ENAE 483/788D LECTURE #05 (RELIABILITY) PROBLEMS – FALL, 2021

(1) There were 135 space shuttle launches. What are the odds that no two of them were launched on the same date? (Ignore leap years)

The odds that one random date doesn't fall on the same date as another random event is $\frac{364}{365} = 0.9973$ *(neglecting leap years). The odds that the first launch (which happens to be April 12, but that isn't relevant to the solution) doesn't fall on the same date any of the other 134 launches is* $\left(\frac{364}{365}\right)^{134} = 0.3684$. *Of the 134 remaining launches, the odds that the second launch doesn't share a date with the other 133 is* $\left(\frac{363}{364}\right)^{133} = 0.3684$. *(The first person took away one day for their birthday, so the remaining people can't have the birthdays of the first or second person. Therefore for each of them there are 363 possible days out of 364 to have a birthday that isn't shared with the second person.) The probability no one shares a birthday with person 1 or person 2 is then (0.7898)(0.7915)=0.1357. Continuing in this mode, the probability that launch i doesn't share a date with a launch not already considered is* $\frac{365-i}{365-i+1}$. *The probability that person i doesn't share a birthday with anyone out of an original population of N people is* $\left(\frac{365-i}{365-i+1}\right)^{(N-i)}$. *The likelihood that nobody in a population of N shares a birthday is*

$$\prod_{i=1}^{N}\left(\frac{365-i}{365-i+1}\right)^{(N-i)}$$

Plug and chug with Excel or Matlab. For your case of N=135, the overall probability that no two launches share the same date is $\boxed{2.95 \times 10^{-59}}$ *- in other words, there's no practical possibility that there's no overlap of launches on the same date.*

(2) Super Heavy, the first stage of the SpaceX Starship launch vehicle, has 33 Raptor engines. If each of these are 99.5% reliable, what is the overall propulsive reliability of the stage?

Let R_i *be the propulsive reliability of the stage with i engines failed, r be the reliability of a single engine, and n be the number of engines*

$$R_0 = r^n = 0.995^{33} = \boxed{0.8475}$$

(3) How does your answer change if it can tolerate one engine failure?

$$R_1 = nr^{n-1}(1-r) = 33(0.995)^{32}(1-0.995) = 0.1405$$

Since the stage works successfully with either 0 or 1 engine failures,

$$R_{1faulttolerant} = R_0 + R_1 = \boxed{0.9881}$$

(4) How many engine failures must it be able to tolerate for a propulsive reliability >0.999?

$$R_2 = \frac{n(n-1)}{2}r^{n-2}(1-r)^2 = \frac{33(32)}{2}(0.995)^{31}(1-0.995)^2 = 0.0113$$

$$R_{2faulttolerant} = R_0 + R_1 + R_2 = \boxed{0.9994}$$

Since this reliability is greater than 0.999, the condition is met if the stage is two-fault tolerant

(5) If Super Heavy is two-failure tolerant on engines, what is the reliability if there is a 10% intercorrelation rate on failures?
For an intercorrelation rate of f,

$$R_{2\ fault\ tolerant} = R_0 + (1-f)R_1 + (1-f)^2 R_2 = 0.8475 + (0.9)0.1405 + (0.9)^2 0.0113 = \boxed{0.9832}$$

(6) If Super Heavy is three-fault tolerant on engines, what is the maximum intercorrelation rate it could accept and still reach 0.999 propulsive reliability?

$$R_3 = \frac{n(n-1)(n-2)}{2(3)} r^{n-3}(1-r)^3 = \frac{33(32)(31)}{6}(0.995)^{30}(1-0.995)^3 = 0.0005868$$

$$R_{3\ fault\ tolerant} = R_0 + (1-f)R_1 + (1-f)^2 R_2 + (1-f)^3 R_3$$

$$= 0.8475 + (1-f)0.1405 + (1-f)^2 0.0113 + (1-f)^3 0.0005868$$

Use a solver routine to numerically find the value of f= $\boxed{0.005813}$

(7) If Raptor engines have to operate for three minutes during a Starship launch and have a reliability of 0.995, what is their required mean time between failures?

$$R = e^{-t/MTBF} \Longrightarrow MBTF = -\frac{t}{\ln R} = -\frac{3}{\ln 0.995} = \boxed{598.5 \text{ min}}$$

(8) SpaceX thinks that ultimately they may fly 10 Starship missions per day. If the LOV rate is 1/1000 flights, the system is down for a month following a failure, and all payloads are retained, what surge rate would be required for the system to be marginally resilient?

$$r = 3650\ flts/yr;\ rd = 300\ flts\ in\ backlog; m = 1000\ flts\ between\ failures; k = 1$$

$$For\ resiliency,\ m = \frac{Srkd}{S-1} \Longrightarrow S = \frac{m}{m - rkd} = \frac{1000}{1000 - 100} = \boxed{1.111}$$