ENAE 483/788D MIDTERM – FALL, 2020 – *NAME:*

This is an open book test. I suspect some of you will need to look at lecture slides on your computer, but there is no reason to do extensive typing (other than possibly asking me a question on Zoom chat). Please turn on your camera if you have one and leave it on throughout the test.

The test starts at 11am and will end at 12:25 (class time plus an extra 10 minutes to scan and upload your answers.) If you have an accommodation, keep working and get it in by your accommodation time plus 10 minutes. Don't chat (by voice or keyboard), use a calculator for necessary computations - I suspect you know the drill by now. Please keep your upload scans neat so I can read it to grade it. Mark your answers with a box or some other clear designation on the page. Each question or subquestion is worth 5 points unless otherwise noted. Good luck!

Some possibly useful numbers:

$\mu_{Earth} = 398,604\ \frac{km^3}{sec^2}$, $r_{Earth} = 6378\ km$,

$\mu_{Mars} = 42,970\ \frac{km^3}{sec^2}$, $r_{Mars} = 3393\ km$

$\mu_{Moon} = 4667.9\ \frac{km^3}{sec^2}$, $r_{Moon} = 1738\ km$

(1) The planned NASA Lunar Gateway station will be in a near-rectilinear halo orbit (NRHO) around the Moon, but for the sake of this problem assume it is in a circular equatorial orbit around Earth at a radius of 300,000 km.

(a) Starting from a circular low Earth orbit at an altitude of 300 km, calculate the total Δv for a Hohmann-type transfer to Gateway, assuming the orbits are coplanar.

$$r_1 = r_{Earth} + h_1 = 6378 + 300 = 6678\ km$$

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right] = \sqrt{\frac{398604}{6678}}\left[\sqrt{\frac{2(300000)}{6678+300000}} - 1\right] = 3.081\ km/sec$$

$$v_{c2} = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{398604}{300000}} = 1.153\ km/sec$$

$$v_a = v_{c2}\sqrt{\frac{2r_1}{r_1+r_2}} = 3.072\sqrt{\frac{2(6678)}{6678+300000}} = 0.241\ km/sec$$

$$\Delta v_2 = v_{c2} - v_a = 1.153 - 0.241 = 0.912\ km/sec$$

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 = 3.081 + 0.912 = \boxed{3.992\text{ km/sec}}$$

(b) Repeat (a) assuming the original orbit is at 28.5° inclination, and that all plane change is accomplished at apogee.

Δv_1 *is unchanged.*

$$\Delta v_2 = \sqrt{v_{c2}^2 + v_a^2 - 2v_{c2}v_a\cos\Delta i} = \sqrt{1.153^2 + 0.241^2 - 2(1.153)(0.241\cos(28.5^o)} = 0.948\ km/sec$$

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 = 3.081 + 0.948 = \boxed{4.029\text{ km/sec}}$$

(c) For the complete maneuver in (b), how much time would elapse from low Earth orbit departure to arrival at the Gateway orbit?

$$a = \frac{r_1 + r_t}{2} = \frac{6678 + 300000}{2} = 153339\ km$$

$$t = \frac{P}{2} = \frac{1}{2}\left(2\pi\sqrt{\frac{a^3}{\mu}}\right) = \pi\sqrt{\frac{298784^3}{398604}} = \boxed{95{,}106 \text{ sec=82h59m44s}}$$

(2) The upper stage of the SpaceX Falcon Heavy has an inert mass of 4.5 mt, a propellant mass of 111.5 mt, a specific impulse of 348 sec, and carries a 63.8 mt payload to low Earth orbit.

(a) What is the Δv for this stage?

$$m_{tot} = m_{in} + m_{pr} + m_{pl} = 4.5 + 111.5 + 63.8 = 175.3\ mt$$

$$\Delta v = -gI_{sp}\ln\left(\frac{m_{in} + m_{pl}}{m_{tot}}\right) = -9.8(348)\ln\left(\frac{4.5 + 63.8}{175.3}\right) = \boxed{3215 \text{ m/sec}}$$

(b) They would like to replace this stage with a LOX/LCH4 stage, with an inert mass fraction $\delta = 0.045$ and a specific impulse of 385 sec. To minimize the impact on the Falcon Heavy program, the new second stage/payload combination must have the same total mass as the current second stage /payload combination as in (a), and must have the same total Δv. How much payload could this stage carry to orbit under these constraints?

$$from\ (a),\ m_{tot} = 175.3\ mt \Rightarrow m_{in} = \delta m_{tot} = 7.889\ mt$$

$$r = e^{\frac{-\Delta v}{gI_{sp}}} = e^{\frac{-3215}{9.8(385)}} = 0.4266$$

$$m_{pr} = (1-r)m_{tot} = (1-.4266)175.3 = 100.5\ mt \Rightarrow m_{pl} = m_{tot} - m_{in} - m_{pr} = 175.3 - 7.889 - 100.5 = \boxed{66.89 \text{ mt}}$$

(3) The SpaceX *Starship/Super Heavy* launch vehicle will have 28 Raptor engines in the first stage, and 6 Raptor engines in the second stage. Assume the individual reliability of a Raptor engine (=R_e) is 99.9%.

(a) If all engines must functional nominally to have a successful launch, what is the overall reliability of this launch vehicle?

$$R_{total} = R_e^{28} R_e^6 = 0.999^{34} = \boxed{96.66\%}$$

(b) What is the launch vehicle reliability if you can tolerate one failure on the first stage?

$$R_{total} = R_{stage1} R_{stage2} = \left[0.999^{28} + 28(0.999)^{27}(1 - 0.999)\right] 0.999^6 = \boxed{99.36\%}$$

(c) What is the launch vehicle reliability if you can lose one engine on the first stage *or* one engine on the second stage, but not both?

$$R_{total} = R(no\ failures) + R(1\ first\ stage\ failure) + R(1\ second\ stage\ failure)$$

$$R(no\ failures) + R(1\ first\ stage\ failure) = 0.9936\ [from\ (3)(b)]$$

$$R(1\ second\ stage\ failure) = R_e^{31}\left[6R_e^5(1-R_e)\right] = 0.999^{28}\left[6(0.999)^5(1-0.999)\right] = 0.00581$$

$$R_{total} = 0.9862 + 0.00581 = 0.9974 = \boxed{99.95\%}$$

(d) How does your answer to (3)(b) [Note: *not* (3)(c)!] change if you have a 25% rate of intercorrelated failures?

$$R_{total} = R_{stage1}R_{stage2} = \left[0.999^{28} + (1-.25)28(0.999)^{27}(1-0.999)\right]0.999^6 = \boxed{99.28\%}$$

(4) If you are following an 90% learning curve, how much does the fifth unit produced cost compared to the first one?

$$p = \frac{\ln LC}{\ln 2} = \frac{\ln 0.9}{\ln 2} = -0.1520$$

$$\frac{c_5}{c_1} = 5^p = 5^{-0.1520} = 0.5278 = \boxed{78.30\%}\ of\ the\ first\ unit\ cost$$

(5) On an 80% learning curve, how many units do you need to produce before the unit cost falls below 25% of the first unit cost?

$$p = \frac{\ln LC}{\ln 2} = \frac{\ln 0.8}{\ln 2} = -0.3219$$

$$\frac{c_n}{c_1} = n^p \Longrightarrow n = \left(\frac{c_n}{c_1}\right)^{1/p} = (0.25)^{1/-0.3219} = 74.2$$

So the cost falls below 25% of the first unit cost on the $\boxed{75^{th}\ unit}$

(6) A solar sail is a large flat plate of highly reflective material which produces thrust by the reflection of solar photons. A prototype solar sail launched in 2019 by the Planetary Society, *LightSail 2*, has a surface area of 32 m^2 and is in a 720 km altitude orbit. The atmospheric density in low Earth orbit is $\rho = 3.875 \times 10^{-9} e^{-\frac{h}{59.06}}$ where h is the orbital altitude in km and ρ is in kg/m^3.

(a) Assuming worst case where the flat plate is perpendicular to the velocity vector, what is the drag force on LightSail 2? ($c_d = 4$)

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398604}{(6378+720)}} = 7.494\ km/sec$$

$$\rho = 3.875 \times 10^{-9} e^{-\frac{h}{59.06}} = 3.875 \times 10^{-9} e^{-\frac{720}{59.06}} = 1.967 \times 10^{-14}\ kg/m^3$$

$$D = \frac{1}{2}\rho v^2 A c_D = \frac{1}{2} 1.967 \times 10^{-14} (7494)^2 (32) 4 = \boxed{7.069 \times 10^{-5} \text{ N}}$$

(b) If the reflective side (facing the sun) has an absorptivity α of 0.05 and an emissivity ϵ of 0.7, and the opposite side has characteristics of $\alpha = 0.02$ and $\epsilon = 0.08$, what is the equilibrium temperature of LightSail 2 while in sunlight?
Since this is from a later lecture, it will not be graded

(c) How would your answer to (b) change if LightSail 2 were at Ceres (2.77 AU)?
Since this is from a later lecture, it will not be graded

(d) Referencing the chart attached at the end of the exam, what is the largest impact you would expect to see in the two years LightSail 2 has been in low Earth orbit? (Ignore the difference in orbital altitude from that specified in the figure.)

$$flux = \frac{1}{(area)(time)} = \frac{1}{(32)(2)} = 0.01563\ hits/m^2/yr$$

As seen by the lines on the attached chart, this corresponds to approximately a $\boxed{\textit{0.06 cm}}$ *diameter impact.*

(7) Extra credit! (1 point each)

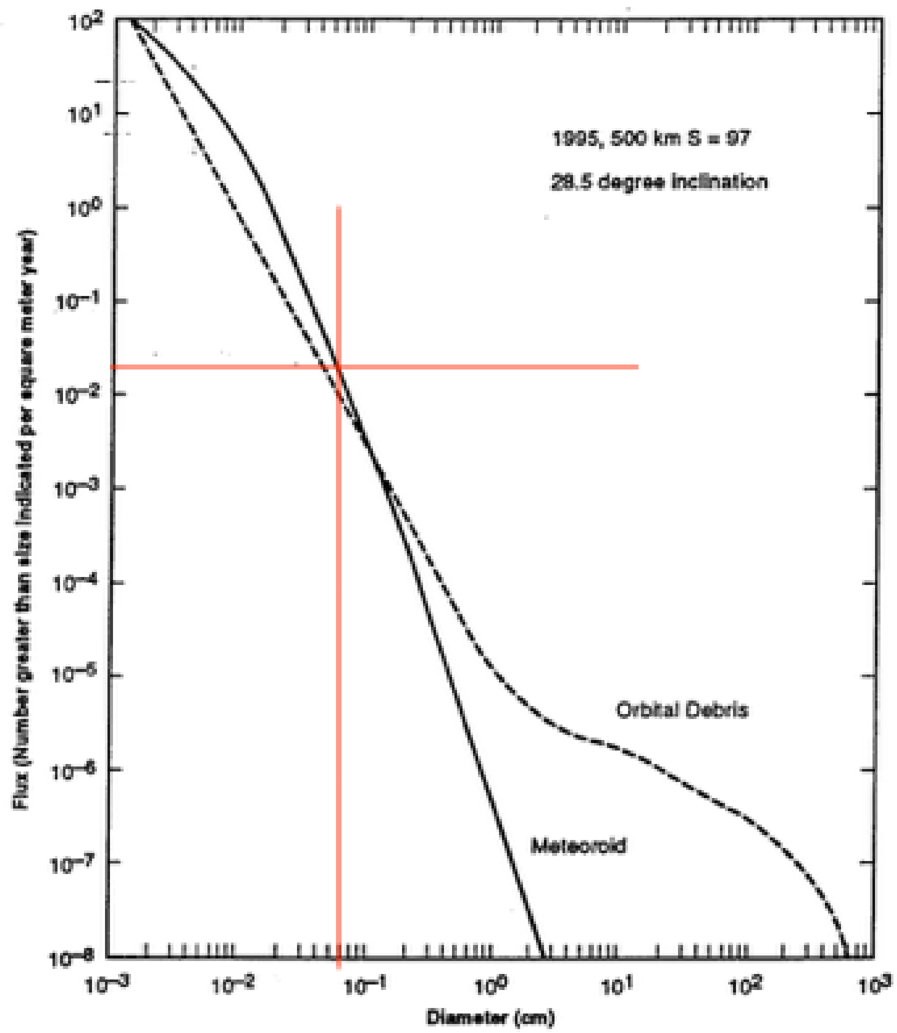
(a) Name an astronaut who walked on the moon other than Neil Armstrong or Buzz Aldrin
Charles "Pete" Conrad, Alan Bean, Alan Shepard, Ed Mitchell, Dave Scott, Jim Irwin, John Young, Charlie Duke, Gene Cernan, Jack Schmidt

(b) There were six space shuttle orbiters. Name any two.
Enterprise, Columbia, Challenger, Discovery, Atlantis, Endeavour

(c) Next years there will be five rovers on Mars (not all of them working). Name any two.
Pathfinder (or Sojourner), Spirit, Opportunity, Curiosity, Perseverance

(d) What is the name of NASA's current program "to put the first woman and the next man on the Moon"?
Artemis

(e) What was the name of the astronaut left behind on Mars in *The Martian*?
Dr. Mark Watney: Botanist/Space Pirate


1995, 500 km S = 97
28.5 degree inclination
Orbital Debris
Meteoroid
Flux (Number greater than size indicated per square meter year)
Diameter (cm)
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3