ENAE 483/788D POWER, PROPULSION, AND THERMAL SPECIALTY PROBLEMS – FALL, 2020

(1) A rocket engine on the first stage of a launch vehicle has a combustion chamber pressure of 2000 psi and an exit pressure of 8 psi. The ratio of specific heats γ in the exhaust is 1.2. The exhaust velocity at ideal conditions ($P_e = P_a$) is 3300 m/sec.

(a) Calculate the expansion ratio of this engine's nozzle

$$\frac{A_t}{A_e} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}} \left(\frac{p_e}{p_0}\right)^{\frac{1}{\gamma}} \sqrt{\frac{\gamma+1}{\gamma-1}\left[1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{A_t}{A_e} = \left(\frac{2.2}{2}\right)^{\frac{1}{0.2}} \left(\frac{8}{2000}\right)^{\frac{1}{1.2}} \sqrt{\frac{2.2}{0.2}\left[1-\left(\frac{8}{2000}\right)^{\frac{0.2}{1.2}}\right]}$$

$$= (1.611)(0.01004)(2.572) = 0.04159 \Longrightarrow \frac{A_e}{A_t} = \boxed{24.04}$$

(b) If you wanted to lower the exit pressure to 1 psi, what expansion ratio would you need?

$$\frac{A_t}{A_e} = \left(\frac{2.2}{2}\right)^{\frac{1}{0.2}} \left(\frac{1}{2000}\right)^{\frac{1}{1.2}} \sqrt{\frac{2.2}{0.2}\left[1-\left(\frac{1}{2000}\right)^{\frac{0.2}{1.2}}\right]}$$

$$= (1.611)(0.001775)(2.811) = 0.008034 \Longrightarrow \frac{A_e}{A_t} = \boxed{124.5}$$

(c) How would the engine's exhaust velocity change with the new nozzle, if all other conditions (including ideal exit matching conditions $P_e = P_a$) in the engine were held constant?

$$V_e \propto \sqrt{1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$V_{e2} = V_{e1}\sqrt{\frac{1-\left(\frac{p_{e2}}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}{1-\left(\frac{p_{e1}}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}} = 3300\sqrt{\frac{1-\left(\frac{1}{2000}\right)^{\frac{0.2}{1.2}}}{1-\left(\frac{8}{2000}\right)^{\frac{0.2}{1.2}}}} = 3300\sqrt{\frac{0.7183}{0.6016}} = \boxed{3606 \text{ m/sec}}$$

(2) You are designing a base on Phobos to monitor the surface of Mars. Your solar arrays are designed to track the sun, but are eclipsed by both nighttime on Phobos and eclipsed each orbit by Mars. The orbital period of Phobos is 7 hrs 32 min (in fact, as viewed from the surface, Phobos rises in the west and sets in the east - almost three times per day!) Starting at dawn on your Phobos base, the solar arrays are illuminated for 89 min, until they are eclipsed by Mars. Darkness on the back side of Mars lasts for 51 min, then sunlight is available for 89 min until local sundown. The nighttime duration is then 230 min. The human base will need a continual power level of 60 kW.

r_{Mars}=3393 km, r_{Phobos}=9376 km, mean radius 11.27 km, P=7h39.2m

(a) How much energy storage do you need in your batteries to maintain power during dark periods?

$$Per\ orbit,\ (60\ kW)(230+51\ min) = 16,860\ kW\text{-}min = \boxed{\text{281 kW-hrs}}$$

(b) If the batteries are 90% efficient at charging (i.e., only 90% of the power you supply for recharge will be stored in the battery) and 95% efficient at discharge (i.e., 5% of the capacity of the battery is unavailable for use), how does your answer to (a) change?

$$E_{req} = 281\ kW\text{-}hrs\left(\frac{1}{0.95}\right)\left(\frac{1}{0.9}\right) = \boxed{\text{328.7 kW-hrs}}$$

(c) If the batteries have a specific energy of 250 W-hrs/kg, what is the mass of the batteries (as defined by your answer to (b))?

$$m_{batt} = \frac{328.7\ kW\text{-}hrs}{250\ W\text{-}hrs/kg} = \boxed{\text{1315 kg}}$$

(d) How much power is required from the photovoltaic arrays to charge the batteries with a 20% energy margin?

$$Daylight/orbit = 178\ min \Longrightarrow P_{recharge} = \frac{(1.2)328.7\ kW\text{-}hrs}{178\ min/60\ \frac{min}{hr}} = \boxed{\text{132.9 kW}}$$

(e) The PV arrays you have chosen have a conversion efficiency of 24% and a mass of 2.5 kg/m^2. What are the area and mass of the PV arrays? What is the total mass of your power system?

$$P_{total} = P_{ops} + P_{recharge} = 60 + 132.9 = 192.9\ kW$$
$$r_{Mars}(worst\ case) = 1.666\ AU$$
$$I_{s,Mars} = I_{s,Earth}\left(\frac{r_{Earth}}{r_{Mars}}\right)^2 = 1394\left(\frac{1}{1.666}\right)^2 = 502.2\ W/m^2$$
$$A_{array} = \frac{P_{total}}{I_s\eta} = \frac{192,900\ W}{502.2\ W/m^2(0.24)} = \boxed{\text{1601 m}^2}$$
$$m_{array} = (1601\ m^2)(2.5\ kg/m^2) = \boxed{\text{4002 kg}}$$
$$m_{total} = m_{array} + m_{batt} = 4002 + 1315 = \boxed{\text{5317 kg}}$$

(f) In the event of a failure of your PV arrays, you need to support minimum base functions at 25 kW for 48 hours to give you time to safely evacuate the crew to orbital spacecraft. How does this change the capacity and mass of the batteries?

$$E_{req} = (25\ kW)(48\ hrs) = 1200\ kW\text{-}hrs\left(\frac{1}{0.95}\right)\left(\frac{1}{0.9}\right) = 1404\ kW\text{-}hrs$$

$$m_{batt} = \frac{1404\ kW\text{-}hrs}{250\ W\text{-}hrs/kg} = \boxed{\text{5616 kg}}$$

(g) You need to be able to recharge the total battery pack following a PV failure and evacuation to allow you to reopen the base for operations. if the station needs to run at the 25 kW (continuous) level while recharging the batteries, how long will it take to reach a full charge?

$$P_{avg} = P_{total}\frac{t_{daylight}}{t_{orbit}} = 192.9\frac{178}{459} = 74.81\ kW$$
$$P_{charge,avg} = P_{avg} - P_{ops} = 74.81 - 25 = 49.81\ kW$$
$$t_{charge} = \frac{E_{req}}{P_{charge,avg}} = \frac{1404\ kW\text{-}hrs}{49.81\ kW} = \boxed{28.19 \text{ hrs}}$$

(3) You are designing a solar dynamic power system for a large space station. The goal is to generate 500 kW continuously.

(a) If the system were 100% efficient, how large would the solar concentrator have to be to collect 500 kW of incoming solar power at 1 AU? Assume the station is in a solar orbit and is never eclipsed.

$$A = \frac{P}{I_s} = \frac{500,000\ W}{1394\ W/m^2} = \boxed{358.7 \text{ m}^2}$$

(b) The best thermodynamic efficiency possible is that of the Carnot cycle, $\eta = 1 - \frac{T_{cold}}{T_{hot}}$. If T_{hot} =2100K and T_{cold} =450K, what is the efficiency? What is the new required solar concentrator area?

$$\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{450}{2100} = \boxed{0.7857}$$
$$A = \frac{P}{\eta I_s} = \frac{500,000\ W}{0.7857(1394\ W/m^2)} = \boxed{456.5 \text{ m}^2}$$

(c) Given your answer to (b) above, how must waste heat do you need to radiate away?

$$P_{waste} = AI_s(1-\eta) = (456.5\ m^2)(1394\ W/m^2)(1-0.7857) = 136,400\ W = \boxed{136.4 \text{ kW}}$$

(d) If the radiator is a flat plate ($\alpha = 0.25;\ \epsilon = 0.9$) that radiates to deep space on both sides without any other heat loads, what area would the radiator have to be?

$$P_{rad} = A\epsilon\sigma T^4 \implies A = \frac{P_{rad}}{\epsilon\sigma T^4} = \frac{136,400}{0.9(5.67\times10^{-8})(450)^4} = 65.17\ m^2$$

$$\textit{Since a flat plate has two sides, } A_{rad} = \boxed{32.59 \text{ m}^2}$$

A_{rad} is the physical size of the radiator assembly, which radiates out of both sides.

(e) If the same radiator design is in low Earth orbit such that one side radiates to Earth (T_{Earth} =280K) and the other side is illuminated by the sun, what would be the new required radiator area?

$$A_{rad}\alpha I_s + P_{rad} = A_{rad}\epsilon\sigma T_{rad}^4 + A_{rad}\epsilon\sigma\left(T_{rad}^4 - T_{Earth}^4\right)$$
$$A_{rad} = \frac{P_{rad}}{\epsilon\sigma T_{rad}^4 + \epsilon\sigma\left(T_{rad}^4 - T_{Earth}^4\right) - \alpha I_s}$$

$$A_{rad} = \frac{136,400}{0.9(5.67 \times 10^{-8})(450)^4 + 0.9(5.67 \times 10^{-8})\left(450^4 - 280^4\right) - 0.25(1394)}$$

$$A_{rad} = \frac{136,400}{2093 + 1779 - 348.5} = \boxed{38.71 \text{ m}^2}$$