ENAE 483/788D PPT/LSM SPECIALTY PROBLEMS – FALL, 2022

(1) A rocket engine on the first stage of a launch vehicle has a combustion chamber pressure of 2500 psi and an exit pressure of 7 psi. The ratio of specific heats γ in the exhaust is 1.2. The exhaust velocity at ideal conditions ($P_e = P_a$) is 3200 m/sec.

(a) Calculate the expansion ratio of this engine's nozzle

$$\frac{A_t}{A_e} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}} \left(\frac{p_e}{p_0}\right)^{\frac{1}{\gamma}} \sqrt{\frac{\gamma+1}{\gamma-1}\left[1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{A_t}{A_e} = \left(\frac{2.2}{2}\right)^{\frac{1}{0.2}} \left(\frac{7}{2500}\right)^{\frac{1}{1.2}} \sqrt{\frac{2.2}{0.2}\left[1-\left(\frac{7}{2500}\right)^{\frac{0.2}{1.2}}\right]}$$

$$= (1.611)(0.007458)(2.621) = 0.03149 \Longrightarrow \frac{A_e}{A_t} = \boxed{31.75}$$

(b) If you wanted to lower the exit pressure to 0.8 psi, what expansion ratio would you need?

$$\frac{A_t}{A_e} = \left(\frac{2.2}{2}\right)^{\frac{1}{0.2}} \left(\frac{0.8}{2500}\right)^{\frac{1}{1.2}} \sqrt{\frac{2.2}{0.2}\left[1-\left(\frac{0.8}{2500}\right)^{\frac{0.2}{1.2}}\right]}$$

$$= (1.611)(0.001224)(2.850) = 0.005618 \Longrightarrow \frac{A_e}{A_t} = \boxed{178.0}$$

(2) You are designing a propellant depot in low Earth orbit to support extended exploration of the Moon and Mars. The depot is in a circular orbit at an altitude of 400 km. The depot will need a continual power level of 50 kW to maintain temperature of the cryogenic propellants and prevent boil-off. In this orbit, the depot is in eclipse for 39% of the orbital period.

(a) How much energy storage do you need in your batteries to maintain power during eclipse periods?

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{(6378+400)^3}{398604}} = 5553\ sec = 92.56\ min = 1.543\ hrs$$

$$Per\ orbit,\ (50\ kW)(0.39)(1.543\ hrs) = \boxed{30.09 \text{ kW-hrs}}$$

(b) If the batteries are 90% efficient at charging (i.e., only 90% of the power you supply for recharge will be stored in the battery) and 95% efficient at discharge (i.e., 5% of the capacity of the battery is unavailable for use), how does your answer to (a) change?

$$E_{req} = 30.09\ kW\text{-}hrs\left(\frac{1}{0.95}\right)\left(\frac{1}{0.9}\right) = \boxed{35.19\text{ kW-hrs}}$$

(c) If the batteries have a specific energy of 300 W-hrs/kg, what is the mass of the batteries (as defined by your answer to (b))?

$$m_{batt} = \frac{35.19\ kW\text{-}hrs}{300\ W\text{-}hrs/kg}1000\ W/kW = \boxed{117.3\text{ kg}}$$

(d) How much power is required from the photovoltaic arrays to operate the depot and charge the batteries with a 20% energy margin?

$$Daylight/orbit = 0.9412\ hrs \Longrightarrow P_{recharge} = \frac{(1.2)35.19\ kW\text{-}hrs}{0.9412\ hr} = \boxed{44.86\text{ kW}}$$

(e) The PV arrays you have chosen have a conversion efficiency of 24% and a mass of 2.5 kg/m^2. What are the area and mass of the PV arrays? What is the total mass of your power system?

$$\begin{aligned}
P_{total} &= P_{ops} + P_{recharge} = 50 + 35.19 = 85.19\ kW \\
A_{array} &= \frac{P_{total}}{I_s\eta} = \frac{85,190\ W}{1394\ W/m^2(0.24)} = \boxed{254.6\text{ m}^2} \\
m_{array} &= (254.6\ m^2)(2.5\ kg/m^2) = \boxed{101.9\text{ kg}} \\
m_{total} &= m_{array} + m_{batt} = 117.3 + 101.9 = \boxed{219.2\text{ kg}}
\end{aligned}$$

(3) You are designing a solar dynamic power system for a large space station. The goal is to generate 500 kW continuously.

(a) If the system were 100% efficient, how large would the solar concentrator have to be to collect 500 kW of incoming solar power at 1 AU? Assume the station is in a solar orbit and is never eclipsed.

$$A = \frac{P}{I_s} = \frac{500,000\ W}{1394\ W/m^2} = \boxed{358.7\text{ m}^2}$$

(b) The best thermodynamic efficiency possible is that of the Carnot cycle, $\eta = 1 - \frac{T_{cold}}{T_{hot}}$. If T_{hot} =2100K and T_{cold} =450K, what is the efficiency? What is the new required solar concentrator area?

$$\begin{aligned}
\eta &= 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{450}{2100} = \boxed{0.7857} \\
A &= \frac{P}{\eta I_s} = \frac{500,000\ W}{0.7857(1394\ W/m^2)} = \boxed{456.5\text{ m}^2}
\end{aligned}$$

(c) Given your answer to (b) above, how must waste heat do you need to radiate away?

$$P_{waste} = AI_s(1-\eta) = (456.5\ m^2)(1394\ W/m^2)(1-0.7857) = 136,400\ W = \boxed{136.4\text{ kW}}$$

(d) If the radiator is a flat plate ($\alpha = 0.25$; $\epsilon = 0.9$) that radiates to deep space on both sides without any other heat loads, what area would the radiator have to be?

$$P_{rad} = A\epsilon\sigma T^4 \Longrightarrow A = \frac{P_{rad}}{\epsilon\sigma T^4} = \frac{136,400}{0.9(5.67\times10^{-8})(450)^4} = 65.17\ m^2$$

$$Since\ a\ flat\ plate\ has\ two\ sides,\ A_{rad} = \boxed{32.59\ \mathrm{m}^2}$$

A_{rad} is the physical size of the radiator assembly, which radiates out of both sides.

(e) If the same radiator design is in low Earth orbit such that one side radiates to Earth (T_{Earth} =280K) and the other side is illuminated by the sun, what would be the new required radiator area?

$$A_{rad}\alpha I_s + P_{rad} = A_{rad}\epsilon\sigma T_{rad}^4 + A_{rad}\epsilon\sigma\left(T_{rad}^4 - T_{Earth}^4\right)$$

$$A_{rad} = \frac{P_{rad}}{\epsilon\sigma T_{rad}^4 + \epsilon\sigma\left(T_{rad}^4 - T_{Earth}^4\right) - \alpha I_s}$$

$$A_{rad} = \frac{136,400}{0.9(5.67\times10^{-8})(450)^4 + 0.9(5.67\times10^{-8})\left(450^4 - 280^4\right) - 0.25(1394)}$$

$$A_{rad} = \frac{136,400}{2093 + 1779 - 348.5} = \boxed{38.71\ \mathrm{m}^2}$$

(4) A propellant tank for a reaction control system contains N_2O_4 ($\rho = 1450$ kg/m^3) and is pressurized to 2 MPa. The tank is spherical, 1 meter in diameter and 1 mm in thickness. It is made of Ti-6Al-4V, with a yield stress of 924 MPa and an ultimate stress of 1000 MPa and a density of 4430 kg/m^3. From pg. 14 of the lecture notes, the required factors of safety for a metallic pressurized propellant tanks are 1.1 for yield and 1.4 for ultimate.

(a) If the only load source on the tank is due to the pressure, find the stress on the tank material.

$$For\ a\ sphere,\ \pi r^2 P = 2\pi r t\sigma \Rightarrow \sigma = \frac{Pr}{2t}$$

$$\sigma = \frac{Pd}{4t} = \frac{2\times10^6(1)}{4(.001)} = \boxed{500\ \mathrm{MPa}}$$

(b) Find the margins of safety for yield and ultimate failure cases.

$$MoS = \frac{allowable\ stress}{FoS \times limit\ applied\ stress} - 1$$

$$MoS_{yield} = \frac{924}{1.1\times500} - 1 = \boxed{0.6800}$$

$$MoS_{ult} = \frac{1000}{1.4\times500} - 1 = \boxed{0.4286}$$

(c) In addition to pressure, the tank must also contain the inertial load of the propellants under acceleration. During launch, the tank will be fully loaded with propellant and can expect to experience 4.5 (Earth) gravities of acceleration. Find the total maximum pressure load on the propellant tank in this case.

$$P_{inertial} = (density)(depth)(acceleration) = 1450\ kg/m^3(1\ m)(4.5\times9.8\ m/sec^2) = 63,950Pa$$

$$P_{tot} = P + P_{inertial} = 500,000,000 + 63,950 = \boxed{500.1\ \mathrm{MPa}}\ (to\ appropriate\ precision)$$

(d) Repeat (b) for this case.

$$MoS_{yield} = \frac{924}{1.1 \times 500.1} - 1 = \boxed{0.6797}$$

$$MoS_{ult} = \frac{1000}{1.4 \times 500.1} - 1 = \boxed{0.4283}$$