Entry Aerothermodynamics

- Review of basic fluid parameters
- Heating rate parameters
- Stagnation point heating
- Heating on vehicle surfaces
- Slides from 2012 NASA Thermal and Fluids Analysis Workshop:
  https://tfaws.nasa.gov/TFAWS12/Proceedings/Aerothermodynamics%20Course.pdf
The kinetic energy of an entry vehicle is dissipated by transformation into thermal energy (heat) as the entry system decelerates. The magnitude of this thermal energy is so large that if all of this energy were transferred to the entry system it would be severely damaged and likely vaporize. Harvey Allen - the blunt body concept. Only a small fraction of this thermal energy is transferred to the entry system. The thermal transfer fraction is dependent on vehicle shape, size, aerodynamic regime and velocity. Near peak heating, 1% to 5% of the total thermal energy is transferred to the entry system. Example: at the peak heating point the freestream energy transfer for Pathfinder was $\dot{Q}_\infty = \frac{1}{2} \rho V^3 \approx 4,000 \text{ W/cm}^2$ but only about 110 W/cm$^2$ (2.7%) was actually transferred to the surface.
Energy density: \[ \frac{E}{m} = \frac{V^2}{2} + g_o h \]

<table>
<thead>
<tr>
<th>Entry</th>
<th>V (km/s)</th>
<th>E/m (MJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MER</td>
<td>5.6</td>
<td>16</td>
</tr>
<tr>
<td>Apollo</td>
<td>11.4</td>
<td>66</td>
</tr>
<tr>
<td>Mars Return</td>
<td>14.0</td>
<td>98</td>
</tr>
<tr>
<td>Galileo</td>
<td>47.4</td>
<td>1130</td>
</tr>
</tbody>
</table>

Note that:
- Water boils @ 2.3 MJ/kg
- Carbon vaporizes @ 60.5 MJ/kg

In each case $g_o h$ is about 1% of total
Side Note: What Can We Test?

Missions of Interest
Live here
Blunt Body Rationale

• Why is a blunt body used for planetary entry?
  – Slender body: low drag, highly maneuverable
  – Blunt body: high drag, not very maneuverable

• Blunt bodies generate strong shock waves
  – Efficient energy dissipation. Shock waves convert kinetic energy to internal energy. Result is: heating of the gas, dissociation, ionization
  – Most of this energy is convected into the vehicle wake rather than transported to the surface
  – Intuitively, blunter is better (more bluntness equals stronger shock). Hold that thought; we will come back to it…
Blunt Body Rationale (2)

• Normal shock heats the gas to many thousands of degrees

• Much of this heat is conducted into the vehicle wake and propagated downstream

• Can be tracked as a “velocity deficit” and persists long downstream of the vehicle
Definitions

• **Heat Rate (q)**
  – Instantaneous heat flux at a point on the vehicle (W/cm²)

• **Heat Load (Q)**
  – Integration of heat rate with time over a trajectory (J/cm²)

• **Convective Heating**
  – Heat flux to the vehicle from conduction (κ gradT)

• **Catalytic Heating**
  – Heat flux to the vehicle due to surface facilitated chemical reactions
  – Commonly lumped with convective heating by convention

• **Radiative Heating**
  – Heat flux to the vehicle from radiation produced by excited atoms and molecules in the shock layer
What is Aerothermodynamics?

- Accurate and conservative prediction of the heating environment encountered by an Earth or planetary entry vehicle

- Aerothermal modeling is coupled and entwined with Thermal Protection System (TPS) design
  - The TPS is designed to withstand the predicted environment with risk-appropriate margin
  - For ablative systems, the flowfield and TPS interact with each other in non-reversible manner; the physics themselves are coupled

- At its core, aerothermodynamics becomes the study of an energy balance at the surface of the material
  - Heat flux (with pressure & shear) used to select TPS material
  - Heat load determines TPS thickness
Principles of Aerothermal Models

**Planetary Atmospheres**
- Mars & Venus: CO\textsubscript{2}/N\textsubscript{2}
- Titan: N\textsubscript{2}/CH\textsubscript{4}
- Giants: H\textsubscript{2}/He
- Earth: N\textsubscript{2}/O\textsubscript{2}

**Hot Shock Layer (up to 20000 K)**
Thermochemical nonequilibrium, Ionization, Radiation

**Boundary Layer (2–6000 K)**
Transport properties, Ablation product mixing, Radiation blockage

**“Cool” Surface (2–3000 K)**
Surface kinetics, Ablation

**Design Problem:** Minimize conduction into vehicle to minimize TPS mass/risk
\[ q_{\text{cond}} = q_c + q_{\text{rad}} - q_{\text{rerad}} - q_{\text{mdot}} \]

**Incident Aeroheating**

**Material Response**

**Thermal Protection System (TPS)**
Current State of the Art: CFD

- The current SOA involves the steady solution of the reacting Navier-Stokes equations via CFD or DSMC methods
- Full 3D simulations possible in hours to days
- Longer time required for the simulation of OML details (steps, gaps, seals, windows, etc.)
Pushing the Current State of the Art

- DES, DNS, LES
- Unsteady RANS (URANS) simulations of Supersonic Retro-Propulsion flowfields; going on right now…
With present computational abilities, why use engineering methods?

• CFD is a powerful tool, but high-fidelity simulations remain time (and resource) consuming

• Some applications of simple relationships for calculating non-ablating convective and radiative heating
  – Negligible computation time
  – Included in most atmospheric trajectory codes-stag. pt. heating
  – Initial estimates of heating rates and loads for use during conceptual design stage

• But most important:
  ➔ In this day of commodity supercomputers it is all too easy to run simulations without truly understanding the physics involved or the trends that are expected. *The fact that it “converged” doesn’t make it right.* Engineering methods are based on sound approximations to theory and provide a valuable sanity check on CFD results
• Pioneering engineering theories were developed in the 1950’s (missile technology)


• Extensions to higher velocities were required to account for chemistry and ionization

• Many extensions and simplifications followed for specific applications, non-Earth atmospheres
Early correlations for convective heating have the form:

\[ \dot{q}_s \sim V^3 \left( \frac{\rho}{R_n} \right)^{\frac{1}{2}} \]

Why?

At first cut, one might expect heat flux to the surface to be proportional to freestream energy flux \( \frac{1}{2} \rho V^3 \)

From previous discussion one would expect convective heat flux to decrease as bluntness (\( R_n \)) increases, but with what functionality?

(insert brief derivation here)
Aerothermodynamic Heating

$\dot{q} \equiv \text{heating rate per unit area} \langle J/sec/m^2 \rangle$

$$\dot{q} \equiv \frac{dq}{dt} = k(T_r - T_w)$$

$k \equiv \text{convective heat transfer coefficient} \langle \frac{J}{m^2 sec K} \rangle$

$T_r \equiv \text{recovery temperature} \langle K \rangle$

$T_w \equiv \text{wall temperature} \langle K \rangle$

$$T_r = T_{\infty} \left(1 + \frac{\gamma - 1}{2} M^2 \right)$$
Wall Temperature

\( T_{wl} \equiv \text{local wall temperature} \)

\[
(T_r - T_w)_l = (T_\infty - T_{wl}) + T_\infty \frac{\gamma - 1}{2} M^2
\]

For high Mach numbers,

\[
(T_r - T_w)_l \approx T_r - T_{wl} \approx T_\infty \frac{\gamma - 1}{2} M^2
\]
Mach Number Manipulation

By definition,

\[ M^2 = \frac{V^2}{\alpha^2} = \frac{V^2}{\gamma RT} \]

\[ M^2 T = \frac{V^2}{\gamma R} = \frac{V^2}{(c_p/c_v)(c_p - c_v)} = \frac{V^2}{c_p(\gamma - 1)} \]

\[ T_r - T_wl = \frac{V^2}{2c_p} \]
**Prandtl Number**

\[ Pr = \mu \frac{C_p}{K} \]

where \( C_p \equiv \) specific heat at constant pressure

\( K \equiv \) thermal conductivity

\( \mu \equiv \) viscosity

\[ Pr \propto \frac{\text{frictional dissipation}}{\text{thermal conduction}} \]

\[ Pr \approx 0.715 \text{ for air at standard conditions} \]
Sutherland’s Law (empirical)

- Viscosity depends on temperature

\[
\frac{\mu}{\mu_{ref}} = \left( \frac{T}{T_{ref}} \right)^{3/2} \frac{T_{ref} + S}{T + S}
\]

For air: \( \mu_{ref} = 1.789 \times 10^{-5} \frac{kg}{m \cdot sec} \)

\( T_{ref} = 288 \; K \)

\( S = 110 \; K \)

- Good to several thousand degrees
Stanton Number

- applies to boundary layer problems

\[ S_T = \frac{\dot{q}_w}{\rho_e v_e (H_{aw} - H_w)} \]

\( H \equiv \text{enthalpy} \)

\( = C_p T \text{ for perfect gas} \)

\( H_w = \text{enthalpy at the wall} \)

\( H_{aw} = \text{enthalpy at an adiabatic wall} \)

for \( H_{aw} \), \( \left( \frac{\partial T}{\partial z} \right)_w = 0 \)
Approximating $H_{aw}$

\[
H_o = H_e + \frac{u_e^2}{2} \quad \iff \text{total enthalpy at edge of boundary layer}
\]

\[
H_{aw} = H_e + r \frac{u_e^2}{2} \quad r \equiv \text{recovery factor}
\]

\[
H_{aw} = H_e + r (H_o - H_e)
\]

for incompressible flow, $r \approx \sqrt{Pr}$

\[
= 0.845 \text{ for std. air}
\]

$r$ decreases only 2.4% from $M=0$ to $M=16$

\[
\implies \text{fairly constant!}
\]
Reynold’s Analogy

Relates the convective heat transfer coefficient $k$ to the local skin friction coefficient $c_f$.

$$c_f = \tau_w \frac{\rho_\infty}{2} V^2$$

$\tau_w \equiv$ local wall stress (Pa)

$$k = \frac{1}{2} c_f c_p\rho V_l$$

$Q \equiv$ total heat transfer rate

$$Q = \int_{S_{wet}} \dot{q} \, ds = \int_{S_{wet}} k(T_r - T_w) \, ds$$
Reynold’s Analogy

\[ \frac{S_T}{c_f} = \frac{1}{2} Pr^{-2/3} \]

\( c_f \equiv \text{skin friction coefficient} \)

since \( Pr \approx 1 \), \( S_T \approx \frac{c_f}{2} \) \( \Leftarrow \) Reynold’s Analogy
Fay-Riddell Method

Convective: derived from boundary layer and stagnation point theories

Fay & Riddell (1958):
Boundary layer eqns, similarity transformation

\[
\dot{q}_w = \frac{0.763}{(Pr_w)^{0.6}} (\rho_e \mu_e)^{0.4} (\rho_w \mu_w)^{0.1} \left[ (h_o)_e - h_w \right] \left[ 1 + (Le^{0.52} - 1) \frac{h_d}{(h_o)_e} \right] \left[ \frac{d u_e}{dx} \right]^{0.5}
\]

Velocity gradient from mod. Newtonian theory \( \sim (1/R_n) \)

\[
\frac{d u_e}{dx} = \frac{1}{R} \sqrt{\frac{2(p_e - p_\infty)}{\rho_e}}
\]

Significant advance, but still requires many quantities that are not readily available to designer

Allows for chemistry effects, non-unity Pr, Le (Prandtl, Lewis numbers)
**Chapman Equation (Earth):**

\[ q_s = 1.63 \times 10^{-4} \left( \frac{\rho}{R_n} \right)^{\frac{1}{2}} V^3 \left( 1 - \frac{h_w}{h_{\infty}} \right) \]

\[ h_{\infty} = \int_0^T C_p T dt + \frac{1}{2} V_{\infty}^2 \]

“hot wall correction” can frequently be neglected in hypersonic flow \( (h_w << h_{\infty}) \)

**Sutton Graves:**

\[ q_s = k \left( \frac{\rho}{R_n} \right)^{\frac{1}{2}} V^3 \]

- \( k = 1.7415 \times 10^{-4} \) (Earth)
- \( k = 1.9027 \times 10^{-4} \) (Mars)

(SI units)

- Calculated for specific atmosphere (Earth or Mars), accounting for thermodynamics.
- Above assume a fully catalytic surface; equivalent expressions for non catalytic wall are available.
Hot Wall Correction Term

- Negligible above about 100 W/cm² assuming radiative equilibrium
- Actual effect is smaller than this for ablative TPS

\[ HWC = \left( 1 - \frac{h_w}{h_\infty} \right) \]
Generalized Chapman Method

\[ q_{c,0} = \frac{C}{\sqrt{R_n}} (\rho_\infty)^m (V_\infty)^n \left[ 1 - \frac{h_w}{h_\infty} \right], \]

Earth: \( m = 0.5, \quad n = 3 \)
Mars: \( m = 0.5, \quad n = 3.04 \)

\( C \) is derived for problem of interest

Powerful design tool - can be used to approximate heating from a small number of CFD “anchor points” even away from the stagnation point by letting \( C, m, \) and \( n \) be curve fit coefficients
Puoricorrelations are straightforward and require only readily available quantities

However, there is a nuance. All are dependent on the effective nose radius of the vehicle under investigation

For a hemisphere, $R_{\text{eff}} = R_n$, but corrections are required for other vehicle shapes.

For example, Apollo was a truncated sphere, with an effective radius almost twice the base radius of the capsule. MER/MSL use sphere-cones, where the conical flank increases the effective radius of the nose

For bodies with a rounded corner, Zoby and Sullivan have computed tables of effective radius as a function of $R_b/R_n$ and $R_c/R_b$:

When does it matter?
Can the flow “tell” that the nose is finite?

45° Sphere-Cone
Supersonic Oblique Shock
$R_{\text{eff}} = R_n$

60° Sphere-Cone
Subsonic Shock
$R_{\text{eff}} > R_n$
• Theory is less intuitive, more involved
• Atoms or molecules are excited by collisions. Excited species can emit a photon that carries energy with it
• Photons are emitted isotropically, and travel effectively instantaneously
• Radiative heating is the integration of those photons that hit the surface times the energy they carry; intuitively should be proportional to the size of the radiating volume
• Partition functions for excited states imply a near exponential dependence on temperature
• Radiation is coupled to the fluid mechanics for two reasons:
  • Emitted photons carry energy out of control volume (adiabatic cooling)
  • Photons can be absorbed in the boundary layer and heat the gas

\[
\frac{N_i}{N} = g_i e^{\frac{E_i}{kT}} = g_i e^{\frac{E_i}{kT}} \sum_j g_j e^{\frac{E_j}{kT}}
\]

\[
hv = \Delta E = E_U - E_L
\]

LTE-Plasma
Relative Importance of Radiation vs Convection

Adapted from Anderson, Hypersonic and High Temperature Gas Dynamics, Fig. 18.10

Martin:

\[ \dot{Q}_r \sim r_n^{1.0} \rho_\infty^{1.6} V_\infty^{8.5} \]

- Earth

Tauber-Sutton:

\[ \dot{Q}_r = C_i r_n^a \rho_\infty^m f_i(V_\infty) \]

- based on tabulated data, equilibrium shock theory

Earth: \( a \sim 1, \quad m \sim 1.2 \)
Mars: \( a = 0.526, \quad m \sim 1.2 \)

\( f_i \) are tabulated, near exponential at moderate velocity

Theory is less intuitive, more involved. Typically relies on table lookups and has limited range of validity

Fortunately, radiation is not a major issue for many problems of interest: Mars (moderate velocity), LEO return, Titan
The shock layer is cooled by the emission of photons. Clearly this effect will become more important as a larger fraction of the total shock layer energy is converted to photons.

Tabular or engineering expressions for stagnation point radiation typically include the radiative cooling effect.

However it is very important to recognize this phenomenon when computing radiation from CFD data (inherently uncoupled operation).

Goulard proposed a non-dimensional parameter that is essentially the ratio of total energy flux to that lost to radiation:

\[ \Gamma = \frac{2q_{R,unc}}{\frac{1}{2} \rho V^3} \]

The net radiative heating can then be computed from (Tauber-Wakefield):

\[ q_{R, coup} = \frac{q_{R, unc}}{1 + \kappa \Gamma^{0.7}} \]

Where \( \kappa \) is an atmosphere-specific constant:
- \( \kappa = 2 \) for Titan
- \( \kappa = 3.45 \) for Earth
- \( \kappa \sim 3 \) for Mars/Venus

Importance of Radiative Cooling
Example - Galileo Probe

Adapted from Anderson, Hypersonic and High Temperature Gas Dynamics, Fig. 18.16
Wall Temperature Estimation

• How hot does the TPS surface get?

• A body radiates heat at a rate proportional to the 4th power of its temperature.

• Stefan-Boltzmann Law: \( q_{\text{rerad}} = \varepsilon \sigma T^4 \)
  • where \( \varepsilon \) is the emissivity of the TPS (\( \varepsilon = 1 \) for a blackbody), \( \sigma \) is the Stefan-Boltzmann constant (\( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{/K}^4 \)), and \( T \) is the wall temperature (assumes the ambient temperature is much lower).

• The wall heat flux balance is in general given by the sum of heat into the material minus reradiation, conduction, and material response. A primary function of TPS is to minimize conduction (good insulator), and thus, neglecting material response we can assume that:

\[ q_{\text{rerad}} \sim q_{\text{conv}} + q_R \]

which can readily be solved for \( T_w \).

• Examples:
  • Orbiter peak heating (\( T_w = 1600 \text{ K} \))
  • MER peak heating (\( T_w = 1725 \text{ K} \))
  • Orion peak heating (\( T_w = 3360 \text{ K} \))
    – by this point we are overpredicting by \( \sim 20\% \) due to material response effects.
Engineering Methods

Distributed Heating - Sphere

• It can be shown that the heat transfer rate along the body varies according to

\[
\frac{q}{q_{stag}} \approx \cos \theta
\]

for angles as large as 45° (in theory) and 70° (in practice).

• This expression permits us to integrate the total heat flux into a spherical nose as

\[
\int qdA = q_{stag} \int \cos \theta dA
\]

\[
dA = 2\pi y R_n d\theta = 2\pi R_n^2 \sin \theta d\theta
\]

\[
\int qdA = 2\pi R_n^2 q_{stag} \int_0^{\pi/2} \sin \theta \cos \theta d\theta = \pi R_n^2 q_{stag}
\]

• For a laminar boundary layer, the heat input to a hemisphere is ~ equal to the product of stag. point heating times the projected area.
Figure 5. - Heat-transfer distribution on hemisphere cylinder. (From ref. 28; reprinted with permission of The American Institute of Aeronautics and Astronautics.)

Figure 6. - Comparison of predicted and measured heat flux distributions on a circular cylinder normal to a stream.
Local Similarity - Flat Faced Cylinder

• Local similarity methods (see e.g. Anderson) can be extended to other geometries

• Take for example a flat-faced cylinder with a rounded corner

• For this case, local similarity theory (and more sophisticated methods) show that the stagnation point is not the highest heating location; rather heating is higher on the corner
  – Physically, the large favorable pressure gradient causes the boundary layer to thin. This increases the magnitude of $\nabla h$, which increases heat transfer per previous arguments. The magnitude of increase is inversely related to the radius of curvature.
Figure 7.-- Heat-transfer distribution on flat-nosed body. (From ref. 28; reprinted with permission of The American Institute of Aeronautics and Astronautics.)
Many other approximate methods have been developed for the calculation of heating on other geometries, e.g. wings, attachment lines.

Detailed assessment is beyond the scope of these lectures, but the interested student can read further in:


which is included as a handout for this course.
Trajectory Effects

- The discussion up to now has focused on the calculation of an instantaneous heat flux (primarily at the stagnation point).
- However, the heating on the vehicle is obviously coupled to the trajectory flown, and thus it is important to develop expressions that quantify the relationship between heating and trajectory.
- You have already learned two basic trajectory equations (Allen-Eggers and Equilibrium Glide); let's start with Allen-Eggers.
- For simplicity, let's use the simplest of convective heating relationships:

\[ q_s \sim (\rho)^{\frac{1}{2}} V^3 \]
Intuition (1)

- Two identical ballistic vehicles enter the atmosphere. One is on a steep entry trajectory and one is on a shallow entry trajectory. Which has the higher peak heat flux? Load?
Intuition (2)

- Two ballistic vehicles enter the atmosphere on an identical flight path angle. One has a higher ballistic coefficient. Which has the higher peak heat flux? Load?
Allan-Eggers Trajectory Equation

\[ V = V_{atm} \exp\left[ C e^{-h/H} \right] = V_{atm} \exp\left[ C \frac{\rho}{\rho_o} \right] \]

\( V_{atm} = \) Velocity at atmospheric interface
\( \beta = m/C_D A \)
Exponential atmosphere assumed
Ballistic entry

- Substitute above for \( V \) into approximate heating equation:

\[ q_s \sim \left( \rho \right)^{1/2} \left( V_{atm}^3 \exp\left[ 3 C \frac{\rho}{\rho_o} \right] \right) \]

- Differentiate w.r.t density:

\[ \frac{1}{2} \left( \rho \right)^{-1/2} \left( V_{atm}^3 \exp\left[ 3 C \frac{\rho}{\rho_o} \right] \right) + \left( \rho \right)^{1/2} \left( \frac{3 C}{\rho_o} \right) \left( V_{atm}^3 \exp\left[ 3 C \frac{\rho}{\rho_o} \right] \right) \cdot \frac{dV_{atm}}{d\rho} = \frac{dq_s}{d\rho} \]
Allan-Eggers Trajectory Equation (2)

• Looking for a maximum of $q_s$, which should occur when $dq_s/d\rho = 0$:

$$1 + 6C \frac{\rho}{\rho_o} = 0$$

• So the density of maximum convective heating is:

$$\rho_{q_{\text{max}}}^* = -\frac{\rho_o}{6C} = \frac{\beta \sin \gamma}{3H}$$

• For a given atmospheric scale height, the density (altitude) of peak heating increases with ballistic coefficient and flight path angle
Allan-Eggers Trajectory Equation (3)

- So, in the exponential atmospheric model

\[
\frac{\beta \sin \gamma}{3H} = \rho_o e^{-h^*/H}
\]

\[
- \frac{h^*}{H} = \ln \left( \frac{\beta \sin \gamma}{3H \rho_o} \right)
\]

- The altitude and velocity of peak heating are given by:

\[
h_{q \text{ max}}^* = -H \ln \left( \frac{\beta \sin \gamma}{3H \rho_o} \right)
\]

\[
V_{q \text{ max}}^* = V_{atm} \exp \left[ \frac{C}{\rho_o} \left( \frac{-\rho_o}{6C} \right) \right] = V_{atm} e^{-1/6} = 0.846 V_{atm}
\]
Allan-Eggers Trajectory Equation (4)

- As in the case of the previously derived expression for the velocity at peak deceleration, the velocity at peak heating is a function only of the entry velocity.
- Recall that $V_{g,\text{max}} = 0.606V_{\text{atm}}$. Therefore, peak heating occurs earlier in the entry than peak deceleration. In fact, it can be shown that
  \[ h_{q,\text{max}}^* \approx 1.1 h_{g,\text{max}}^* \]
- We are now in the position of being able to calculate the peak stagnation point convective heat rate for a ballistic entry vehicle.
- Substitute the evaluated expressions for $V_{q,\text{max}}$ and $\rho_{q,\text{max}}$ into the Sutton-Graves Equation:
  \[ q_{s,\text{max}} = k \left( \frac{1}{R_n} \right)^{\frac{1}{2}} \left( \frac{\beta \sin \gamma}{3H} \right)^{\frac{1}{2}} (0.6055V_{\text{atm}}^3) \]
- In addition to the nose radius dependence shown earlier, we now see that peak heating rate increases with increasing ballistic coefficient and flight path angle.
Heat Load

• Stagnation point heat load is just the time integration of the heat flux

\[ Q_s = \frac{k}{\sqrt{R_n}} \int \rho^{\frac{1}{2}} V^3 dt \]

• How do we convert this to an integral that we now how to evaluate (redefine \( dt \) through change of variables)? Lets borrow some logic from the Equations of Motion:

\[ \sin \gamma = -\frac{dh}{ds} ; \quad V = \frac{ds}{dt} \]

\[ dt = \frac{ds}{V} = -\frac{dh}{V \sin \gamma} \]

• Using the exponential atmosphere model we can write this in terms of \( d\rho \)
Heat Load (2)

- Exponential atmosphere model
  \[ \rho = \rho_o e^{-h/H} \]

- Differentiate:
  \[ \frac{d\rho}{dh} = -\frac{\rho_o}{H} e^{-h/H} = -\frac{\rho}{H} \]

- Substitute into \( dt \):
  \[ dt = \frac{Hd\rho}{\rho V \sin \gamma} \]

- Now we can substitute into the heat load integral:
  \[ Q_s = \int q_s dt = \frac{k}{\sqrt{R_n}} \frac{V^2}{\sin \gamma} \int_0^\rho \rho^{-\frac{1}{2}} \exp \left[ \frac{2C\rho}{\rho_0} \right] d\rho \]

  \[ Q_s \sim kV_{atm}^2 \left[ \frac{\beta}{R_n \sin \gamma} \right]^{\frac{1}{2}} \]

  After some manipulation…
Heat Rate vs Heat Load

- Quantitative expression can be derived from approximate evaluation of the integral:

\[
Q_s \approx kV_{atm}^2 \left[ \frac{\beta}{\rho_o \left( \frac{\pi H}{R_n \sin \gamma} \right)} \right]^\frac{1}{2}
\]

\(k\) is the Sutton-Graves constant

- Compare the derived expressions for heat rate and heat load:

\[
q_{s,\text{max}} = k \left( \frac{1}{R_n} \right)^{\frac{1}{2}} \left( \frac{\beta \sin \gamma}{3H} \right)^{\frac{1}{2}} \left( 0.6055V_{atm}^3 \right)
\]

\[
Q_s = kV_{atm}^2 \left[ \frac{\beta}{\rho_o \left( \frac{\pi H}{R_n \sin \gamma} \right)} \right]^\frac{1}{2}
\]

- Heat rate increases with both \(\beta\) and \(\gamma\), while heat load increases with \(\beta\), but decreases with \(\gamma\)

- This leads to a second mission design trade (the first was \(R_n\) and its impact on drag, convective heating, and radiative heating):

- The selection of \(\gamma\) becomes a trade between peak heat rate (TPS material selection), and total heat load (TPS thickness and mass)
So, Did you get it Right? (1)

- Two identical ballistic vehicles enter the atmosphere. One is on a steep entry trajectory and one is on a shallow entry trajectory. Which has the higher peak heat flux? Load?
Two ballistic vehicles enter the atmosphere on an identical flight path angle. One has a higher ballistic coefficient. Which has the higher peak heat flux? Load?

So, Did you get it Right? (2)

- Higher Peak Heat Load
- Higher Peak Heat Flux
Mars Entry Heating - Example

Entry Flight Path Variation

\[ \beta = 90 \text{ kg/m}^2; \quad V_i = 5.5 \text{ km/s} \]

Heat rate falls and heat load grows as FPA decreases.
Mars Entry Heating - Example

Ballistic Coefficient Variation
\[ \gamma = -12 \text{ deg}; \ V_i = 5.5 \text{ km/s} \]

Rising ballistic coefficient raises heat rate and load
Equilibrium Glide Entry

- Can perform the same analysis of an equilibrium glide (lifting) entry
- Details are left as an exercise for the student

\[ V_{q_{\text{max}}}^* = \sqrt{\frac{2}{3}} V_c \quad \text{(for } V_{\text{atm}} \geq \sqrt{\frac{2}{3}} V_c) \]

\[ q_{s_{\text{max}}} = 1.94 \times 10^4 \left[ \frac{1}{R_n} \left( \frac{\beta}{L/D} \right) \right]^{\frac{1}{2}} \]

\[ Q_s \approx 2.05 \times 10^7 \left[ \frac{\beta}{R_n} \left( \frac{L}{D} \right) \right]^{\frac{1}{2}} \sin^{-1} \left( \frac{V_{\text{atm}}}{V_c} \right) - \frac{V_{\text{atm}}}{V_c} \left( 1 - \left( \frac{V_{\text{atm}}}{V_c} \right)^2 \right)^{\frac{1}{2}} \]

- Compare to Allen-Eggers; similar dependence on \( \beta \), but a lifting body (L/D > 1) will have heat flux inversely dependent on L/D and heat load directly dependent on L/D
Numerical Example: MER

• What is the peak stagnation point heating for the MER example previously examined \((R_n = 0.5R_b)\)?

  • At peak heating:
    \[ V_{q_{\text{max}}} = 0.846 \times 5.45 = 4.61 \text{ km/s} \]
    \[ R_n = \frac{2.65}{4} = 0.6625 \text{ m} \]
    \[ h = 40.87 \text{ km} \]
    \[ \rho = 3.11 \times 10^{-4} \text{ kg/m}^3 \]

  • From the Allen-Eggers expressions derived herein:
    \[
    q_s = k \left( \frac{\rho}{R_n} \right)^{\frac{1}{2}} V^3 = 1.9027 \times 10^{-4} \left( \frac{3.11 \times 10^{-4}}{0.6625} \right)^{\frac{1}{2}} (4610)^3 = 40.4 \text{ W/cm}^2
    \]
    \[
    T_w = \left( \frac{q_w}{\varepsilon \sigma} \right)^{\frac{1}{4}} = \left( \frac{40.4 \times 10^4}{0.8 \times 5.67 \times 10^{-8}} \right) = 1727K
    \]

    (literature quoted values range from 40-44 W/cm² based on CFD)
Other Trajectory Effects

- Prior discussion focused on impact of trajectory on stagnation point heating

- However, trajectory selection has other aerothermal impacts as well

- Transition to turbulence
  - Can dramatically increase heating levels away from stagnation point (4-6 times laminar levels)
  - Governed by Reynolds number \((\rho uL/\mu)\), therefore exacerbated by large entry bodies, steeper flight path angle, higher entry velocity, higher ballistic coefficient

- Heat soak
  - Longer trajectory time increases the amount soak of energy into the TPS, which increases the amount of TPS required to protect the structure (a given TPS tends to be less efficient as peak heat flux drops but heat load stays constant)
Orbiter Thermal Imagery

Stagnation Point (Laminar)

Turbulent flow from wing BLT protuberance

Turbulent flow from unknown origin

STS-119
Mach ~ 8.5
Mar 28, 2009
CFD Process for Entry Vehicle Design

- High fidelity CFD tools based on 20-year old methodologies
- Recent advances in parallel computing, efficient implicit algorithms have enabled rapid turnaround capability for complex geometries
- Full body three-dimensional CFD is an integral part of the design of all planetary and Earth entry TPS
Identification of Aerothermal Modeling Needs for Entry Missions

♦ Needs are both physics and process driven
  • process improvements are important for modeling complex geometries - not covered in this presentation
  • physical model improvements are important across the spectrum of NASA missions

♦ Gaps are destination and mission specific
  • shock layer radiation in particular will dominate aeroheating for some missions and be unimportant for others
  • sensitivity analysis must be performed for each candidate mission

♦ Gaps can be divided into general categories
  • reacting gas physical models
  • surface kinetics
  • transition and turbulence
  • afterbody heating
  • shock layer radiation modeling
  • coupling between radiation/material response/fluid dynamics/aerodynamics
  • unsteady separated flows (wakes, control surface shock-BL interaction)
  • geometry effects
Turbulence: The Eyeball...

70° Sphere-Cone:
Hypersonic Flight in Ballistic Range

T (K)
Orbiter Thermal Imagery

- Stagnation Point (Laminar)
- Turbulent flow from wing BLT protuberance
- Turbulent flow from unknown origin

STS-119
Mach ~ 8.5
Mar 28, 2009
Transition and Turbulence
Status and Remaining Gaps

- Transition is less of a concern for blunt capsules
  - shorter trajectories, smaller surface area leads to less heat load augmentation
  - single use ablative TPS can withstand heating if mass penalty not large – design to fully turbulent

- Conclusion: Transition cannot be accurately predicted for most problems of interest. Designs must rely on testing and conservatism.

- Acreage turbulent heating predictions generally within 25% for orbital Earth entries (RANS), but additional developments are required for chemistry, blowing, roughness

- DNS, LES, DES type models under development to replace current RANS
Previous discussion centered on smooth wall turbulence

However, all ablators develop a roughness pattern that can augment heating

Analysis for MSL based on correlations from WT experiments and DoD RV data
- 1mm roughness → potential for up to 50% augmentation to baseline smooth wall predictions
- if true, roughness has eaten up entire turbulent heating uncertainty!

Roughness can also lead to a positive feedback loop → vortical structures are generated that augment roughness
No validated model exists for Mars:

\[
\text{CO} + \text{O} \quad \text{O} + \text{O} \quad \text{CO} + \text{O}_2
\]

As a consequence, Mars entry vehicles are designed assuming a worst case scenario – so called “supercatalytic” wall.

For MSL there is a factor of four difference in heating between the various models.

What are the key gaps?

- quantum chemistry to determine reaction rates (gas phase and gas-surface)
- MD simulations of key GSI processes
- experimental data on TPS materials at relevant conditions
Wake flows are much harder to simulate than forebody
- separated, low density, unsteady, nonequilibrium flowfield
- significant code-to-code differences still exist

Current uncertainty levels ~50-300%
- primary reason: lack of validation; we have not quantified how good (or bad) we are

What are the key gaps?
- additional ground test data (including free flight or stingless models)
- explore advanced methods (DES, LES) for hypersonic separated flows
- advocate for additional flight data
Now throw OML singularities (such as RCS thrusters) into the wake flow
   – does not make things easier!

MSL is actively guided; thrusters must fire during hypersonic entry
   – predicted locally high heating rates necessitated a late change in backshell TPS for MSL (with significant cost and mass penalty)

MSL backshell design requires canted thrusters for control authority

Thrusters sticking into the flow; must be designed to withstand aerothermal environment
   – no validation of our methods for this application
Shock Layer Radiation

- Shock layer radiation is highly non-equilibrium, non-blackbody
  - Titan analysis showed order of magnitude differences between equilibrium & accurate model

- Not important for Mars missions to date, but critical for HMMES
  - Importance increases with velocity & vehicle size
  - Primary radiator, CO(4+) emits in UV

What are the key gaps?
- Obtain additional shock tube data for Mars entries
- Build collisional-radiative models for all atomic and molecular radiators
- Compute excitation rates from QM
- Develop medium-fidelity methods for design
- Develop models for coupling to fluid dynamics
Flowfield-Radiation-Ablation Coupling

- **Flowfield-Radiation (adiabatic cooling)**
  - Engineering approximation
  \[ \Gamma = \frac{2q_{\text{rad}}}{\left(\frac{1}{2} \rho_\infty V^3\right)} \quad \frac{q_{\text{coup}}}{q_{\text{unc}}} = \frac{1}{1 + \nu \Gamma^{0.7}} \]
  - Loose coupling is also possible
  - More accurate answer requires simultaneous solution of the Navier-Stokes and radiative transfer equations; not possible except for limiting cases

- **Flowfield-Ablation**
  - Blowing reduces heat transfer
  - Ablation products mix with boundary layer gases
  - Typically solved via loose-coupling approximation

- **Radiation-Ablation**
  - Injected ablation products can absorb/emit radiation

- **Ablation-Trajectory**
  - Significant ablation can lead to changes in aerodynamics/trajectory/GN&C
  - Primarily a concern for RV’s
We have already discussed gas-surface and ablation coupling, but other interactions are important

Ablation induced distributed roughness
• Surface roughness generated on TPS surface as a consequence of ablation.
• Strong interaction with boundary layer - increased heating and shear stress result
• Heating augmentation from zero to factor of three possible over turbulent smooth wall

Discrete roughness
• Due to gaps, repairs, geometrics singularities, etc.
• Generate local heating and shear augmentation factors which must be accommodated

For MSL:
• Distributed roughness adds about 20% to heating (pattern roughness not expected)
• Discrete roughness adds another 40% locally in areas of gaps or repairs
Melt layer interactions

- One class of ablators uses a glassy substrate material
- Energetically favorable; glass vaporization is highly endothermic
- Can cause strongly coupled instabilities in environments where glass melts but does not vaporize
- Interactions or instabilities can range from minor to catastrophic

What to do?

- Simple solution: don’t fly glassy ablators in such environments
- Better long term solution: develop models of the boundary layer surface interaction

Research topic: Better models for all aspects of material / fluid interactions
The primary reason we continue to use 70° sphere cones for Mars entry is “heritage”

- argument is weak: clear finding of MSL aerothermal peer review last summer

Non-optimal from aerothermal perspective

- expansion around nose leads to boundary layer instabilities, early transition, high heating levels

Modified ellipsoidal aeroshell has significant advantages with same aerodynamics

- for Mars aerocapture this shape led to 50% lower heat flux, potential 67% TPS mass savings

For large entry masses other shapes (e.g. ellipsled, biconic, bent biconic) should be explored as well

A full shape optimization study should be part of any future Mars systems analysis
Validation: AS-202 Flight Data

**Problem:** Current uncertainty on afterbody heating predictions is very high

**Goal:** reduce uncertainty levels by validation with flight data

**Afterbody Calorimeter Placement**

- Computations generally agree with flight data to within ±20% uncertainty at 15 of 19 calorimeter locations.

Surface Oilflow  
\[ t = 4900 \text{ s}, Re_D = 7.6 \times 10^5 \]

⇒ Computations generally agree with flight data to within ±20% uncertainty at 15 of 19 calorimeter locations.

Ref: AIAA 2004-2456
Flight Data: MER-B Heatshield

- Unique opportunity to observe in-situ flight hardware during Opportunity extended mission
- Multiple images of (inverted) heatshield made with cameras and micro-imager
- Work ongoing to compare visualized material response to predictions

Image courtesy Christine Szalai, JPL

Flight data are the gold standard for final model validation
HQ approval for MSL instrumentation suite!

- High TRL sensors to be installed in seven locations on heatshield
- Flight data obtained will go a long way toward validating ARMD-developed tools to drive down uncertainties discussed herein
- No backshell instrumentation (backshell is on critical path)
Thermal Protection Systems

John A. Dec
NASA Langley Research Center
Outline

♦ Background Information
  – What is TPS?
  – Selecting the Right Material for the Mission

♦ Ablative TPS Modeling
  – Ablator Characteristics
  – Surface Recession
  – In-Depth Models

♦ TPS Sizing and Margin

♦ TPS Testing

♦ Look to the Future
Three Kinds of TPS

Passive (Reusable)
Rely on reradiation to reject heat, low thermal conductivity to limit penetration
Coatings to increase emissivity, reduce catalycity
Limited by reusable temperatures of common materials
Uses: Shuttle Orbiter, X33, X34

Active (Reusable)
Rely on active cooling for heat rejection
Plumbing systems, active transpiration
Very complex; seldom considered; very low technology readiness

Ablative (Non-Reusable)
Combine reradiation with ablation and pyrolysis for heat rejection
Can be considered passive transpiration cooling
Ideal for high heat flux/load entries, particularly when reusability not required

The focus of today’s lecture is on ablative systems; baseline for all planetary EDL to date
Ablation

• Definition:
  – The term ablation is encountered in many fields of science and engineering
    • In the medical field it refers to the surgical removal of a body part or tissue
    • In glaciology it refers to the removal of ice and snow from the surface of a glacier
  – In space physics, ablation is the process of absorbing energy by removal of surface material by melting, vaporization, sublimation, or chemical reaction
Why Ablative Materials?

Courtesy Bernie Laub, NASA Ames
How is TPS Chosen?

• Heat rate, along with pressure and shear, determine type of TPS to employ
  • Material classes have clear performance limits marked by poor performance/material failure

• Heat load determines overall thickness of TPS material

• Other design features play a role
  • Need for tiles, forebody penetrations, compression pads, structural loads, etc. can impact material selection and TPS design
  • RF transparency for materials that protect antennae
Ablative TPS: History of Success, Little Recent Development

No Human Rated Ablative TPS Available Today!
CEV/Orion is working to develop Avcoat, for a human rated system - Very Close to Achieving This Goal!

Courtesy Bernie Laub, NASA Ames
## Available Materials

<table>
<thead>
<tr>
<th>Material Name</th>
<th>Manufacturer</th>
<th>Density (kg/m³)</th>
<th>Limit (W/cm²)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLA-561V</td>
<td>Lockheed-Martin</td>
<td>256</td>
<td>~ 200</td>
<td>Not viable for high shear</td>
</tr>
<tr>
<td>FM 5055 Carbon Phenolic</td>
<td>Fibercote (formerly US Polymeric), Hitco Inc.</td>
<td>1450</td>
<td>&gt; 10,000</td>
<td>No source of heritage Rayon</td>
</tr>
<tr>
<td>MX4926N Carbon Phenolic</td>
<td>Cytec (pre-preg), ATK, HITCO</td>
<td>1450</td>
<td>&gt; 10,000</td>
<td>Flown on Shuttle SRM, never as a heat shield</td>
</tr>
<tr>
<td>PhenCarb-20,24,32</td>
<td>Applied Research Associates (ARA)</td>
<td>320-512</td>
<td>~ 750</td>
<td>Never flown</td>
</tr>
<tr>
<td>PICA (Phenolic Impregnated Carbon Ablator)</td>
<td>Fiber Materials, Inc. (FMI)</td>
<td>265</td>
<td>&gt; 1500</td>
<td>Must be tiled above 1m diameter</td>
</tr>
<tr>
<td>Avcoat 5026 (Apollo)</td>
<td>Textron Systems</td>
<td>513</td>
<td>~1000</td>
<td>Recreated for CEV</td>
</tr>
<tr>
<td>ACC</td>
<td>Lockheed-Martin</td>
<td>1890</td>
<td>~ 1500</td>
<td>Heavy, not readily extendible above 2m</td>
</tr>
</tbody>
</table>
Wow! This is going to be hard w/o a significant improvement to the state of the art.
What Are They Made Of?

**Large Cell Phenolic Honeycomb**

**Organic Fiber Reinforced Phenolic**

- **Silica Microballoons**
- **Largest ~100 Microns**
Pyrolyzing Ablators

Substrate Material (e.g. fibers, cloth)
- Desire ability to withstand high temperatures (reradiation)
- Carbon is best; glass also good (heat of vaporization)

Organic Resins (e.g. phenolics)
- Pyrolyzing ablators only
- When heated resin generates gas and leaves carbon residue
- What are they good for?
  - in-depth and surface transpiration
  - endothermic reactions absorb energy
  - carbon char for reradiation

Additives (e.g. microballoons, cork)
- Density & thermal conductivity control

Added Reinforcement (e.g. honeycomb)
- Structural integrity, bond verification (adds mass)
How do they Work?
Surface Ablation Mechanisms

- Melting – common ablation mechanism, but doesn’t absorb much energy

- Vaporization – absorbs significant amount of energy

- Oxidation – exothermic process that adds energy

- Sublimation – Can be significant energy absorber

- Spallation – Mass loss with minimal energy absorption (Thermostructural Failure – HIGHLY UNDESIRABLE)
Oxidation

- Oxidation is an \textit{exothermic} process
- Example:
  \[ 2C(s) + O_2(g) \rightarrow 2CO(g) \]
  \[ \Delta H_{\text{comb}} \approx -4170 \text{ kJ/g}_{\text{carbon}} \]

- Note: the B' curve for carbon in air was generated with assumptions of thermochemical equilibrium, equal diffusion coefficients, etc.
- The “equilibrium” assumption allows the diffusion-limited plateau to extend to \textit{unrealistically low} surface temperatures

Other exothermic surface chemistry is possible ("nitridation" and "hydridation") but these are not typically significant players

Courtesy Bernie Laub, NASA Ames
Other Mechanisms

• Material decomposition …aka pyrolysis
  – Endothermic reactions absorb energy
  – Convection of pyrolysis gas through the char

• Conduction through the material
  – Transfer energy to structure or heat sink

• Re-radiation from the surface
  – Largest percentage of energy is dissipated through this mechanism
Modeling Approach

• In the mid to late 1960’s, Kendall, Rindal, and Bartlett, and Moyer and Rindal extended the work by Kratsch et. al.
  – Included unequal heat and mass transfer coefficients
  – Non-unity Lewis and Prandtl numbers
  – Corrected in-depth energy equation:
    • to account for the energy of the pyrolysis gas convection and generation within the solid
    • to account for grid motion due to a coordinate system that is attached to the receding surface

\[
\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_s} \left( k \frac{\partial T}{\partial x_s} \right) + \left( h_g - h \right) \frac{\partial \rho}{\partial t} + \dot{S} \rho c_p \frac{\partial T}{\partial x_s} + \dot{m}_g \frac{\partial h_g}{\partial x_s}
\]

\[ -k \frac{dT}{dx} = \rho_e U_e C_H \left( H_{sr} - h_{sw} \right) + \rho_e U_e C_M \left( \sum_i \left( Z_{ie}^* - Z_{iw}^* \right) h_i^0 + B'_c h_c + B'_g h_g - B'_h_w \right) - q^* + q_{rad_{out}} - \alpha q_{rad_{in}} \] (13)

• If the diffusion coefficients are assumed equal and the Le=Pr=1.0, the surface energy balance simplifies to

\[ -k \frac{dT}{dx} = \rho_e U_e C_H \left( H_{sr} - h_{sw} + B'_c h_c + B'_g h_g - B'_h_w \right) - q^* + q_{rad_{out}} - \alpha q_{rad_{in}} \] (14)
Derivation of the Governing Differential Equations

1-Dimensional control volume

\[
\left( m_{g_x} h_g + \dot{m}_g \frac{u^2}{2} + \dot{m}_g gZ \right)_x
\]

\[ m_{cv} \]

\[ \dot{E}_{st} \]

\[ dx \]

\[ m_{g_{x+dx}} \]

\[ q_x \]
Conservation of Mass

- Pyrolysis gas flows from the pyrolysis zone through the porous char to the heated surface
  - Assume gas flow is 1-D and normal to the heated surface
  - Assume dp~0 across the char (neglect the momentum eqn)

\[
\frac{\partial m_{cv}}{\partial t} = m_{in} - m_{out}
\]  \hspace{1cm} (15)

Where \( m_{cv} = \rho A dx \) \hspace{1cm} (16) \hspace{1cm} \dot{m}_{g_{x+dx}} = \dot{m}_{g_{x}} + \frac{\partial m_{g_{x}}}{\partial x} dx \) \hspace{1cm} (17)

\[
A \frac{\partial \rho}{\partial t} dx \left( \dot{m}_{g_{x}} + \frac{\partial \dot{m}_{g_{x}}}{\partial x} dx \right) - \dot{m}_{g_{x}} \hspace{1cm} \frac{\partial \rho}{\partial t} = - \frac{\partial \dot{m}_{g_{x}}''}{\partial x} \]  \hspace{1cm} (18)

\[
\frac{\partial \rho}{\partial t} \hspace{1cm} \text{Determined experimentally and modeled with an Arrhenius fit}
\]

\[
\dot{m}_{g_{x}}'' \hspace{1cm} \text{Mass flow rate per unit area}
\]
Conservation of Energy

- Two energies associated with this control volume
  - Pyrolysis gas flow
  - Heat conduction.

- Pyrolysis gas flow assumptions
  - Pyrolysis gas is in thermal equilibrium with the charred material within the control volume
  - Pyrolysis gas residence time within the control volume is small.
  - Potential energy of the pyrolysis gas may be neglected since the change in height across the control volume is negligible.
  - The kinetic energy of the pyrolysis gas may be neglected since it is of small magnitude relative to its enthalpy
Conservation of Energy

- **1st Law of Thermodynamics**

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_{in}(e + Pv)_{in} - \dot{m}_{out}(e + Pv)_{out}] \quad (19)
\]

Where \( e \) is the total energy per unit mass and includes kinetic, potential, and internal energy.

The internal energy and flow work may be expressed in terms of the enthalpy by, \( h = u + Pv \)

Rewriting equation 19 in a simplified form gives,

\[
\frac{dE_{cv}}{dt} = \dot{E}_{in} - \dot{E}_{out} \quad (20)
\]
Conservation of Energy

- The energy entering and leaving the control volume can be expressed as

\[ \dot{E}_{in} = q_x + \left( \dot{m}_g h_g \right)_{x+dx} \]

\[ \dot{E}_{out} = q_{x+dx} + \left( \dot{m}_g h_g \right)_x \]  \hspace{1cm} (21)

- Expressing the incremental heat conduction leaving and the convection of energy by the pyrolysis gas entering the control volume as Taylor series expansions gives, dropping H.O.T

\[ q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} \, dx \]

\[ \left( \dot{m}_g h_g \right)_{x+dx} = \left( \dot{m}_g h_g \right)_x + \frac{\partial}{\partial x} \left( \dot{m}_g h_g \right)_x \, dx \]  \hspace{1cm} (22)
Conservation of Energy

- The rate of energy storage within the control volume can be expressed in terms of the density and enthalpy of the solid as

$$\frac{dE_{cv}}{dt} = \frac{\partial}{\partial t} (\rho h) A dx \quad (23)$$

- Substituting eq 21 into eq 20, and using the definitions in eqns 22 and 23 gives

$$\frac{\partial}{\partial t} (\rho h) A dx = \left[ q_x + (m_g h_g)_x + \frac{\partial}{\partial x} (m_g h_g)_x \right] dx - \left[ q_x + \frac{\partial q_x}{\partial x} dx + (m_g h_g)_x \right] \quad (24)$$
Conservation of Energy

• Canceling like terms, dividing by $Adx$, and using Fourier’s law of heat conduction eqn 24 reduces to,

$$\frac{\partial}{\partial t} \left( \rho h \right) = \frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial x} \left( \dot{m}_{g_x} h_g \right) \quad (25)$$

Where,

- $\rho$ : density of the solid
- $h$ : enthalpy of the solid
- $h_g$ : pyrolysis gas enthalpy
- $k_x$ : thermal conductivity in the x-direction
- $T$ : temperature
- $\dot{m}_{g_x}$ : local gas flow rate in the x-direction
- $x$ : coordinate direction

• Physically,
  - Term I represents energy storage
  - Term II represents conduction through the material
  - Term III represents convection due to pyrolysis gas flow
Transforming the Governing Equations to a Moving Coordinate System

• The control volume is not fixed in space, it is tied to the receding surface
  – Requires transforming eqns 18 and 25 into a moving coordinate system
  – After some elaborate calculus and algebraic manipulation we arrive at,

\[
\frac{\partial \rho}{\partial t} \bigg|_{x_s} = \dot{S} \frac{\partial \rho}{\partial x_s} \bigg|_{t} + \frac{\partial \rho}{\partial t} \bigg|_{x}
\]

(26)

Conservation of mass in a moving coordinate system

\[
\frac{\partial \rho h}{\partial t} \bigg|_{x_s} = \frac{\partial}{\partial x_s} \left( k_x \frac{\partial T}{\partial x_s} \right) \bigg|_{t} + \frac{\partial}{\partial x_s} \left( \dot{m} \frac{h_g}{g_x} \right) \bigg|_{t} + \dot{S} \frac{\partial}{\partial x_s} (\rho h) \bigg|_{t}
\]

(27)

Conservation of energy in a moving coordinate system

Where terms I-III are the same as in eqn 25 and term IV is the convection of energy due to coordinate system movement
Final Form of the Energy Equation

- It is convenient to express the $(\rho h)$ terms in equation 27 in terms of material properties rather than the thermodynamic quantity of enthalpy.

- Performing some algebra and defining a new quantity, $\bar{h}$, the energy equation takes the following form:

\[
\begin{align*}
\rho c_p \frac{\partial T}{\partial t} &= \frac{\partial}{\partial x_s} \left( k_x \frac{\partial T}{\partial x_s} \right) + \left( h_g - \bar{h} \right) \frac{\partial \rho}{\partial t} \bigg|_x + \dot{S} \rho c_p \frac{\partial T}{\partial x_s} + \dot{m}'' \rho c_p \frac{\partial h_g}{\partial x_s} \\
\text{where} \quad \bar{h} &= \left[ \frac{\rho_v H_v - \rho_c H_c}{\rho_v - \rho_c} \right] \\
H_v &= h_v^0 + \int_0^T c_{p_v} dT \\
H_c &= h_c^0 + \int_0^T c_{p_c} dT \\
\rho_v &\quad : \text{virgin material density} \\
\rho_c &\quad : \text{charred material density} \\
H_v &\quad : \text{total enthalpy of the virgin material} \\
H_c &\quad : \text{total enthalpy of the charred material}
\end{align*}
\]
Final Form of the Energy Equation

\[
\rho c_p \frac{\partial T}{\partial t} = \left( k_x \frac{\partial T}{\partial x} \right) + \left( h_g - \bar{h} \right) \frac{\partial \rho}{\partial t} + \dot{S} \rho c_p \frac{\partial T}{\partial x} + m_g \frac{\partial h_g}{\partial x} \tag{28}
\]

- Each term in equation 28 has physical significance
  - Term I
    - rate of sensible energy storage
  - Term II
    - net conduction through the material
  - Term III
    - creation of sensible energy due to pyrolysis (ie the heat of decomposition)
  - Term IV
    - energy convected due to coordinate system movement
  - Term V
    - energy convected away due to pyrolysis gas generation at that point
TPS Sizing Approach

- Baseline (zero-margin) sizing computed assuming nominal environments and response model to hit given bondline temperature limit

- Margin process then applied to account for various sources of uncertainty

- Appropriate factors of safety be applied to trajectory dispersions, aerothermal loads, initial conditions, and material variabilities

- Primary (thermal) margin is applied directly to the TPS design criterion (e.g. maximum bondline temperature)
  - The impact of this margin on TPS thickness is material-dependent since the sensitivity of bondline temperature to thickness is material-dependent

- Secondary (recession) margin is also employed
  - Bondline is insensitive to excessive recession until it is too late

- Various independent sources of error are RSS’ed to avoid stacked conservatism

- Additional program imposed thickness factor of safety is recommended to account for unknown unknowns

- Other factors (e.g. thermal stress, CTE mismatch, adhesive failure) should also be tracked as possible limiting cases
  - Adhesive failure accounted for by maintaining conservative bondline temperature limit
Simplified Approach

• Return to the simplest form of the in-depth energy equation

\[ \rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) \] (1)

• Here we neglect the affects of decomposition, pyrolysis gas flow and surface recession.

• Additionally, if it is assumed that the solid extends to infinity in all but one direction and is characterized by a single identifiable surface, if a sudden change in conditions is imposed at this surface, transient, one-dimensional conduction will occur within the solid. This is known as the semi-infinite solid approximation.

• This approach is for illustrative purposes only and should not be used beyond conceptual design.
Simplified Approach

- Using the semi-infinite solid approximation, closed-form analytical solutions to the in-depth energy equation can be derived.
- For a thick slab which has a constant surface temperature at any instant in time, the temperature at a depth \( x \) within the solid at time \( t \) is given by,

\[
T(x, t) = \text{erf} \left( \frac{x}{2 \sqrt{\alpha t}} \right) (T_i - T_s) + T_s
\]

where,

- \( T_i \) is the initial temperature
- \( k = \text{thermal conductivity} \)
- \( T_s \) is the surface temperature
- \( c_p = \text{specific heat} \)
- \( \text{erf} \) is the gaussian error function
- \( \rho = \text{density} \)
- \( \alpha \) is the thermal diffusivity = \( \frac{k}{\rho c_p} \)

\[(29)\]
Simplified Approach

• In this simplified approach, the amount of material required for insulation and the amount of material required for recession are calculated separately.

• To calculate the recession in an approximate way, use the data correlation parameter known as the heat of ablation ($Q^*$) and solve for recession rate

\[
\dot{S} = \frac{\dot{Q}_{cw} \left( \frac{H_r - H_{T_w}^{T_w}}{H_r} \right) - \sigma \varepsilon T_w^4}{\rho Q^*}
\]  

(5)
Simple Finite Difference Approach

- To increase the fidelity, a finite difference approximation of equation 1 can be written incorporating a simplified surface energy balance

\[
\frac{\rho c_p}{\partial t} \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) \quad (1)
\]

\[
\dot{q}_{\text{conv}} + \alpha \dot{q}_{\text{rad}} - \dot{q}_{\text{cond}} - \epsilon \sigma T_w^4 = 0 \quad (30)
\]

\[
\left(1 + \frac{2k \Delta t}{\rho c_p \Delta x_i^2} \right) T_{i+1}^{n+1} - \frac{2k \Delta t}{\rho c_p \Delta x_i^2} T_{i+1}^n = T_i^n + \alpha \dot{q}_{\text{rad}} \frac{2\Delta t}{\rho c_p \Delta x_i} + \dot{q}_{\text{conv}} \frac{2\Delta t}{\rho c_p \Delta x_i} - \epsilon \sigma T_i^4 \frac{2\Delta t}{\rho c_p \Delta x_i} \quad (31a)
\]

\[
T_i^n = -\frac{k \Delta t}{\rho c_p \Delta x_i^2} T_{i+1}^{n+1} + \left(1 + \frac{2k \Delta t}{\rho c_p \Delta x_i^2} \right) T_{i+1}^{n+1} - \frac{k \Delta t}{\rho c_p \Delta x_i^2} T_{i+1}^{n+1} \quad (31b)
\]
Stagnation Point Sizing Example

- Ballistic Earth entry
  - Ballistic coefficient = 60 kg/m², entry velocity = 12.6 km/s
  - 60° sphere cone, 0.8 m diameter, \( r_n = 0.23 \) m
  - At the stagnation point, \( H_r \) can be approximated by \( \frac{V^2}{2} \)

- PICA heat shield

\[ \rho = 265.0 \ \frac{kg}{m^3} \]
\[ k = 1.6 \ \frac{W}{m \cdot K} \]
\[ C_p = 1592.0 \ \frac{J}{kg \cdot K} \]

- Radiative equilibrium temperature

\[ T_w = \sqrt[4]{\frac{\dot{q}_{cw}}{\varepsilon \sigma} + T_{sur}^4} \]
Stagnation Point Sizing Example

- Comparing the simplified approach, the simple FD approach, and the high fidelity code CMA

![Graph](image-url)
Typical Approach to Modeling Materials

- **Decouple surface recession from in-depth conduction**
  - Use steady-state surface energy balance expression
  - Employ equilibrium thermodynamic tables for surface recession, corrected as required for finite-rate chemistry, spallation, melt flow
  - Validate surface model with arc jet data

- **Once recession model is working, develop in-depth pyrolysis model**
  - Thermochemical data from materials testing
  - Validate model by arc jet data; use first thermocouple as “truth model” boundary condition
  - Tweak char thermal conductivity as required

- **Add additional physics as required for the problem**
  - Multi-dimensional conduction, Darcy’s Law, etc.
Material Modeling

Thermochemical Properties:

1. Conduct Thermogravimetric Analysis (TGA experiments) in inert gas, low temperature rise rates, usually 10° C/min. Residual mass fraction defines char yield. Data fits provide decomposition kinetic constants for the Arrhenius equation.

2. Conduct digital scanning calorimeter (DSC experiments) in inert gas, low temperature rise rates, 10° C/min. Data provides heat of reaction for pyrolysis reactions as function of temperature.

3. Measure elemental composition of virgin material, by mass spectrometry.

4. Measure heat of combustion of virgin material and derive heat of formation.

5. Derive elemental composition of char from known constituents and char yield data. Can be problematic to measure thermal conductivity (explained later).

6. Derive heat of formation of char from known constituents and existing data.

7. Derive elemental composition of pyrolysis gases. Develop model(s) for pyrolysis gas enthalpy using combination of thermochemical equilibrium calculations and measured heat of pyrolysis data.
Material Modeling

8. Measure specific heat of virgin material as function of temperature.

9. Measure thermal conductivity of virgin material as function of temperature (and orientation, if appropriate).

10. Derive specific heat of char from known (or derived) composition using method of mixtures.

11. Measure optical properties of virgin material

12. Derive optical properties of char from known composition and properties of similar materials (or determine experimentally)

13. Measure thermal conductivity of char as function of temperature (and orientation, if appropriate).

Assertion: the thermal conductivity of the char cannot be measured in standard lab facilities!
Char Thermal Conductivity

Traditional practice has been to bake the material in an oven and measure the thermal properties of the resulting “char.” Studies conducted under the Apollo heat shield program (and re-validated in other programs) demonstrated that the cellular structure of “oven chars” was different than the cellular structure of chars formed in ground test or flight.
Example

Tape-wrapped carbon phenolic
Arc jet test in N₂/He mixture

900 W/cm² 10° ply orientation

Temperature (°C)

Time (s)

Tape-wrapped carbon phenolic
Arc jet test in N₂/He mixture

900 W/cm² 90° ply orientation

Temperature (°C)

Time (s)

Courtesy Bernie Laub, NASA Ames
Margin vs Nominal (Zero Margin)

• The purpose of the TPS thickness margin is to capture two things:
  • Uncertainties in operating conditions
  • Uncertainties in baseline (nominal) sizing required to meet operational requirements (including abort)

• As such, the TPS margin captures implicitly the fidelity and level of uncertainty in the underlying TPS design tools employed to determine the baseline sizing

• Research is underway to calculate TPS margins probabilistically, this requires knowledge of the uncertainties in the input parameters for all analysis codes being used; aerothermal, trajectory, thermal response
Excess Recession Lien
Localized Application

Material Property Uncertainty
Arc-Jet Statistics
Ground-to-Flight Traceability

Result from incomplete knowledge of operating environment, inability to test in flight environment, and/or deficiencies in underlying physical models.

Courtesy Mike Wright, NASA Ames
• No ground facility can reproduce all aspects of the flight environment; every test is a compromise

• Facility classes: arc jets, combustion plasma, lasers, radiant lamps, the atmosphere of the Earth (flight tests)

• Best facility for a given test depends upon the objectives:
  • Materials screening
  • Materials characterization and model development
  • Performance limit evaluation (failure modes)
  • Materials qualification
  • Material interface evaluation (gaps, seals, etc.)
  • System level testing
Diagnostic Instrumentation

• Instrumentation is critical to the success of the test
• Possible Types:
  • Flowfield diagnostics (calorimetry, null points, LIF)
    – absolutely essential in arc jets to characterize freestream
  • Surface temperature (pyrometry)
    – validate recession model, detect local anomalies; global result
  • Film or video
    – evaluate transient performance, detect failures, recession (PRM)
  • In-situ
    – thermocouples, both bondline (qualification) and in-depth (material characterization
    – recession sensors
    – strain gauges (system level testing)
Arc Jets

- Workhorse facility for TPS testing

- Can put flight-like $q, h, p, \tau$ on sample for long duration (but usually not more than two at a time)

- Limitations include:
  - sample size; subscale testing only
  - combined radiative/convective heating (no facility exists)
  - non-Earth gas mixture (no domestic facility exists)
  - difficult to simulate time-varying (trajectory based) conditions
  - freestream characterization (what are we testing in anyway?)
Arc Jets
Sandia Solar Tower

Up to ~200 W/cm²
Concentrated solar radiation

**Advantages:**
Large models possible
Good for system level testing

**Disadvantages:**
No flow (other than wind)
Non flight like application of heat flux (only matching one of $q,h,p,\tau$)
Only works on sunny, cloudless days (but it is in desert!)

Courtesy Bill Congdon, ARA
Sandia Solar Tower

Midday Test of SRAM-20 Panel 490 at 151 W/cm² for 230 Sec – 2-15-07

National Geographic Photo – Oct. 2007 Issue – Pg.118.

Courtesy Bill Congdon, ARA
Sandia Solar Tower

1 meter aeroshell test (ISP program)

Within Seconds of Exposure

Near End of 210-sec Exposure

Steve Moon Test Photos

Courtesy Bill Congdon, ARA
LHME/L Laser Facility

Up to 100 kW on user-defined spot size for up to 100 seconds
CO₂ radiation

**Advantages:**
Large models possible
High throughput
Very low uncertainty in applied heat flux

**Disadvantages:**
No flow (other than wind)
Non flight like application of heat flux (only matching one of $q, h, p, \tau$)
Current Research

• Current modeling research is geared towards making improvements
  – Multi-dimensional geometry
  – Orthotropic material properties
  – Loose coupling to CFD codes
  – Loose coupling to grid and trajectory codes
  – Coupled ablator thermochemistry
  – Coupled thermal stress
  – Multi-dimensional pyrolysis gas flow
  – Non-equilibrium surface thermochemistry
  – Probabilistic heat shield sizing
Current Research

Deployable/Inflatable Entry Systems

Trailing ballute concept
(Courtesy Ball Aerospace)

IRVE-3 Vehicle
Flexible Thermal Protection Systems

- Flexible TPS materials are a mission enabler for large mechanically deployable or inflatable entry system aeroshells

- Large aeroshell diameter reduces ballistic coefficient and therefore peak aerodynamic heating
Thermal Protection Systems

• Flexible TPS consist of multiple layers of different materials
  • Outer reinforcing fabric
  • Inner insulation
  • Impermeable gas barrier

In-depth
• Solid conduction
• Gas conduction
• Internal radiative transport

Pressure and temperature dependant

• Material decomposition
• Pyrolysis gas flow
• Pyrolysis gas convective heat transfer
  (not included in the model yet)
The current generation of ablative TPS models must be significantly improved to support the next generation of complex NASA entry missions.

Advanced modeling and new systems will be a key component of reducing mass while increasing system reliability.

Improvements required:
- Finite-rate gas-surface interaction capability
- Loose coupling to CFD codes
- Loose coupling to grid and trajectory codes
- True multidimensional analysis, including gaps, seams and other interfaces
- Coupled ablator thermochemistry
- Built in models for melt flow (glassy ablators), mechanical erosion, etc.
- Robust models for multi-layer ablative systems
Nomenclature
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>area, $m^2$</td>
</tr>
<tr>
<td>$B_i$</td>
<td>pre-exponential factor for the $i$th resin component</td>
</tr>
<tr>
<td>$B'_c$</td>
<td>non-dimensional charring rate</td>
</tr>
<tr>
<td>$B'_g$</td>
<td>non-dimensional pyrolysis gas rate at the surface</td>
</tr>
<tr>
<td>$B'$</td>
<td>total non-dimensional blowing rate</td>
</tr>
<tr>
<td>$C_H$</td>
<td>Stanton number for heat transfer</td>
</tr>
<tr>
<td>$C_M$</td>
<td>Stanton number for mass transfer</td>
</tr>
<tr>
<td>$C_p$</td>
<td>solid material specific heat, $J/kg-K$</td>
</tr>
<tr>
<td>$C_{p_g}$</td>
<td>pyrolysis gas specific heat, $J/kg-K$</td>
</tr>
<tr>
<td>$E_{a_i}$</td>
<td>activation energy for the $i$th resin component, $J/kg$-mole</td>
</tr>
<tr>
<td>$\dot{E}_{st}$</td>
<td>rate of energy storage in the control volume, $W$</td>
</tr>
<tr>
<td>$H_r$</td>
<td>recovery enthalpy, $J/kg$</td>
</tr>
<tr>
<td>$H_w$</td>
<td>wall enthalpy, $J/kg$</td>
</tr>
<tr>
<td>$H_{air}^{T_w}$</td>
<td>enthalpy of air evaluated at the wall temperature, $J/kg$</td>
</tr>
<tr>
<td>$H_g$</td>
<td>pyrolysis gas enthalpy, $J/kg$</td>
</tr>
<tr>
<td>$h_i^0$</td>
<td>enthalpy of formation of species $i$, $J/kg$</td>
</tr>
<tr>
<td>$h_{ref}$</td>
<td>reference enthalpy at 298K, $J/kg$</td>
</tr>
<tr>
<td>$h_g$</td>
<td>enthalpy of pyrolysis gas, $J/kg$</td>
</tr>
<tr>
<td>$h_c$</td>
<td>enthalpy of char, $J/kg$</td>
</tr>
<tr>
<td>$h_w$</td>
<td>enthalpy of the boundary layer edge gas evaluated at the wall temperature, $J/kg$</td>
</tr>
</tbody>
</table>
# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>node index, or resin component index (A,B,C)</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity, W/m-K</td>
</tr>
<tr>
<td>$m_g$</td>
<td>mass flow rate of pyrolysis gas, kg/s</td>
</tr>
<tr>
<td>$m_g''$</td>
<td>mass flux of pyrolysis gas, kg/m$^2$-s</td>
</tr>
<tr>
<td>$m_c''$</td>
<td>mass flux of char, kg/m$^2$-s</td>
</tr>
<tr>
<td>$m_{cv}$</td>
<td>mass stored in the control volume, kg</td>
</tr>
<tr>
<td>$q$</td>
<td>source term in the general heat equation</td>
</tr>
<tr>
<td>$q^*$</td>
<td>condensed phase energy removal, W/m$^2$</td>
</tr>
<tr>
<td>$\dot{q}_{rad}$</td>
<td>stagnation point radiative heat flux, W/m$^2$</td>
</tr>
<tr>
<td>$\dot{q}_{conv}$</td>
<td>stagnation point convective heat flux, W/m$^2$</td>
</tr>
<tr>
<td>$\dot{q}_{cond}$</td>
<td>conductive heat flux, W/m$^2$</td>
</tr>
<tr>
<td>$\dot{q}_{cw}$</td>
<td>cold wall heat flux, W/m$^2$</td>
</tr>
<tr>
<td>$\dot{q}_{hw}$</td>
<td>hot wall heat flux, W/m$^2$</td>
</tr>
<tr>
<td>$Q^*$</td>
<td>thermochemical heat of ablation, J/kg</td>
</tr>
<tr>
<td>$Q$</td>
<td>also hot wall heat of ablation, J/kg</td>
</tr>
<tr>
<td>$R$</td>
<td>universal gas constant, J/kg-mole-°K</td>
</tr>
<tr>
<td>$\dot{s}$</td>
<td>recession rate, m/s</td>
</tr>
<tr>
<td>ss</td>
<td>steady state</td>
</tr>
</tbody>
</table>
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>temperature, °C or K</td>
</tr>
<tr>
<td>$T_w$</td>
<td>wall temperature, °C or K</td>
</tr>
<tr>
<td>$T_0, T_i$</td>
<td>initial temperature, °C or K</td>
</tr>
<tr>
<td>$T_s$</td>
<td>surface temperature, °C or K</td>
</tr>
<tr>
<td>$T_{surr}$</td>
<td>surrounding, or ambient temperature, °C or K</td>
</tr>
<tr>
<td>$t$</td>
<td>time, sec</td>
</tr>
<tr>
<td>$u_e$</td>
<td>boundary layer edge gas velocity, m/s</td>
</tr>
<tr>
<td>$x$</td>
<td>distance measured from the original surface of the ablating material, m</td>
</tr>
<tr>
<td>$x_s$</td>
<td>distance measured from the moving surface of the ablating material, m</td>
</tr>
<tr>
<td>$Z^*_{ie}$</td>
<td>diffusion driving potential at the boundary layer edge</td>
</tr>
<tr>
<td>$Z^*_{iw}$</td>
<td>diffusion driving potential at the wall</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>solar absorptivity, or thermal diffusivity m$^2$/s</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>emissivity</td>
</tr>
<tr>
<td>$\eta$</td>
<td>transpiration coefficient</td>
</tr>
</tbody>
</table>
Nomenclature

\[ \Delta H_v \] enthalpy of vaporization, J/kg
\[ \Delta H \] enthalpy difference, J/kg
\[ \Delta H_d \] heat of decomposition, J/kg
\[ \Delta T \] temperature difference, °C
\[ \Gamma \] resin volume fraction
\[ \rho_r \] residual density, kg/m³
\[ \rho_s \text{ or } \rho_s \] solid material density, kg/m³
\[ \rho_e \] boundary layer edge gas density, kg/m³
\[ \rho_{\text{resin}} \] density of resin component, kg/m³
\[ \rho_{\text{fiber}} \] density of fiber reinforcement, kg/m³
\[ (\rho v)_w \] total mass flux entering the boundary layer, kg/m² - s
\[ \sigma \] Stephan-Boltzmann constant, W/m² - K⁴
\[ \psi_i \] density exponent factor
\[ \phi \] transpiration correction factor
References
References - Background


References - Response Models


