**ENAE 483/788D MIDTERM – FALL, 2018 – *NAME:***

*One 8.5" x 11" piece of paper allowed for notes (both sides). No Internet-enabled devices allowed. Put your name on the cover page, and on each page if you disassemble the quiz package. Please write neatly, and put boxes around your answers. Every question or subquestion is worth 5 points.*

*Some possibly useful numbers:*

$\mu_{Earth} = 398,604 \ \frac{km^3}{sec^2}$, $r_{Earth} = 6378 \ km$,

$\mu_{Mars} = 42,970 \ \frac{km^3}{sec^2}$, $r_{Mars} = 3393 \ km$

$\mu_{Moon} = 4667.9 \ \frac{km^3}{sec^2}$, $r_{Moon} = 1738 \ km$

(1) We would like to consider the possibility of using an areosynchronous orbit - that is, a circular orbit that would remain above a single point on Mars' equator. The length of a Mars day is 24h39m33s.

(a) What is the radius of this orbit?

$$P = 2\pi\sqrt{\frac{a^3}{\mu_{Mars}}} \Longrightarrow a = \left[\left(\frac{P}{2\pi}\right)^2 \mu_{Mars}\right]^{\frac{1}{3}}$$

$$P = a = \left[\left(\frac{88,773}{2\pi}\right)^2 42970\right]^{\frac{1}{3}} = \boxed{\text{20,470 km}}$$

(b) If you inserted into a circular orbit which was one km too high, how far would you have to move along the equator to keep the spacecraft directly overhead after one sol? In which direction?

$$P = 2\pi\sqrt{\frac{a^3}{\mu_{Mars}}} = 2\pi\sqrt{\frac{20471^3}{42970}} = 88,778 \ sec$$

*So after one sol, the subspacecraft point would have gone* $\frac{88773}{88778} = 99.9943\%$ *of the way around, so it would be*

$$2\pi r_{Mars}\left(1 - \frac{88773}{88778}\right) = \boxed{\text{1.221 km West}} \ \textit{of the original starting point}$$

(2) It was decided instead that the initial parking orbit would be circular with an altitude of 17,000 km. You would like to perform a Hohmann transfer from this orbit to the orbit of Phobos ($r_{Phobos} = 9376$ km).

(a) If you are using a low-thrust stage to do this maneuver, what is the $\Delta v$ required?

$$r_{parking} = h_{parking} + r_{Mars} = 17000 + 3393 = 20,393 \ km$$

$$\Delta v = |v_{c,Phobos} - v_{c,parking}| = \left|\sqrt{\frac{\mu_{Mars}}{r_{Phobos}}} - \sqrt{\frac{\mu_{Mars}}{r_{parking}}}\right| = \left|\sqrt{\frac{42970}{9376}} - \sqrt{\frac{42970}{20393}}\right| = \boxed{0.6892 \ \frac{km}{sec}}$$

(b) If you use a high-thrust stage to do a transfer from your initial orbital radius to Phobos' orbit, what $\Delta v$ will this require?

$$v_{c,parking} = \sqrt{\frac{\mu_{Mars}}{r_{parking}}} = \sqrt{\frac{42970}{20393}} = 1.452 \ \frac{km}{sec}$$

$$v_a = v_{c,parking}\sqrt{\frac{2r_{Phobos}}{r_{Phobos}+r_{parking}}} = 1.452\sqrt{\frac{2\times 9376}{9376+20393}} = 1.152\ \frac{km}{sec}$$

$$\Delta v_1 = v_{c,parking} - v_a = 1.452 - 1.152 = 0.2999\ \frac{km}{sec}$$

$$v_{c,Phobos} = \sqrt{\frac{\mu_{Mars}}{r_P hobos}} = \sqrt{\frac{42970}{9376}} = 2.141\ \frac{km}{sec}$$

$$v_p = v_{c,Phobos}\sqrt{\frac{2r_{parking}}{r_{parking}+r_{Phobos}}} = 1.353\sqrt{\frac{2\times 20393}{20393+9376}} = 2.506\ \frac{km}{sec}$$

$$\Delta v_2 = v_a - v_{c,Phobos} = 1.622 - 1.353 = 0.3650\ \frac{km}{sec}$$

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 = 0.2999 + 0.3650 = \boxed{0.6649\ \frac{km}{sec}}$$

(3) You need to design a system to bring a payload consisting of a 5000 kg crew module from low lunar orbit (LLO) to a landing on the lunar surface (LS), and return it to LLO.
$\Delta v_{landing} = 2706$ m/sec, $\Delta v_{launch} = 2334$ m/sec, $I_{sp} = 445$ sec, $\delta = 0.1$ for all vehicles

(a) We will start with expendable stages for landing and launch. Calculate $m_o$, $m_{in}$, and $m_{prop}$ for the stage to launch the payload from the lunar surface back into lunar orbit.

$$r_{ascent} = e^{-\frac{\Delta v_{launch}}{gI_{sp}}} = e^{-\frac{2334}{9.8(445)}} = 0.5856$$

$$\lambda_{ascent} = r_{ascent} - \delta = 0.5856 - 0.1 = 0.4856$$

$$m_o = \frac{m_{PL}}{\lambda_{ascent}} = \frac{5000}{0.4856} = \boxed{10{,}298 \text{ kg}}$$

$$m_{in} = \delta m_o = 0.1(10298) = \boxed{1030 \text{ kg}}$$

$$m_{prop} = m_o(1 - r_{ascent} = 10298(1 - 0.5856) = \boxed{4267 \text{ kg}}$$

(b) Calculate $m_o$, $m_{in}$, and $m_{prop}$ for the stage to land both the payload and the fully fueled ascent stage on the lunar surface

$$r_{descent} = e^{-\frac{\Delta v_{landing}}{gI_{sp}}} = e^{-\frac{2706}{9.8(445)}} = 0.5377$$

$$\lambda_{descent} = r_{descent} - \delta = 0.5377 - 0.1 = 0.4377$$

$$m_o = \frac{m_{PL}}{\lambda_{descent}} = \frac{10298}{0.4377} = \boxed{23{,}529 \text{ kg}}$$

$$m_{in} = \delta m_o = 0.1(23529) = \boxed{2353 \text{ kg}}$$

$$m_{prop} = m_o(1 - r_{descent} = 23529(1 - 0.5377) = \boxed{10{,}877 \text{ kg}}$$

(c) Alternately, we would like to consider a reusable vehicle which both lands and launches from the lunar surface. Carrying the same 5000 kg payload both ways, calculate $m_o$, $m_{in}$, and $m_{prop}$ for this reusable propulsion stage.

$$\Delta v_{total} = \Delta v_{landing} + \Delta v_{launch} = 2334 + 2706 = 5040\ m/sec$$

$$r_{round\ trip} = e^{-\frac{\Delta v_{total}}{gI_{sp}}} = e^{-\frac{5040}{9.8(445)}} = 0.3148$$

$$\lambda_{total} = r_{round\ trip} - \delta = 0.3148 - 0.1 = 0.2148$$

$$m_o = \frac{m_{PL}}{\lambda_{total}} = \frac{5000}{0.2148} = \boxed{23{,}273\text{ kg}}$$

$$m_{in} = \delta m_o = 0.1(23273) = \boxed{2327\text{ kg}}$$

$$m_{prop} = m_o(1 - r_{round\ trip}) = 23273(1 - 0.3148) = \boxed{15{,}947\text{ kg}}$$

(d) There will be times we want to land payload (like, say, a habitat or rover) on the lunar surface which does not return to orbit. Calculate the maximum payload the vehicle you designed in (3)(c) can carry to the lunar surface if it ascends with no payload.

*To launch empty from the moon back to LLO, you only have inert and propellant masses.*

$$r_{ascent} = \frac{m_{in}}{m_{in} + m_{pr,ascent}} \implies m_{pr,ascent} = \frac{1 - r_{ascent}}{r_{ascent}} m_{in} = \frac{1 - 0.5856}{0.5856} 2327 = 1647\ kg$$

$$r_{descent} = \frac{m_{PL} + m_{in} + m_{pr,ascent}}{m_{PL} + m_{in} + m_{pr}} \implies m_{PL} = \frac{r_{descent}}{1 - r_{descent}} m_{pr} - \frac{1}{1 - r_{descent}} m_{pr,asc} - m_{in}$$

$$m_{PL} = \frac{0.5377}{1 - 0.5377} 15947 - \frac{1}{1 - 0.5377} 1649 - 2353 = \boxed{12{,}628\text{ kg}}$$

(4) You need to evaluate the safety of using long-range rovers on the Moon.

(a) Assume the rover has a 0.9999 reliability for each km it travels. Find the MKBF (Mean Kilometers Between Failures) for the rover. [*Hint:* it's where your chance of still being functional is $1/e$.]

$$\frac{1}{e} = R^{MKBF} \implies MKBF = \frac{-1}{\ln R} = \frac{-1}{\ln 0.9999} = \boxed{9999.5\text{ km}}$$

(b) If you have a requirement of 0.999 chance of crew survival, how far can you allow the rover to drive?

$$P = e^{-\frac{d}{MKBF}} \implies d = -lnPMKBF = -ln0.9999999.5 = \boxed{10.00\text{ km}}$$

(c) Let's assume you have two rovers, and either can successfully bring the entire crew back. What is the required reliability for each rover to have an aggregate total reliability of 0.999?

$$R = 1 - (1 - P)^2 \implies P = 1 - \sqrt{1 - R} = 1 - \sqrt{0.001} = \boxed{0.9684}$$

(d) Given your answer to (4)(c), how far can you let the rovers drive now before your crew risk exceeds the allowed 0.999?

$$d = -lnPMKBF = -ln0.96849999.5 = \boxed{321.3 \text{ km}}$$

(e) You plan to have three rovers on the Moon, with at least one required to ensure crew survival under all conditions. If each rover is 90% reliable, what is the probability of loss of crew?

$$P(3\ operational) = R^3 = 0.9^3 = 0.7290$$
$$P(2\ operational) = 3R^2(1-R) = 3(0.9^2)(1-0.9) = 0.2430$$
$$P(1\ operational) = 3R(1-R)^2 = 3(0.9)(1-0.9)^2 = 0.0270$$
$$P(at\ least\ 1\ operational) = P(3\ operational) + P(2\ operational) + P(1\ operational) = \boxed{0.9990}$$

(f) How does your answer to (4)(e) change if there is a 10% intercorrelation rate?

$$P(3\ operational) = R^3 = 0.9^3 = 0.7290$$
$$P(2\ operational) = 3R^2(1-R)(1-f) = 3(0.9^2)(1-0.9)(1-0.1) = 0.2187$$
$$P(1\ operational) = 3R(1-R)^2(1-f)^2 = 3(0.9)(1-0.9)^2(1-0.1)^2 = 0.0219$$
$$P(at\ least\ 1\ operational) = P(3\ operational) + P(2\ operational) + P(1\ operational) = \boxed{0.9696}$$

(5) You need to estimate the cost for the crew module from question (3). According to Arney and Wilhite, the nonrecurring and first unit production costs of a crew module can be estimated by

$$c_{NR} < \$M >= 285.57(m < kg >)^{0.2667}; \quad c_{R1} < \$M >= 49.923(m < kg >)^{0.2409}$$

(a) Calculate the nonrecurring cost of the crew module
*From question (3), the crew module (=payload) = 5000 kg*

$$c_{NR} < \$M >= 285.57(5000)^{0.2667} = \boxed{\$2768\text{M}}$$

(b) Calculate the first unit production cost of the crew module

$$c_{R1} < \$M >= 49.923(5000)^{0.2409} = \boxed{\$388.5\text{M}}$$

(c) Assuming a 78% learning curve, calculate the production costs for units 2 through 4

$$p = \frac{\ln LC}{\ln 2} = \frac{\ln 0.78}{\ln 2} = -0.3585$$
$$c_{R1} = c_{R1}2^p = 388.5(2^{-0.3585}) = \boxed{\$303.0\text{M}}$$
$$c_{R3} = c_{R1}3^p = 388.5(3^{-0.3585}) = \boxed{\$265.0\text{M}}$$
$$c_{R4} = c_{R1}4^p = 388.5(4^{-0.3585}) = \boxed{\$236.4\text{M}}$$

(d) If you were selling the four crew modules to NASA, what price would you have to charge per module to make an overall 10% profit?

$$c_{tot} = c_{NR} + c_{R1} + c_{R2} + c_{R3} + c_{R4} = 2768 + 388.5 + 303.0 + 265.0 + 236.4 = \$3958M$$
$$Price = \frac{1.1c_{tot}}{N} = \frac{1.1(3958)}{4} = \boxed{\$1088\text{M/module}}$$

(6) A solar sail is in low Earth orbit at an altitude of 300 km. The vehicle is a circular disk with a diameter of 300m, and is flying such that the surface of the disk is perpendicular to the velocity vector. The density of Earth's atmosphere at this altitude can be approximated by

$$\rho = 3.875 \times 10^{-9} e^{-h<km>/59.06} \text{ (in kg/m}^3\text{)}$$

(a) What is the drag force on the solar sail?

$$\rho = 3.875 \times 10^{-9} e^{-h<km>/59.06} = 3.875 \times 10^{-9} e^{-300/59.06} = 2.411 \times 10^{-11}\ kg/m^3$$

$$In\ circular\ orbit,\ v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398604}{6378+300}} = 7726\ m/sec$$

*As a flat plate perpendicular to the velocity vector, the drag coefficient* $c_D$*=4. The area of the ship perpendicular to the velocity vector is (1 km)*$^2$*, or* $1 \times 10^6\ m^2$*.*

$$D = \frac{1}{2}\rho v^2 A c_D = \frac{1}{2} 2.411 \times 10^{-11}(7726)^2(1 \times 10^6)4 = \boxed{2878\text{ N}}$$

(b) At this altitude, the flux for orbital debris impacts by particles of a centimeter in diameter is $10^{-5}$ hits/m$^2$/year. How long, on average, could the solar sail stay in this orbit before it could expect to have an impact of this size?

$$(Flux)(Area) = 10^{-5}(6 \times 10^6) = 60\ hits/year \Longrightarrow 0.01667\ years/hit = \boxed{6.088\text{ days}}$$