ENAE 483/788D MIDTERM – FALL, 2015 – *NAME:*

Much of this exam (okay, pretty much *all* of this exam) comes from the movie "The Martian". I am, however, endeavoring to prevent any out-and-out spoilers for those of you who haven't seen it yet. If I do inadvertently post a spoiler, I apologize. And, for those of you who have seen it, if something I say doesn't correspond to something you remember from the movie, that's because (a) I took it from the book, (b) I changed it for the purpose of the exam, or (c) I just made it up. Deal with it as presented here.

Average: 77.1; std.dev.: 13.5; high score: 99

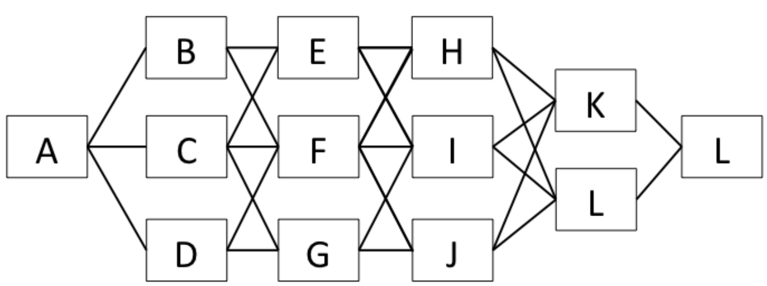
(1) Mark Watney is marooned on Mars and has a number of tasks he has to accomplish to survive long enough to get rescued. Being well-trained as an aerospace engineer as well as a botanist, his first thought is to list all of his tasks and their precedents so he can use Systems Engineering techniques to plan his work and monitor his performance.

Task	Preceding tasks	Time (sols)
A	–	10
B	A	20
C	A	30
D	A	10
E	B, C	30
F	B, C, D	20
G	C, D	10
H	E, F	10
I	E, F, G	30
J	F, G	20
K	H, I, J	20
L	H, I , J	30
M	K, L	20

(a) How many sols does it take to accomplish all of his tasks?
150 sols

(b) What are the tasks on the critical path?
A–C–E–I–L–M

(c) Draw Watney's PERT chart (you might want to do this before answering (a) and (b))



(2) Immediately after performing an emergency ascent and rendezvous with Hermes, the crew of Ares III performs a burn to put themselves in a Hohmann orbit to Earth. Useful parameters: $\mu_{Sun} = 1.327 \times 10^{11}$ km^3/sec^2, μ_{Earth}=389,604 km^3/sec^2, r_{EO}=149,500,000 km, r_{MO}=228,000,000 km, r_{Earth}=6378 km. ("EO" and "MO" refer to the radius of Earth and Mar's orbits around the sun, respectively.)

(a) How long (in days) will it take the Hermes to arrive at Earth?

$$a = \frac{1}{2}(r_{EO} + r_{MO}) = \frac{1}{2}(149,500,000 + 228,000,000) = 188,750,000\ km$$

$$P = 2\pi\sqrt{\frac{a^3}{\mu_{Sun}}} = 2\pi\sqrt{\frac{188,750,000^3}{1.327 \times 10^{11}}} = 44,728,000\ sec = 517.7\ days$$

The Hermes will take half an orbital period to return to Earth, so t_{flight}= $\boxed{\textit{22,364,000 sec=258.8 days}}$

(b) What will the perihelion velocity be at Earth?

$$v_{p,arrival} = \sqrt{\frac{\mu_{Sun}}{r_{EO}}}\left(\sqrt{\frac{2r_{MO}}{r_{EO} + r_{MO}}}\right) = \sqrt{\frac{1.327 \times 10^{11}}{149,500,000}}\left(\sqrt{\frac{2(228,000,000)}{149,500,000 + 228,000,000}}\right)$$

$$= 29.79(1.0991) = \boxed{32.74\ \frac{km}{sec}}$$

(c) In the patched conic method of multibody orbital mechanics, the perihelion velocity for a Hohmann transfer is the same as the hyperbolic excess velocity v_h. *Yeah, okay, it's definitely not the same. The hyperbolic excess velocity v_h is the difference between the perihelion velocity of the spacecraft upon Earth arrival and the circular orbital velocity of the Earth around the sun. This was a very poorly phrased question, and I'm going to give everyone full credit for this part of the question, and extra credit if you got it right regardless.* If the Hermes wanted to enter a circular low Earth orbit at an altitude of 500 km, what would the required Δv be for an impulsive braking maneuver?

$$v_h = v_{p,arrival} - v_{EO} = v_{p,arrival} - \sqrt{\frac{\mu_{Sun}}{r_{EO}}} = 32.74 - \sqrt{\frac{1.327 \times 10^{11}}{1.495 \times 10^8}} = 2.949\ \frac{km}{sec}$$

$$v_{arrival} = \sqrt{v_h^2 + \frac{2\mu_{Earth}}{r_{LEO}}} = \sqrt{2.949^2 + \frac{2(398604)}{6378 + 500}} = 11.16\ \frac{km}{sec}$$

$$v_{LEO} = \sqrt{\frac{\mu_{Earth}}{r_{LEO}}} = \sqrt{\frac{398604}{6378 + 500}} = 7.613\ \frac{km}{sec}$$

$$\Delta v = v_{arrival} - v_{LEO} = 11.16 - 7.613 = \boxed{3.547\ \frac{km}{sec}}$$

(d) Instead, the discovery that Watney is still alive means that Hermes must get back to Mars as quickly as possible. Rather than perform another Hohmann transfer, Hermes will not slow down into Earth orbit, but will perform an impulsive maneuver at Earth perigee which puts it on a trajectory with perihelion at Earth's orbit and which intersects Mars' orbit at the end of the parameter of the ellipse (or "semi-latus rectum", or the point where $\theta = \pi/2$ in the orbit. Find the eccentricity e and semi-major axis a for this new orbit.

$$r_p = r_{EO} = a(1-e);\ p = r_{MO} = a(1-e^2)$$

$$\frac{p}{r_p} = \frac{r_{MO}}{r_{EO}} = 1 + e \implies e = \frac{r_{MO}}{r_{EO}} - -1 = \frac{228}{149.5} - 1 = \boxed{0.5251}$$

$$a = \frac{r_{EO}}{1-e} = \frac{149,500,000}{1-0.5251} = \boxed{314{,}790{,}000\ km}$$

(e) What Δv at Earth encounter is required to put the Hermes into the new orbit? Thought question (not really looking for rigorous analysis here): Would you rather do this maneuver as close to Earth as possible, or as far away as possible? (Note: I'm not considering gravitational "slingshot" maneuvers here, just basic dynamics.) *This is another very poorly phrased question, as the text implies the whole question is the "thought question", although I did want the numerical answer. Again, full credit for this question and extra credit if you got it right anyway.*

$$v_{p,depart} = \sqrt{\mu_{Sun}\left(\frac{2}{r_{EO}} - \frac{1}{a}\right)} = \sqrt{1.327\times10^{11}\left(\frac{2}{1.495\times10^8} - \frac{1}{3.1479\times10^8}\right)} = 36.79\ \frac{km}{sec}$$

In (d) above, we found the velocity of the Hermes at arrival in LEO. If, instead of braking into LEO it accelerates to the desired $v_{p,departure}$ to get onto the return orbital trajectory, it would have to reach a velocity in LEO of

$$v_{h,depart} = v_{depart} - \sqrt{\frac{\mu_{Sun}}{r_{EO}}} = 36.79 - \sqrt{\frac{1.327\times10^{11}}{1.495\times10^8}} = 6.997\ \frac{km}{sec}$$

The required velocity at LEO altitude to achieve this hyperbolic excess velocity is

$$v_{depart} = \sqrt{v_{h,depart}^2 + \frac{2\mu_{Earth}}{r_{LEO}}} = \sqrt{6.997^2 + \frac{2(398604)}{6878}} = 12.84\ \frac{km}{sec}$$

$$\Delta v = v_{depart} - v_{arrival} = 12.84 - 11.16 = \boxed{1.680\ \frac{km}{sec}}$$

Note that if this were just a maneuver at perihelion and the Earth was out of the picture, the Δv would be $v_{p,depart} - v_{p,arrival}$=36.79-32.74=4.05 km/sec. This shows the great advantage of the "Oberth" effect of performing the maneuver at a higher energy state low in Earth's gravity well.

(f) How long will it take for the Hermes to return to Mars on this new trajectory? *At arrival at Mars, $\theta = \pi/2$ or 90^o. Since we know e, we can calculate the eccentric anomaly E as*

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2} = \sqrt{\frac{1-0.5251}{1+0.5251}}\tan\frac{\pi}{4} = 0.5580 \implies E = 1.018\ rad$$

$$n = \sqrt{\frac{\mu_{Sun}}{r_{EO}^3}} = \sqrt{\frac{1.327\times10^{11}}{(1.495\times10^8)^3}} = 1.992\times10^{-7}\ \frac{rad}{sec}$$

$$t = \frac{1}{n}(E - e\sin E) = \frac{1.018 - 0.5251\sin 1.018}{1.992\times10^{-7}} = \boxed{2{,}865{,}800\ sec{=}33.17\ days}$$

(3) The water recycler in the Ares III habitat was designed for a 30-sol mission with a reliability of 99.99%. A sol is approximately 24.5 hours long.

(a) What MTBF would be required (in hours) to achieve this reliability?

$$r = e^{-\frac{t}{MTBF}} \implies MTBF = -\frac{t}{\ln r} = -\frac{30(24.5)}{\ln .9999} = \boxed{7{,}350{,}000\ hrs}$$

(b) Mark Watney had to survive on Mars for 549 sols. What was the probability that the water recycler would work all that time without a failure?
549 sols is 13,450 hours

$$R = e^{-t/MTBF} = e^{-\frac{24.5(549)}{7350000}} = \boxed{99.82\%}$$

(4) Watney's rover has six wheels, each of which has a drive motor.

(a) If the reliability of each wheel motor is 99%, what is the overall reliability of the rover if all six wheels must be functional for the rover to work?

$$R_{rover} = R_{wheel}^6 = 0.00^6 = \boxed{0.9415}$$

(b) How does your answer to (a) change if the rover can drive with one failed wheel?

$$R_{rover} = R_{wheel}^6 + 6R_{wheel}^5(1 - R_{wheel}) = 0.99^6 + 6(0.99)^5(0.01) = 0.9415 + 0.0571 = \boxed{0.9985}$$

(c) How does your answer to (b) change if the rover can drive with one failed wheel or one on each side, but not two failures on the same side?

$$R_{left} = R_{right} = R_{wheel}^3 + 3R_{wheel}^2(1 - R_{wheel}) = 0.99^3 + 3(0.99)^2(0.01) = 0.97030.0294 = 0.9997$$

$$R_{rover} = R_{left}R_{right} = 0.9997^2 = \boxed{0.9994}$$

(d) How does your answer to (b) [not (c)!] change if there is a 30% intercorrelated failure rate?

$$R_{rover} = R_{wheel}^6 + 6R_{wheel}^5(1 - R_{wheel})(1 - f) = 0.99^6 + 6(0.99)^5(0.01)(0.7) = 0.9415 + 0.0571(0.7) = \boxed{0.9815}$$

(5) The MAV (Mars Ascent Vehicle) that the Ares missions use has an empty mass (inert and payload masses together) of 12,600 kg and a propellant mass of 19,397 kg.

(a) To reach the nominal low Martian orbit (LMO) for return to Hermes, the MAV must accomplish a Δv of 4100 m/sec. What specific impulse is required from the propellant for this maneuver?

$$r = \frac{m_{final}}{m_{initial}} = e^{-\frac{\Delta v}{gI_{sp}}} \implies I_{sp} = -\frac{\Delta v}{g \ln\left(\frac{m_{final}}{m_{initial}}\right)} = -\frac{4100}{9.8 \ln\left(\frac{12600}{12600+19397}\right)} = \boxed{448.9\ sec}$$

(b) To rendezvous with Ares in a hyperbolic fly-by maneuver, the MAV must instead achieve a Δv of 5800 m/sec. How much inert mass must Watney remove from the MAV to accomplish this performance?
Even if you didn't calculate the I_{sp} in the previous problem, you can solve this using the rocket equation for the two cases.

$$\frac{\Delta v_1}{\Delta v_2} = \frac{\ln r_1}{\ln r_2} \implies \ln r_2 = \frac{\Delta v_2}{\Delta v_1} \ln r_1 = \frac{5800}{4100} \ln 0.3938 = -1.3183 \implies r_2 = 0.2676$$

$$r = \frac{m_{in}}{m_{in} + m_{pr}} \implies m_{in2} = \frac{r}{1-r} m_{pr} = \frac{0.2676}{1-0.2676} 19,397 = 7087\ kg$$

$$\Delta m_{in} = 12600 - 7087 = \boxed{5513\ kg}$$

(6) The MAV empty mass, as noted earlier, is 12,000 kg. *Okay, as noted earlier, it was 12,600 kg. Mea culpa - I should have proofread more closely. I'll stick with the number I gave you here.* Using the Arney and Wilhite cost estimating relations for cryogenic ascent vehicle nonrecurring and first unit production costs (in \$M2012)

$$C_{NR} = 405.62 m_{empty}^{0.2151}; \quad C_{R1} = 92.715 m_{empty}^{0.1606}$$

(a) What is the nonrecurring cost for the MAV?

$$C_{NR} = 405.62 m_{empty}^{0.2151} = 405.62(12000)^{0.2151} = \boxed{\$3.0588\text{B}}$$

(b) What is the cost for the first unit?

$$C_{R1} = 92.715 m_{empty}^{0.1606} = 92.715(12000)^{0.1606} = \boxed{\$419.1\text{M}}$$

(c) At an 83% learning curve rate, what are the costs for units 2, 3, 4, and 5?

$$p = \frac{\ln LC\%}{\ln 2} = \frac{\ln 0.83}{\ln 2} = -0.2688$$

$$C_{Rn} = C_{R1} n^p \implies C_{R2} = C_{R1} 2^p = (419.1)2^{-0.2688} = \boxed{\$347.9\text{M}}$$

$$C_{R3} = (419.1)3^{-0.2688} = \boxed{\$311.9\text{M}} \quad C_{R4} = (419.1)4^{-0.2688} = \boxed{\$288.7\text{M}}$$

$$C_{R5} = (419.1)5^{-0.2688} = \boxed{\$271.9\text{M}}$$

(d) Assume all nonrecurring costs for the MAV are paid in year 0, and individual vehicle production costs are paid every four years with the missions which occur in years 4, 8, 12, 16, and 20. What is the total net present value for the entire cost of the MAV program at a 10% discount rate?

$$NPV_{Total} = C_{NR} + C_{R1}(1+r)^{-4} + C_{R2}(1+r)^{-8} + C_{R3}(1+r)^{-12} + C_{R4}(1+r)^{-16} + C_{R5}(1+r)^{-20}$$

$$= 3058.8 + 419.1(1.1)^{-4} + 347.9(1.1)^{-8} + 311.9(1.1)^{-12} + 288.7(1.1)^{-16} + 271.9(1.1)^{-20}$$

$$= 3058.8 + 286.3 + 162.3 + 99.4 + 62.8 + 40.4 = \boxed{\$3.710B}$$

Extra credit

(7) The following questions relate to movies that, as space-oriented aerospace engineers, you should have seen at least once.

from *2001: A Space Odyssey*

(a) There is a shot of Heywood Floyd, on his way to the Moon, intently studying a 10-item checklist. What is the checklist for?

Flushing the toilet

(b) How did the flight attendants get around in microgravity?

Velcro shoes

(c) How many wheels were on the lunar transport vehicle that took Heywood to see the monolith?

None - it used rocket engines to "fly" to the site

from *Silent Running*

(d) What were the spaceship *Valley Forge* and the other spaceships in the fleet doing at the beginning of the movie?

Carrying plants from a polluted Earth

(e) What were the names of the three droids? (Actually, full credit for the name of any of the droids...)

Huey, Dewey, and Louie, but "little Dewey isn't with us anymore..."

from *The Right Stuff*

(f) Who did they suggest using as astronauts just before President Eisenhower insisted on test pilots?

Circus performers

from *Apollo 13*

(g) What is not an option?

Failure, of course

from reality

(h) What was yesterday the 15th anniversary of?

The last time there were no humans in space

(or, the start of continuous habitation of International Space Station.)

(i) Name three astronauts who flew on the space shuttle

Since there were 355 of them, I won't list them all here.

(j) Name someone who walked on the moon who did not fly on Apollo 11

Pete Conrad, Alan Bean, Alan Shepard, Edgar Mitchell, Dave Scott,

Jim Irwin, John Young, Charlie Duke, Gene Cernan, Jack Schmidt