ENAE 483/788D FINALS – FALL, 2017 – *NAME:*

One 8.5" x 11" piece of paper allowed for notes (both sides). No Internet-enabled devices allowed. Put your name on the cover page, and on each page if you disassemble the quiz package. Please write neatly, and put boxes around your answers.

Some possibly useful numbers:

$\mu_{Earth} = 398,604\ \frac{km^3}{sec^2}$, $r_{Earth} = 6378\ km$, $\mu_{Mars} = 42,970\ \frac{km^3}{sec^2}$, $r_{Mars} = 3393\ km$

(1) You're designing a spherical composite pressure vessel for use as a cryogenic fuel storage tank. This carries with it a NASA-mandated factor of safety of 1.5 applied to the maximum design pressure. The maximum design pressure will be 135 psi, the diameter is 8 feet, and the thickness 0.0625 inches. The ultimate tensile strength of the composite material used is 145 ksi.

(a) Calculate the hoop stress

$$\sigma_{hoop} = \frac{Pr}{t} = \frac{135(4 \times 12)}{0.0625} = \boxed{103.7 \text{ ksi}}$$

(b) Calculate the margin of safety for this tank and comment on what you find

$$MS = \frac{\sigma_{allowable}}{\sigma_{applied} \times FOS} - 1 = \frac{145}{103.7 \times 1.5} - 1 = \boxed{\text{-0.0678}}$$

The tank design has a $\boxed{\textit{negative margin of safety}}$*, which indicates that it is underdesigned (slightly)*

(2) In the Apollo-Soyuz Test Project (ASTP) mission, the crew had to transfer from the Soyuz spacecraft (14.7 psi, 21% O_2) to the Apollo spacecraft (5 psi, 100% O_2).

(a) What was the R-value for this transfer?

$$R = \frac{ppN2(before)}{P_{tot}(after)} = \frac{0.79(14.7)}{5} = \boxed{2.323}$$

(b) At the same cabin pressure, what would the percentage of oxygen have to be in the Soyuz spacecraft to make the R-value equal to 1.4?

$$ppN2 = R(P_{tot}) = 1.4(5) = 7\ psi \Longrightarrow ppO2 = 14.7 - 7 = 7.7\ psi \Longrightarrow O_2\% = \frac{7.7}{14.7} = \boxed{52.38\%}$$

(c) What pressure would you have to use in a spacesuit (100% O2) to exit the Soyuz spacecraft at an R value of 1.4 without prebreathing?

$$R = \frac{ppN2(before)}{P_{tot}(after)} \Longrightarrow P_{tot}(after) = \frac{ppN2(before)}{R_{desired}} = \frac{0.79(14.7)}{1.4} = \boxed{8.295 \text{ psi}}$$

(3) The Borg ship from the midterm exam has reappeared. It's still a cube 1km on a side. Through advanced technologies, it behaves thermally as an isothermal black body. The Borg are humanoid, so they would like to maintain an internal temperature of 295K.

(a) The Borg ship starts out in deep space. What is the limit on the internal power generated to maintain an equilibrium temperature of 295K?

$$P_{int} = \sigma A T^4 = 5.67 \times 10^{-8}(6 \times 10^6)(295)^4 = \boxed{2.576{\times}10^9 \text{ W}}$$

(b) The Borg ship moves into solar orbit at 1 AU (I_s=1394 W/m^2). How does their internal power limit change due to the additional heat input from the Sun? (Note: they are creatures of habit, and always keep one face of the cube perpendicular to the incoming sunlight.)

$$P_{int} + AI_s = \sigma A_{rad} T^4 \Longrightarrow P_{int} = \sigma A_{rad} T^4 - AI_s$$

$$= 5.67 \times 10^{-8}(6 \times 10^6)(295)^4 - 1 \times 10^6(1394) = 2.576 \times 10^9\ W = \boxed{1.182{\times}10^9 \text{ W}}$$

(c) How close can they come to the Sun (in AU) before they have to shut down all internal power to maintain their 295K equilibrium temperature?

$$AI_{s,max} = \sigma A_{rad} T^4 \Longrightarrow I_{s,max} = \sigma \frac{A_{rad}}{A_s} T^4 = 5.67 \times 10^{-8} \frac{6 \times 10^6}{1 \times 10^6}(295)^4 = 2576\ W/m^2$$

$$\frac{I_{s,max}}{I_{s,Earth}} = \left(\frac{r_{Earth}}{r_{min}}\right)^2 \Longrightarrow r_{min} = \sqrt{\frac{I_{s,Earth}}{I_{s,max}}} r_{Earth} = \sqrt{\frac{1394}{2576}} 1 = \boxed{0.7356 \text{ AU}}$$

(4) The Merlin 1D rocket engine on the first stage of a SpaceX Falcon 9 has a combustion chamber pressure of 9.7 MPa and an exit pressure of 65.7 KPa. The ratio of specific heats γ in the exhaust is 1.2. The exhaust velocity at ideal conditions ($P_e = P_a$) is 2770 m/sec.

(a) Calculate the expansion ratio of this engine's nozzle

$$\frac{A_t}{A_e} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}} \left(\frac{p_e}{p_0}\right)^{\frac{1}{\gamma}} \sqrt{\frac{\gamma+1}{\gamma-1}\left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{A_t}{A_e} = \left(\frac{2.2}{2}\right)^{\frac{1}{0.2}} \left(\frac{65.7}{9700}\right)^{\frac{1}{1.2}} \sqrt{\frac{2.2}{0.2}\left[1 - \left(\frac{65.7}{9700}\right)^{\frac{0.2}{1.2}}\right]}$$

$$= (1.611)(0.01557)(2.493) = 0.06252 \Longrightarrow \frac{A_e}{A_t} = \boxed{15.99}$$

(b) The Merlin 1Dvac (upper stage engine) is the same as the first-stage Merlin 1D, but has an expansion ratio A_e/A_t=165, which results in an exit pressure of 3.41 KPa. How would the engine's exhaust velocity change with the new nozzle, if all other conditions (including ideal exit matching conditions $P_e = P_a$) in the engine were held constant?

$$V_e \propto \sqrt{1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$V_{e2} = V_{e1}\sqrt{\frac{1-\left(\frac{p_{e2}}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}{1-\left(\frac{p_{e1}}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}} = 2770\sqrt{\frac{1-\left(\frac{3.41}{9700}\right)^{\frac{0.2}{1.2}}}{1-\left(\frac{65.7}{9700}\right)^{\frac{0.2}{1.2}}}} = 2770\sqrt{\frac{0.7343}{0.5650}} = \boxed{3158 \text{ m/sec}}$$

(5) Newly made-up requirements for the Solar Electric Propulsion (SEP) stage in RASC-AL have it entering an areosynchronous orbit – that is, an orbit with a period of one sol around Mars. A sol is 24h39m.

(a) If the Mars orbit is circular, what is the altitude of the orbit above Mars' surface?

$$P = 2\pi\sqrt{\frac{a^3}{\mu_{Mars}}} \Longrightarrow a = \left[\left(\frac{P}{2\pi}\right)^2 \mu_{Mars}\right]^{\frac{1}{3}}$$

$$P = a = \left[\left(\frac{88,740}{2\pi}\right)^2 42970\right]^{\frac{1}{3}} = 20,465\ km\ radius \Longrightarrow \boxed{\text{17,072 km altitude}}$$

(b) Your SEP stage is designed to provide 750 kW of electrical power when at Earth's distance from the sun (1 AU). Mars is currently at aphelion at a solar distance of 1.67 AU. What is the power output of your arrays now?

$$P_{Mars} = P_{Earth}\left(\frac{r_{Earth}}{r_{Mars}}\right)^2 = 750\left(\frac{1}{1.67}\right)^2 = \boxed{268.9 \text{ kW}}$$

(c) Your nominal concept of operations is to use your (high thrust) chemical propulsion system to depart Mars orbit, then switch over to the electrical propulsion system when you have reached Mars escape. What Δv is required for this high-thrust maneuver?

$$\Delta v_{esc} = \sqrt{\frac{2\mu}{r}} - \sqrt{\frac{\mu}{r}} = \left(\sqrt{2}-1\right)\sqrt{\frac{\mu}{r}} = \left(\sqrt{2}-1\right)\sqrt{\frac{42970}{20465}} = \boxed{0.6002 \text{ km/sec}}$$

(d) Your chemical propulsion system has failed, and you will have to use your electric propulsion system to depart Mars orbit. In your current orbit, 5.3% of your orbital period is spent in the shadow of Mars. To efficiently depart Mars under low thrust, you need to accelerate continuously at a constant value, including nighttime periods. You will have to choose a power level that will allow you to thrust at constant power, while recharging batteries during the day for the night passes. Assume the charging and discharge processes in your energy storage system are perfectly efficient. What is the maximum power you can use for the propulsion system while departing Mars orbit?

Assume the generating power is p_{gen} and the steady-state operating power is P_{ops}. We need to equate the energy generated per orbit with the energy used:

$$P_{gen}(1-0.053) = P_{ops}(1) \Longrightarrow P_{ops} = P_{gen}(0.947) = 268.9(0.947) = \boxed{254.6 \text{ kW}}$$

(e) What Δv will be required for this low-thrust Mars departure maneuver?

$$\textit{For low-thrust escape, } \Delta v = v_c = \sqrt{\frac{\mu_{Mars}}{r}} = \sqrt{\frac{42970}{20465}} = \boxed{1.449\ \frac{km}{sec}}$$

(6) A MAV (Mars Ascent Vehicle) to return the crew and samples from Mars' surface to low Mars orbit has an empty mass (inert and payload masses together) of 10,000 kg and a propellant mass of 25,000 kg.

(a) To reach a nominal low Martian orbit (LMO) from the surface, the MAV must accomplish a Δv of 4100 m/sec. What specific impulse is required from the propellant for this maneuver?

$$r = \frac{m_{final}}{m_{initial}} = e^{-\frac{\Delta v}{g I_{sp}}} \Longrightarrow I_{sp} = -\frac{\Delta v}{g \ln\left(\frac{m_{final}}{m_{initial}}\right)} = -\frac{4100}{9.8 \ln\left(\frac{10000}{10000+25000}\right)} = \boxed{334.0\ sec}$$

(b) Instead, your SEP stage is in a 5-sol orbit, which will require 5200 m/sec Δv from the surface to rendezvous. How much less payload can you accommodate on the MAV, all other things being the same?

$$r = e^{-\frac{\Delta v}{g I_{sp}}} = e^{-\frac{5200}{9.8(334)}} = 0.2042 = \frac{m_{final}}{m_{final} + m_{pr}} = \frac{m_{final}}{m_{final} + 25000}$$

$$m_{final} = 25000 \frac{0.2042}{0.7958} = 6415 \Longrightarrow \Delta m_{pay} = \boxed{\text{-3585 kg}}$$

(7) Assume the MAV inert mass is 5500 kg. Using the Arney and Wilhite cost estimating relations for an ascent vehicle with storable propellants, nonrecurring and first unit production costs can be estimated to be (in \$M2012)

$$C_{NR} = 405.62 m_{in}^{0.2151}; \quad C_{R1} = 66.129 m_{in}^{0.1606}$$

(a) What are the nonrecurring and first unit production costs for the MAV?

$$C_{NR} = 405.62 m_{empty}^{0.2151} = 405.62(5500)^{0.2151} = \boxed{\$2.586\text{B}}$$

$$C_{R1} = 92.715 m_{empty}^{0.1606} = 92.715(5500)^{0.1606} = \boxed{\$369.7\text{M}}$$

(b) At an 82.5% learning curve rate, what is the recurring cost for the fifth unit?

$$p = \frac{\ln LC\%}{\ln 2} = \frac{\ln 0.825}{\ln 2} = -0.2775$$

$$C_{R5} = (369.7)5^{-0.2775} = \boxed{\$236.5\ \text{M}}$$