Structural Design and Analysis

• Loads and Load Sources
  – Designing or Critical Loads
  – Load Information / Estimation

• Piece Parts Analysis
  – Margin of Safety Definition
  – Factors of Safety to use
  – Summary Table

• Important Structural Concepts
  – Primary/Secondary Structure
  – Failsafe & Fracture Critical Structure
  – Aerospace Materials
  – Structural Failure
Loads

• "Designing Load" is the load that determines one or more structural characteristic of the part:
  – shape, thickness, strength, stiffness, material...

• Critical Load (somewhat synonymous) is more exactly the load that gives the minimum margin of safety (MS) for a part
  – MS represents the amount of extra structural capability you have over the applied load (elbow room)

• Examples of Critical Loads
  – pressurization loads for a rocket casing
  – launch loads for a spacecraft
  – thermal loads for a propulsion subsystem
  – crash loads for a car
Load Sources

• Where do these loads come from?

• For every part (subsystem) in your design, you should review every phase of its life and identify all loads that have the potential to be critical:
  – manufacturing & assembly
  – test (qualification, proof test)
  – transportation (truck or launch)
  – operation
  – contingencies (crash landing)

• Obtain or estimate loads
  – look up loads in reference books
  – ask other groups to determine loads
  – guesstimate for the purposes of starting analysis

• Calculate all margins of safety
Launch Vehicle Loads

• Max Q - Aerodynamic Loads
  – \( Q = \rho \frac{V^2}{2} \)
  – maximum pressure and bending on vehicle

• Max g's
  – usually occurs at stage burnout
  – maximum axial load on vehicle and payload

• Abrupt environmental & vehicle changes
  – internal and external pressure drop
  – dramatic thermal changes

• Staging shock loads
  – high g's, high frequency

• Random vibration and acoustics
  – equiv. g's = \( \sqrt{\pi \text{ PSD} f_n Q / 2} \)

• Some of these loads apply to payload as well
Launch Vehicle Failures

- LV failures are tied to the following subsystems
  - Propulsion (70%)
  - Avionics (11%)
  - Separation (8%)
  - Electrical (7%)
  - Structural (2%)

- Propulsion or Control System Failure More Common
  - Conestoga, LLV, Ariane V, and Antares
  - Structural Failure Relatively Rare
  - AmRoc, Shuttle, Pegasus, and Falcon 9
Spacecraft On-Orbit Loads

• Accelerations
  – orbital accelerations
  – gravity gradient
  – spinning
  – on-board disturbances
  – thrusting (attitude control, reboost)

• Thermal Loads
  – sun / shadow thermal gradients
  – eclipse effects (thermal snap)

• Other Special Cases
  – EVA loads (corners & edges)
  – rendezvous & docking

• Generally spacecraft are designed by launch loads!
Planetary Vehicle Loads

- Vibration loads from traversing rough terrain
- Launch / landing loads
- Maneuvering loads
  - tight turn
  - driving on an incline
  - losing traction / support on one wheel
- Crash loads
  - driving into a big boulder
  - rolling vehicle in unstable soil
  - safety is primary consideration
Piece Parts Analysis

• Structural analysis of a system consists of at least the following three tasks
  – Load Cycle Modeling (system-level) - iterative process
  – Piece-Part Analysis (static) - minimum margins of safety
  – Fracture and Fatigue Analysis (dynamic) - safe life analysis

• Piece Parts Analysis
  – Identify all loads on each part / subsystem
  – Calculate margins of safety
  – Tabulate minimum margins of safety
Factors & Margins of Safety

• Limit Loads: maximum loads expected (applied loads)

• Yield Load and Ultimate Load

• **Factors of Safety**: numbers imposed by the Customer (or your own good sense) that reflect
  - how uncertain you are of the load or structure
  - how safe you want to be
  - examples: 10 for bridges, 5 for ground handling equip, 2 for a/c

• **Margins of Safety** are calculated as follows:

\[
MS = \frac{Allowable\ Load}{Applied\ Load \times FS} - 1.0
\]

• **Beware**: There are other definitions of these terms in engineering, but the above approach is the most common in Aerospace
### NASA Standard Factors of Safety

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>FACTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YIELD</td>
</tr>
<tr>
<td>General structure</td>
<td>≥ 1.0</td>
</tr>
<tr>
<td>Main propellant tanks:</td>
<td></td>
</tr>
<tr>
<td>Combined loads and pressures</td>
<td>1.1</td>
</tr>
<tr>
<td>Pressure alone</td>
<td>—</td>
</tr>
<tr>
<td>Pressurized windows, doors, and hatches</td>
<td>—</td>
</tr>
<tr>
<td>Pressurized structure:</td>
<td></td>
</tr>
<tr>
<td>Combined loads and pressures</td>
<td>≥ 1.0</td>
</tr>
<tr>
<td>Pressure alone</td>
<td>—</td>
</tr>
<tr>
<td>Pressure vessels (other than main propellant</td>
<td>—</td>
</tr>
<tr>
<td>tanks)</td>
<td></td>
</tr>
<tr>
<td>Pressurized lines and fittings:</td>
<td></td>
</tr>
<tr>
<td>Less than 1.5 in. in diameter</td>
<td>—</td>
</tr>
<tr>
<td>1.5 in. in diameter or larger</td>
<td>—</td>
</tr>
</tbody>
</table>

*In addition to including the design factors in this table, designs for major load-carrying structure, windows, doors, hatches, and tanks should use fracture-control procedures to account for sharp cracks, crack-like flaws, and other stress concentrations in a manner that ensures the structural life meets mission requirements.

**Factor applied to limit load at limit pressure.

Source: NASA-STD-5001b
Table 1—Minimum Design and Test Factors for Metallic Structures

<table>
<thead>
<tr>
<th>Verification Approach</th>
<th>Ultimate Design Factor</th>
<th>Yield Design Factor</th>
<th>Qualification Test Factor</th>
<th>Proof Test Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype</td>
<td>1.4</td>
<td>1.0*</td>
<td>1.4</td>
<td>N/A or 1.05**</td>
</tr>
<tr>
<td>Protolight</td>
<td>1.4</td>
<td>1.25</td>
<td>1.2</td>
<td>N/A or 1.05**</td>
</tr>
</tbody>
</table>

* Structure has to be assessed to prevent detrimental yielding during its design service life, acceptance, or proof testing. ** Propellant tanks and SRM cases only.

Table 2—Minimum Design and Test Factors for Composite/Bonded Structures

<table>
<thead>
<tr>
<th>Verification Approach</th>
<th>Geometry of Structure</th>
<th>Ultimate Design Factor</th>
<th>Qualification Test Factor</th>
<th>Proof Test Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype</td>
<td>Discontinuity Area</td>
<td>2.0*</td>
<td>1.4</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>Uniform Material</td>
<td>1.4</td>
<td>1.4</td>
<td>1.05</td>
</tr>
<tr>
<td>Protolight</td>
<td>Discontinuity Area</td>
<td>2.0*</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Uniform Material</td>
<td>1.5</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

* Factor applies to concentrated stresses. For nonsafety-critical applications, this factor may be reduced to 1.4 for prototype structures and 1.5 for protolight structures.
Primary Structure

• Primary, Secondary, & Tertiary Structure
  – Primary structure is the system's backbone (carries all of the major loads imposed on vehicle)
  – Secondary structure includes all essential appendages and support structures (such as solar arrays, antennas, & fuel tanks)
  – Tertiary structures are less-essential mounting hardware (brackets, component housings, connector panels)

• Example of primary structure
  – Thin-walled cylindrical launch vehicle
  – Challenge is to figure out how to react shear & torsion stresses
  – Buckling of skin is most common failure mode
  – Buckling of a cylindrical section:
    \[ \sigma_{\text{crit}} = \frac{E \ t}{R \sqrt{3(1-\nu^2)}} \]
Critical Structure

• Critical Items List (CIL) contains all parts that
  – are deemed criticality 1 by FMEA (ie, single point failures)
  – are fracture critical (ie, stressed to the point where a flaw will grow to critical size)

• Failsafe & Fracture Critical Structure
  – Catastrophic failure is generally defined by customer
  – Failsafe structure can take redistributed loads after failure (ie, not single point failures); shall release no hazardous mass; shall not change dynamics significantly; shall have no fatigue problems
  – Low-risk structure is not primary structure; has only a remote possibility of failure; will not propagate a crack in 4 lifetimes
    \[ \sigma_{\text{max}} < \frac{F_{\text{tu}}}{[4(1-0.5R)K_t]} \]
  – Fracture critical parts must be labeled and analyzed as such, then inspected, treated, and tracked more carefully than conventional parts

• Crack Growth Analysis (FLAGRO)
  – All FC parts must be shown good for four lifetimes of load cycles with an initial flaw (determined by Non Destructive Investigation (NDI))
Aerospace Materials

• Comparison of specific stiffness, specific strength, and buckling parameter for a variety of aerospace metals and composites

• Definition of Structural Failure
  – Detrimental Yield vs Textbook Yield
    • deformation that detrimentally affects functionality of system
    • 0.2% Tresca yield condition (assumes system linear in first place)
  – Ultimate Failure
    • any material rupture or loss of functionality
Material Strength & Stiffness

- Typical Yield & Ultimate Strengths
  - low strength steel
    - yield: 36 ksi
    - ultimate: 58 ksi
  - high strength steel
    - yield: 102 ksi
    - ultimate: 116 ksi
  - Titanium
    - yield: 134 ksi
    - ultimate: 145 ksi
  - aluminum
    - yield: 37 ksi
    - ultimate: 42 ksi

- Stiffness versus Strength Designs
  - aluminum
    - $\rho: 0.10$
    - $E/\rho: 100$
    - $\sigma_u/\rho: 420$
  - Low $\sigma_u$ steel
    - $\rho: 0.28$
    - $E/\rho: 102$
    - $\sigma_u/\rho: 204$
  - high $\sigma_u$ steel
    - $\rho: 0.29$
    - $E/\rho: 98$
    - $\sigma_u/\rho: 390$
  - titanium
    - $\rho: 0.16$
    - $E/\rho: 109$
    - $\sigma_u/\rho: 906$

Conclusion: for aerospace structures - titanium and aluminum
Structural Analysis

• Some key structural formulas that are handy to have for early (back-of-the-envelope) design analyses:

  – Spring & Beam Stiffnesses
  – Beam Natural Frequencies
  – Euler Buckling Loads
  – Stresses in Simple Pressurized Shell
    \[ \sigma_{\text{hoop}} = \frac{p \, R}{t} > \sigma_{\text{long}} = \frac{p \, R}{2 \, t} \]
  – Random Vibe and Acoustic Equivalent g's

• Get a copy of Roark and Young, *Formulas for Stress and Strain*
  – available in the library or as Excel spreadsheets from www.roarksformulas.com
Structural Analysis

• Definition of Example Problem
• Definition of Load Cases
• Analysis of Stresses
• Tabulation of Margins of Safety
• Identification of Critical Load Case
Close-up of Z1 Truss
Structural Example

- Storage canister for ISS solar array deployment system
- 200 lb tip mass
- Cantilever launch configuration
- Thin-wall aluminum shell structure
Loads Sources

• Launch
  – Accelerations
  – Pressurization
  – Acoustics
  – Random Vibration
  – Thermal

• Crash Landing

• On-Orbit
Structural Parameters

\[ R = 25 \text{ in} \quad \ell = 100 \text{ in} \quad t = 0.10 \text{ in} \quad \rho = 0.10 \frac{\text{lbs}}{\text{in}^3} \]

\[ I = \frac{\pi}{4} \left( R_o^4 - R_i^4 \right) \approx \pi R^3 t = 4800 \text{ in}^4 \]

\[ E = 1 \times 10^7 \text{ psi} \quad \alpha = 13 \times 10^{-6} \frac{\text{in}}{\text{in} \cdot ^\circ \text{F}} \]

\[ W_{\text{canister}} = 2\pi \rho t R \ell = 157 \text{ lbs} \quad W_{\text{tip}} = 200 \text{ lbs} \]

\[ g = 386.4 \frac{\text{in}}{\text{sec}^2} \]

\[ \sigma_{Ty} = 37 \text{ ksi} \quad \sigma_{Tu} = 42 \text{ ksi} \]

\[ A = 2\pi Rt = 15.71 \text{ in}^2 \]
Launch Accelerations

\[ FOS = 1.4 \]

\[ \sigma_{LA} = \frac{MR}{I} + \frac{W_{tip}}{A} g_x \]

\[ M = g_{transverse} \left( W_{canister} h_{CG} + W_{tip} h_{tip} \right) \]

\[ g_{transverse} = \sqrt{5.8^2 + 8.5^2} = 10.3 \, g \]

\[ M = 10.3(157 \cdot 50 + 200 \cdot 100) = 286,900 \, in \cdot lb \]

\[ \sigma_{LA} = \frac{(286900inlb)(25in)}{4800in^4} + \frac{200lb}{15.71in^2} 4.85 \]

\[ \sigma_{LA} = 1494 + 61.75 = 1556 \, psi \]
Pressurization Loads

\[ \sigma_{\text{Hoop}} = \frac{PR}{t} = \frac{(14.7 \text{ psi})(25 \text{ in})}{0.1 \text{ in}} = 3675 \text{ psi} \]

\[ \sigma_{\text{Longitudinal}} = \frac{PR}{2t} = 1838 \text{ psi} \]
Launch Vehicle Vibration Environment

Power Spectral Density ($g^2/\text{Hz}$)

Frequency (Hz)
Random Vibration Loads

Miles’ Equation

\[ RLF_n = \sqrt{\frac{\pi f_n PSD}{4 \xi}} \]

\[ f_1 = \frac{1.732}{2\pi} \sqrt{\frac{E I g}{W_{tip} \ell^3 + 0.236 W_{canister} \ell^3}} = 80 \text{ Hz} \]

<table>
<thead>
<tr>
<th>( f_n )</th>
<th>( \xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;150 Hz</td>
<td>0.045</td>
</tr>
<tr>
<td>150-300 Hz</td>
<td>0.020</td>
</tr>
<tr>
<td>&gt;300 Hz</td>
<td>0.005</td>
</tr>
</tbody>
</table>

\[ FOS = 3.0 \]

\[ RLF = 7.93 \text{ g} \]

\[ M = 220,700 \text{ in} \cdot \text{lbs} \]

\[ \sigma_{RV} = \frac{MR}{I} = 1150 \text{ psi} \]

(repeat for each axis)
Thermal Loads

\[ FOS = 1.4 \]

\[ \Delta l = \alpha \cdot \Delta T \cdot l \]

\[ \Delta l = 13 \times 10^{-6} \cdot -100^\circ F \cdot 100 = .13 \text{ in} \]

Assume support structure shrinks only half as much as canister

\[ \sigma_{\text{Thermal}} = E \varepsilon = 10^7 \cdot \frac{.5 \times .13}{100} = 6500 \text{ psi} \]
Launch Loads Summary

<table>
<thead>
<tr>
<th>Load Source</th>
<th>Limit Stresses</th>
<th>$FOS$</th>
<th>Design Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch Accelerations</td>
<td>1556</td>
<td>1.4</td>
<td>2178</td>
</tr>
<tr>
<td>Pressurization</td>
<td>3675</td>
<td>3.0</td>
<td>11,025</td>
</tr>
<tr>
<td>Random Vibration</td>
<td>1150</td>
<td>3.0</td>
<td>3450</td>
</tr>
<tr>
<td>Thermal</td>
<td>6500</td>
<td>1.4</td>
<td>9100</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>25,750 psi</td>
</tr>
</tbody>
</table>

$$MS = \frac{\text{Allowable Load}}{\text{Design Load}} - 1 = \frac{37,000}{25,750} - 1 = 43.7\%$$
Observations about Launch Loads

- Individual loads could be applied to same position on canister at same times - conservative approach is to use superposition to define worst case
- 43% margin indicates that canister is substantially overbuilt - if launch loads turn out to be critical load case, redesign to lighten structure and reduce mass.