ENAE 483/788D LECTURES #21-23
(STRUCTURAL DESIGN AND ANALYSIS)PROBLEMS – FALL, 2020

(1) Consider the strength design of a **payload interface adapter** to be used for mounting an 10,000 kg spacecraft atop a SpaceX Falcon 9 launch vehicle. The spacecraft's center of gravity (CG) is located 4 m above the interface plane on the centerline, and the adapter can be modeled as an Aluminum 6061 ring with average diameter D = 1575 mm.

(a) Using a Factor of Safety of 1.25, find the minimum thickness, t, required to sustain simultaneous axial and lateral launch accelerations. Treat the spacecraft as a point mass at its CG. Figure 4-1 from the Falcon 9 User's Guide is provided on the last page of this document for convenience.

(i) Calculate the margin of safety on worst case axial design stress. Assume t = 1.0 cm.

First we need some derived geometric parameters, namely A and I. It is recommended that you set up a spreadsheet to automate these calculations.

$$A \approx 2\pi \frac{D}{2} t = 2\pi \frac{1.575}{2} t \approx 4.948t\ m^2$$

$$I \approx 2\pi \left(\frac{D}{2}\right)^3 t = 2\pi \left(\frac{1.575}{2}\right)^3 t \approx 3.069t\ m^4$$

From the payload user's guide, the worst-case acceleration *in each direction* is +6 g axial and +2g lateral.

$$\sigma_{axial} = \frac{P}{A}(FoS)$$

$$\frac{(10000)(+6)(9.81)}{(4.948)(0.01)}(1.25) = 14.87\ MPa$$

$$MoS = \frac{\sigma_{Ty}}{\sigma_{axial}} - 1 = \frac{255100000}{14870000} - 1 = \boxed{16.2}$$

(ii) Calculate the margin of safety on worst case lateral/bending design stress for t = 1.0 cm.

$$\sigma_{bend} = \frac{Mc}{I}(FoS) = \frac{(+2)(9.81)(10000)(4)(1.575)}{(3.069)(0.01)}(1.25) = 50.03\ MPa$$

$$MoS = \frac{\sigma_{Ty}}{\sigma_{bend}} - 1 = \frac{255100000}{50340000} - 1 = \boxed{4.07}$$

(iii) Calculate the margin of safety on the combined loading case.

2.91 using the worst cases above, or 3.05 using the max norm (+6g axial and +0.5g lateral) from the payload users guide. We should probably be considering other launch loads.

(iv) Bonus : ✓+ Try to recreate the payload adapter capability plot from Figure 3-2 and explain the probable limitations (max payload height and payload mass). Note: The geometric assumptions from parts i-iii are not wholly accurate to the vehicle

Write margin of safety equations above explicitly in terms of t (including A(t) and I(t) equations) and M ($m_{payload} \cdot h_{cg}$). With MoS = 0 (or $\sigma_{design} = \sigma_{Ty}$) for a given Al alloy (very likely NOT Al 6061-T6!)

(b) If the Falcon 9's fundamental frequency is 35 Hz, state the required minimum component frequencies for Units A, B, and C. Assume the octave rule as a design guideline.

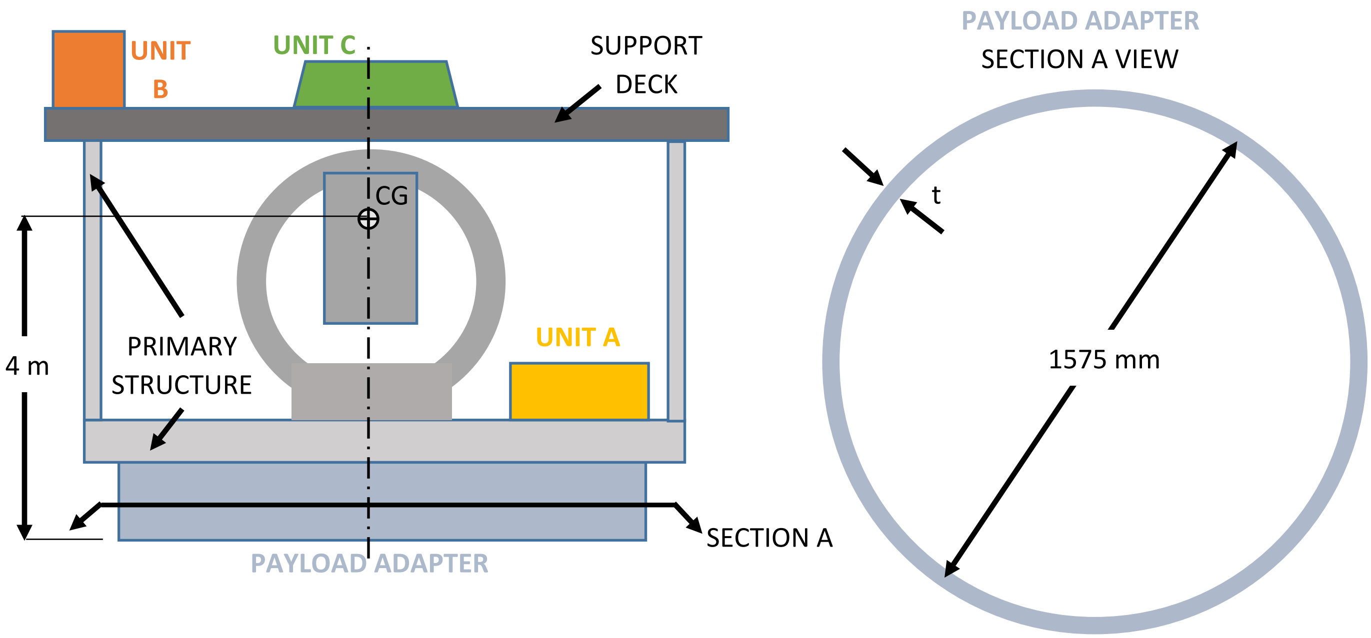
$$f_{n,A} \geq 70\ Hz$$

$$f_{n,B} \geq 70\ Hz$$

(It's basically on the primary load path)

$$f_{n,A} \geq 280\ Hz$$

Support deck would need <140 Hz itself



(2) Consider the stiffness design of a deployable solar array. This deployable system's fundamental frequency is an important factor to consider during the design of the attitude control system, since certain duty cycles (the on-off frequency of actuation) may cause the solar array to resonate and the support structure or mechanism to fail. Both the deployable structure and deployment mechanisms can drive this fundamental frequency.

One method to estimate the fundamental bending frequency of this solar array is to assume the boom that attaches it to the spacecraft structure is a cantilever beam fixed at its root on the spacecraft interface, which yields

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{ML^3}}$$

Figure 1 - Small Communications Satellite with Deployed Solar Arrays
https://www.spacetechexpo.eu/assets/files/images/BRE/artists-impression-leo-constellation.jpg

The deployable fundamental bending frequency may also be estimated by instead treating the entire solar array as a rigid body and considering only the relatively low stiffness of the actuator/hinge joint, which yields

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{J}}$$

In these equations, f_n is the fundamental frequency in [Hz]; (EI) is the beam bending stiffness which depends on the boom material modulus, E, and 2nd moment of area, I; the length from the panel's center of mass to the boom root, L; the panel mass, M, and rotational mass moment of inertia, J; and the actuator/hinge torsional stiffness, k.

(a) Consider a small communications satellite (SCS). The SCS is a 200 kg, 1 m cube-shaped spacecraft with two deployable arrays. The power system requires that the mass of each solar panel (cells + substrate structure) is estimated at 4.75 kg. Packaging requirements dictate that each panel measures 1.0 m x 0.7 m and be mounted on a boom of length 0.5 m for field of view requirements. Assume the simple, low-cost solar array boom is an aluminum 6061 tube with outer diameter 50 mm and wall thickness 0.5 mm. The actuator torsional stiffness is measured to be 10,000 Nm/radian and it's located at the surface of the spacecraft bus.

(i) Calculate the bending stiffness, EI, of the boom.

The 2nd moment of area for the boom tube is

$$I = \frac{\pi\left(d_0^4 - d_i^4\right)}{64} = \frac{\pi(0.050^4 - 0.0049^4)}{64} = \boxed{2.38 \times 10^{-8}\, m^4}$$

(ii) Calculate the bending frequency of the deployed boom + solar array structure using the cantilever approximation.

Assume that the CG is at a distance of 1 m from the spacecraft interface (boom length plus ½ long dimension of panel). The fundamental bending frequency of the deployable structure is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{(3)(68.95 \times 10^9)(2.38 \times 10^{-8})}{(4.75)(1)^3}} = \boxed{5.1\, Hz}$$

Note that this is an upper bound since the equation assumes and ideal structural load path and doesn't account for the added flexibilities of hinges and actuators.

(iii) Calculate the rotational mass moment of inertia, J, of the solar array considering it as a rectangular plate with its longest dimension parallel to the boom and that it rotates about the actuator (i.e. don't forget about the parallel axis theorem).

First calculate the rotational mass moment of inertia, J. Assume the actuator is at the spacecraft interface and the array is a homogeneous panel with dimensions a x b.

$$J_{panel} = \frac{m_{panel}\left(a_{panel}^2 + b_{panel}^2\right)}{12} = \frac{(4.75)(1^2 + 0.75^2)}{12} = 0.59\ kg \cdot m^2$$

but remember that it rotates about the actuator, not its centroid. So, using the parallel axis theorem,

$$J = J_{panel} + m_{panel}d^2 = 0.59 + (4.75)(1^2) = \boxed{5.34\ kg \cdot m^2}$$

(iv) Calculate the bending frequency of the deployed boom + solar array structure using the rigid body approximation.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{10000}{5.34}} = \boxed{6.89\ Hz}$$

(b) Propose two design alternatives that would bestow a higher fundamental frequency by considering all relevant factors (e.g. material and launch costs, machineability, packaging for launch, etc.)

(i) Propose an alternative material for the boom.

e.g. Use Al 7075-T6 or Al 2050 to increase E. Modest cost increase, new $f_n \approx 0.1 - 0.3\ Hz$ higher.

(ii) Propose an alternative configuration for the deployable solar panels.

e.g. Swap the side lengths of the panels to decrease J for a.

$$J = J_{panel} + m_{panel}(0.75)^2 = 3.26\ kg \cdot m^2, f_n \approx 8.8\ Hz$$

(iii) Compare and contrast the original design and the two alternatives to recommend a "best solution". Provide references for material properties and explain your decision with supporting justification, and if you use the word "optimum" be sure to specify "with respect to *what*."

The best solution is probably the combination of the two. However, a low cost satellite may be primarily cost limited, and rotating the panels so they don't stick out as far may be a complicated design change requiring a new actuator choice. Any sensible statements that are backed up by well-reasoned analyses and cited references are acceptable.

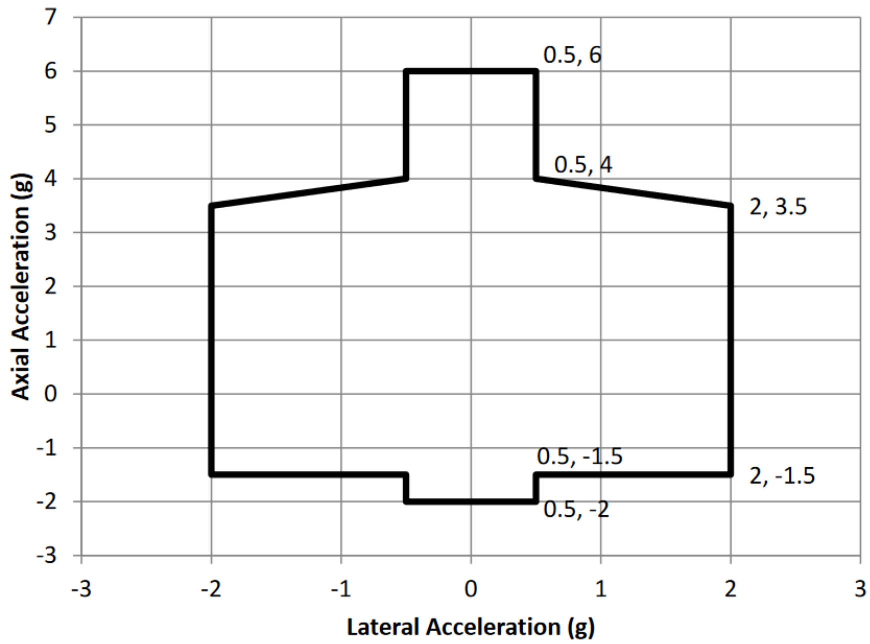


Figure 4-1: Falcon 9 and Falcon Heavy payload design load factors for "standard" mass (over 4,000 lb)

Table 1 - Properties for Al-6061-T6

Young's modulus	$E = 68.95\ GPa$
Mass density	$\rho = 2770\ kg/m^3$
Tensile yield strength	$\sigma_{Ty} = 255.1\ MPa$

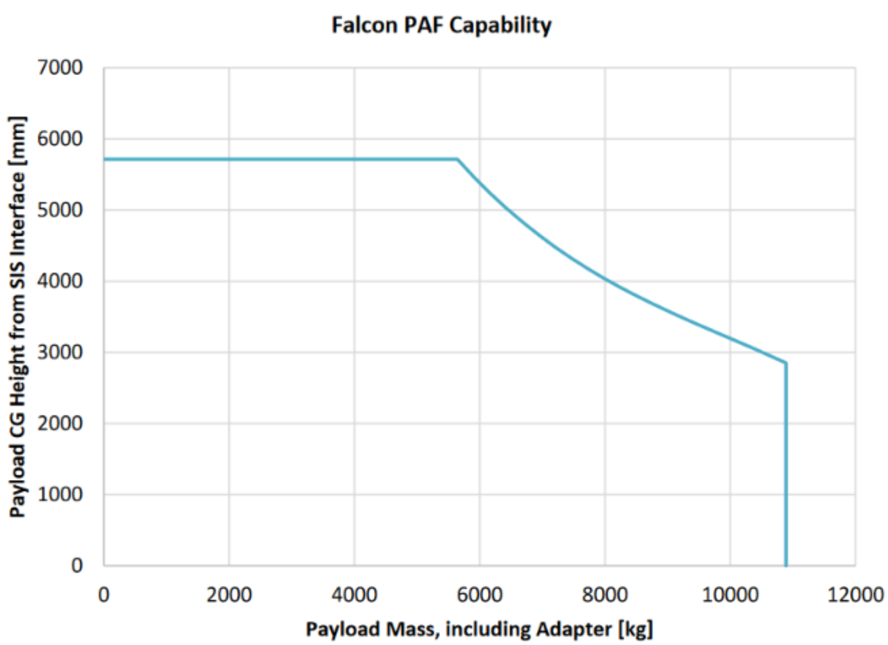


Figure 3-2: Allowable center-of-gravity height above the 1575-mm plane