (LH2 Tanks)
- Linear (LOX Tanks)
Propellant Tank MERs (Volume)

- LH$_2$ tanks

\[ M_{LH_2 \, Tank} \langle kg \rangle = 9.09 V_{LH_2} \langle m^3 \rangle \]

- All other tanks

\[ M_{Tank} \langle kg \rangle = 12.16 V_{prop} \langle m^3 \rangle \]
Propellant Tank MERs (Mass)

- LH₂ tanks
  \[ \rho_{LH_2} = 71 \frac{kg}{m^3} \implies M_{LH_2 Tank} \langle kg \rangle = 0.128 M_{LH_2} \langle kg \rangle \]

- LOX tanks
  \[ \rho_{LOX} = 1140 \frac{kg}{m^3} \implies M_{LOX Tank} \langle kg \rangle = 0.0107 M_{LOX} \langle kg \rangle \]

- RP-1 tanks
  \[ \rho_{RP_1} = 820 \frac{kg}{m^3} \implies M_{RP_1 Tank} \langle kg \rangle = 0.0148 M_{RP_1} \langle kg \rangle \]
Cryogenic Insulation MERs

\[ M_{LH_2 \, Insulation} \langle kg \rangle = 2.88 A_{tank} \langle \frac{kg}{m^2} \rangle \]

\[ M_{LOX \, Insulation} \langle kg \rangle = 1.123 A_{tank} \langle \frac{kg}{m^2} \rangle \]
LOX Tank Design

- Mass of LOX = 116,400 kg
  \[ M_{LOX \ Tank} = 0.0107(116,400) = 1245 \ kg \]

- Need area to find LOX tank insulation mass - assume a sphere

  \[ V_{LOX \ Tank} = \frac{M_{LOX}}{\rho_{LOX}} = 102.1 \ m^3 \]

  \[ r_{LOX \ Tank} = \left( \frac{V_{LOX}}{\frac{4\pi}{3}} \right)^{\frac{1}{3}} = 2.90 \ m \]

  \[ A_{LOX \ Tank} = 4\pi r^2 = 105.6 \ m^2 \]

  \[ M_{LOX \ Insulation} = 1.123\left(\frac{kg}{m^2}\right)(105.6\langle m^2 \rangle) = 119 \ kg \]
LH₂ Tank Design

- Mass of LH₂ = 19,390 kg
  \[ M_{LH₂ \ Tank} \langle kg \rangle = 0.128(19,390) = 2482 \text{ kg} \]
- Again, assume LH₂ tank is spherical

\[ V_{LH₂ \ Tank} = \frac{M_{LH₂}}{\rho_{LH₂}} = 273.1 \text{ m}^3 \]

\[ r_{LH₂ \ Tank} = \left( \frac{V_{LH₂}}{4\pi/3} \right)^{\frac{1}{3}} = 4.02 \text{ m} \]

\[ A_{LH₂ \ Tank} = 4\pi r^2 = 203.6 \text{ m}^2 \]

\[ M_{LH₂ \ Insulation} = 2.88 \langle \frac{kg}{m^2} \rangle (203.6 \langle m^2 \rangle) = 586 \text{ kg} \]
Current Design Sketch

- Masses
  - LOX Tank 1245 kg
  - LOX Tank Insulation 119 kg
  - LH$_2$ Tank 2482 kg
  - LH$_2$ Tank Insulation 586 kg
High-Pressure Gas Tanks

\[
\begin{align*}
y &= 299.8x + 1.9963 \\
R^2 &= 0.9617
\end{align*}
\]

\[
\begin{align*}
y &= 115.29x + 3.1569 \\
R^2 &= 0.7003
\end{align*}
\]
Pressurized Gas Tank MERs

- COPV (Composite Overwrapped Pressure Vessel)

\[ M_{COPV\ Tank}(kg) = 115.3 \ V_{contents}(m^3) + 3 \]

- Titanium tank

\[ M_{COPV\ Tank}(kg) = 299.8 \ V_{contents}(m^3) + 2 \]
Smaller Storable Liquids Tanks

\[
y = 71.168x + 2.8911 \\
R^2 = 0.9726
\]

\[
y = 34.694x + 2.9677 \\
R^2 = 0.9
\]
Small Liquid Tankage MERs

- Bare metal tanks

\[ M_{Bare \ Tank} (kg) = 27.34 \ V_{contents} (m^3) + 2 \]

- Tanks with propellant management devices

\[ M_{PMD \ Tank} (kg) = 34.69 \ V_{contents} (m^3) + 3 \]

- Titanium tanks with positive expulsion bladders

\[ M_{Diaphragm \ Tank} (kg) = 71.17 \ V_{contents} (m^3) + 3 \]
Minimum Cost Lunar Architecture
Orbital Maneuvering Stage (OMS)

- Gross mass 6950 kg
  - Inert mass 695 kg
  - Propellant mass 6255 kg
  - Mixture ratio $\text{N}_2\text{O}_4$/UDMH = 2.0 (by mass)

- $\text{N}_2\text{O}_4$ tank
  - Mass = 4170 kg
  - Density = 1450 kg/m$^3$
  - Volume = 2.876 kg/m$^3$

- UDMH tank
  - Mass = 2085 kg
  - Density = 793 kg/m$^3$
  - Volume = 2.629 kg/m$^3$
N$_2$O$_4$ Tank Sizing

- Need total N$_2$O$_4$ volume = 2.876 m$^3$

- Single PMD tank
  - Radius = 0.882 m
  - Mass = 102.8 kg

- Dual PMD tanks
  - Radius = 0.700 m
  - Mass = 52.9 kg ($\times 2 = 105.8$ kg)

- Triple PMD tanks
  - Radius = 0.612 m
  - Mass = 36.3 kg ($\times 3 = 108.9$ kg)
Tank Configuration Issues
Other Structural MERs

- Fairings and shrouds

\[ M_{\text{fairing}} \langle kg \rangle = 4.95 \left( A_{\text{fairing}} \langle m^2 \rangle \right)^{1.15} \]

- Avionics

\[ M_{\text{avionics}} \langle kg \rangle = 10 \left( M_o \langle kg \rangle \right)^{0.361} \]

- Wiring

\[ M_{\text{wiring}} \langle kg \rangle = 1.058 \sqrt{M_o \langle kg \rangle \ell^{0.25}} \]
External Fairings - First Cut

\[ A_{\text{cone}} = \pi r \sqrt{r^2 + h^2} \]

\[ A_{\text{frustrum}} = \pi (r_1 + r_2) \sqrt{(r_1 - r_2)^2 + h^2} \]

\[ A_{\text{cylinder}} = 2\pi rh \]
External Fairings - First Cut

- Assumptions
  - P/L fairing $h$: 7 m
  - P/L fairing $r$: 2.9 m
  - I/T fairing $h$: 7 m
  - I/T fairing $r_1$: 4.02 m
  - I/T fairing $r_2$: 2.9 m
  - Aft fairing $h$: 7 m
  - Aft fairing $r$: 4.02 m
Fairing Analysis

- **Payload Fairing**
  - Area: 69.03 m²
  - Mass: 645 kg

- **Intertank Fairing**
  - Area: 154.1 m²
  - Mass: 1624 kg

- **Aft Fairing**
  - Area: 176.8 m²
  - Mass: 1902 kg

LOX
  - r = 2.90 m

LH2
  - r = 4.02 m
Avionics and Wiring Masses

- **Avionics**

\[ M_{avionics} \langle kg \rangle = 10 (153,000)^{0.361} = 744 \ kg \]

- **Wiring**

\[ M_{wiring} \langle kg \rangle = 1.058 \sqrt{153,000} (21 \ m)^{0.25} = 886 \ kg \]
Propulsion MERs

- Liquid Pump-Fed Rocket Engine Mass
  
  \[ M_{\text{Rocket Engine}}(\text{kg}) = 7.81 \times 10^{-4} T(N) + 3.37 \times 10^{-5} T(N) \sqrt{\frac{A_e}{A_t}} + 59 \]

- Solid Rocket Motor
  
  \[ M_{\text{Motor Casing}} = 0.135 M_{\text{propellants}} \]

- Thrust Structure Mass
  
  \[ M_{\text{Thrust Structure}}(\text{kg}) = 2.55 \times 10^{-4} T(N) \]
Propulsion MERs (continued)

- Gimbal Mass

\[ M_{Gimbals}(kg) = 237.8 \left( \frac{T(N)}{P_0(Pa)} \right)^{0.9375} \]

- Gimbal Torque

\[ \tau_{Gimbals}(N \cdot m) = 990,000 \left( \frac{T(N)}{P_0(Pa)} \right)^{1.25} \]
Propulsion System Assumptions

- Initial T/mg ratio = 1.3
  - Keeps final acceleration low with reasonable throttling
- Number of engines = 6
  - Positive acceleration worst-case after engine out

\[ \frac{5}{6}(1.3) = 1.083 > 1 \]

- Chamber pressure = 1000 psi = 6897 kN
  - Typical for high-performance LOX/LH2 engines
- Expansion ratio \( A_e/A_t = 30 \)
  - Compromise ratio with good vacuum performance
Propulsion Mass Estimates

- Rocket Engine Thrust (each)
  \[ T(N) = \frac{m_0 g (T / W) \_0}{n_{\text{engines}}} = 324,900 \text{ N} \]

- Rocket Engine Mass (each)
  \[ M_{\text{Rocket Engine}} (\text{kg}) = 7.81 \times 10^{-4} (324,900) + 3.37 \times 10^{-5} (324,900) \sqrt{30} + 59 = 373 \text{ kg} \]

- Thrust Structure Mass
  \[ M_{\text{Thrust Structure}} (\text{kg}) = 2.55 \times 10^{-4} (324,900) = 497 \text{ kg} \]
First Pass Vehicle Configuration

LOX
r=2.90 m

LH2
r=4.02 m
## Mass Summary - First Pass

<table>
<thead>
<tr>
<th>Component</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Inert Mass Estimate</td>
<td>12,240 kg</td>
</tr>
<tr>
<td>LOX Tank</td>
<td>1245 kg</td>
</tr>
<tr>
<td>LH2 Tank</td>
<td>2482 kg</td>
</tr>
<tr>
<td>LOX Insulation</td>
<td>119 kg</td>
</tr>
<tr>
<td>LH2 Insulation</td>
<td>586 kg</td>
</tr>
<tr>
<td>Payload Fairing</td>
<td>645 kg</td>
</tr>
<tr>
<td>Intertank Fairing</td>
<td>1626 kg</td>
</tr>
<tr>
<td>Aft Fairing</td>
<td>1905 kg</td>
</tr>
<tr>
<td>Engines</td>
<td>2236 kg</td>
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<tr>
<td>Thrust Structure</td>
<td>497 kg</td>
</tr>
<tr>
<td>Gimbals</td>
<td>81 kg</td>
</tr>
<tr>
<td>Avionics</td>
<td>744 kg</td>
</tr>
<tr>
<td>Wiring</td>
<td>886 kg</td>
</tr>
<tr>
<td>Reserve</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total Inert Mass</strong></td>
<td><strong>13,052 kg</strong></td>
</tr>
<tr>
<td><strong>Design Margin</strong></td>
<td><strong>-6.22%</strong></td>
</tr>
</tbody>
</table>
Modifications for Second Pass

- Keep all initial vehicle sizing parameters constant
- Pick vehicle diameter and make tanks cylindrical to fit
- Redo MER analysis
Effect of Vehicle Diameter on Mass Margin

Inert Mass Margin (%) vs. Vehicle Diameter (m)

- The graph shows the relationship between vehicle diameter and inert mass margin.
- As the vehicle diameter increases, the inert mass margin decreases.
- The maximum inert mass margin occurs at a vehicle diameter of approximately 2.5 meters.
Effect of Mass-Optimal Diameter Choice

- Mass-optimal vehicle has diameter = 1.814 m
- Mass margin goes from -6.22% to +33.1%
- Vehicle length = 155 m
- Length/diameter ratio = 86 – approximately equivalent to piece of spaghetti
- No volume for six rocket engines in aft fairing
- Infeasible configuration
Effect of Diameter on Vehicle L/D

![Graph showing the effect of vehicle diameter on length/diameter ratio. The graph plots vehicle diameter in meters on the x-axis and length/diameter ratio on the y-axis, with a logarithmic scale. The data trend indicates a decrease in length/diameter ratio as the diameter increases.]
Second Pass Vehicle Configuration

Mass Estimating Relations
ENAE 791 - Launch and Entry Vehicle Design
## Mass Summary - Second Pass

<table>
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<th>Second Pass</th>
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<tr>
<td>Initial Inert Mass Estimate</td>
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<td>12,240 kg</td>
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<td>LOX Tank</td>
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<td>1245 kg</td>
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<td>LH2 Tank</td>
<td>2482 kg</td>
<td>2482 kg</td>
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<tr>
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<td>119 kg</td>
<td>56 kg</td>
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<td>LH2 Insulation</td>
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<td>Payload Fairing</td>
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<td>402 kg</td>
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<td>448 kg</td>
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<td>579 kg</td>
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<td>Engines</td>
<td>2236 kg</td>
<td>2236 kg</td>
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<tr>
<td>Thrust Structure</td>
<td>497 kg</td>
<td>497 kg</td>
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<tr>
<td>Gimbals</td>
<td>81 kg</td>
<td>81 kg</td>
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<tr>
<td>Avionics</td>
<td>744 kg</td>
<td>744 kg</td>
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<tr>
<td>Wiring</td>
<td>886 kg</td>
<td>1044 kg</td>
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<tr>
<td>Reserve</td>
<td>-</td>
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<td>Total Inert Mass</td>
<td>13,052 kg</td>
<td>9960 kg</td>
</tr>
<tr>
<td>Design Margin</td>
<td>-6.22 %</td>
<td>+22.9 %</td>
</tr>
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</table>
Modifications for Iteration 3

- Keep 4 m tank diameter
- Change initial assumption of δ iteratively, with resulting changes in \( m_0 \) and \( m_i \), to reach 30% mass margin
- Modify diameter to keep \( L/D \leq 10 \) and iterate again for optimal initial mass estimate
Vehicle-Level Prelim Design - 3rd Pass

- Single Stage to Orbit (SSTO) vehicle
- \( \Delta V = 9200 \text{ m/sec} \)
- 5000 kg payload
- LOX/LH2 propellants
  - \( I_{sp} = 430 \text{ sec} \)
  - \( V_e = 4214 \text{ m/sec} \)
  - \( \delta = 0.08323 \)
- Diameter = 4.2 m
- \( L/D = 9.7 \)

\[
r = e^{-\frac{\Delta V}{V_e}} = 0.1127
\]

\[
\lambda = r - \delta = 0.0294
\]

\[
M_o = \frac{M_{\ell}}{\lambda} = 169,800 \text{ kg}
\]

\[
M_i = \delta M_o = 14,130 \text{ kg}
\]

\[
M_p = M_o(1 - r) = 150,700 \text{ kg}
\]
# Mass Summary - Third Pass

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<th>Component</th>
<th>Initial</th>
<th>Pass 1</th>
<th>Pass 2</th>
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<td>LOX Tank</td>
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<td>LH2 Tank</td>
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<td>2482 kg</td>
<td>2755 kg</td>
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<tr>
<td>LOX Insulation</td>
<td>119 kg</td>
<td>56 kg</td>
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<td>586 kg</td>
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<td>Payload Fairing</td>
<td>645 kg</td>
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<td>448 kg</td>
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<td>1905 kg</td>
<td>579 kg</td>
<td>626 kg</td>
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<td>Engines</td>
<td>2236 kg</td>
<td>2236 kg</td>
<td>2443 kg</td>
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<tr>
<td>Thrust Structure</td>
<td>497 kg</td>
<td>497 kg</td>
<td>552 kg</td>
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<td>744 kg</td>
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# Mass Budgeting

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<td>62 kg</td>
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<td>Reserve</td>
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<td>1630 kg</td>
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References

- Willie Heineman, Jr., Fundamental Techniques of Weight Estimating and Forecasting for Advanced Manned Spacecraft and Space Stations NASA TN-D-6349, May 1971
- Willie Heineman, Jr., Mass Estimation and Forecasting for Aerospace Vehicles Based on Historical Data NASA JSC-26098, November 1994