ENAE 483/788D LECTURE #03
(ROCKET PERFORMANCE) PROBLEMS – FALL, 2022

NASA says that the first orbital flight of the Space Launch System will occur before you finish ENAE 483. They like to claim that it will be the "largest launch vehicle ever flown", although interestingly enough the Block 1A version of SLS will be both lighter and with less payload than the Saturn V. (The argument for "largest" apparently is based on thrust at lift-off, which exceeds the Saturn V thanks to the two large solid rocket motors of SLS.) Since it hasn't flown yet, let's look more closely at the current champion of launch vehicles, the Saturn V. In its two-stage configuration (as used for launching Skylab) it was capable of putting 108 mt[1] into low Earth orbit (LEO). The masses and performance parameters of the two-stage Saturn V are given in the following table. The first stage used liquid oxygen/kerosine (LOX/RP-1) as propellants, and the second stage used LOX/LH2. Please answer the following questions about the Saturn V performance based on the data provided and your calculations.

Stage	empty mass (mt)	propellant mass (mt)	specific impulse (sec)
First stage	130	2080	264
Second stage	36	450	425

(1) Calculate

(a) Gross mass of the entire vehicle m_o

$$m_o = m_{in,1} + m_{pr,1} + m_{in,2} + m_{pr,2} + m_{pl} = 130 + 2080 + 36 + 450 + 108 = \boxed{2804 \text{ mt}}$$

(b) Inert mass fraction for the first stage δ_1

$$\delta_1 = \frac{m_{in,1}}{m_{o,1}} = \frac{130}{2804} = \boxed{0.04636}$$

(c) Inert mass fraction for the second stage δ_2

$$\delta_2 = \frac{m_{in,2}}{m_{in,2} + m_{pr,2} + m_{pl}} = \frac{36}{36 + 450 + 108} = \boxed{0.06061}$$

(d) Stage inert mass fraction for the first stage ϵ_1

$$\epsilon_1 = \frac{m_{in,1}}{m_{in,1} + m_{pr,1}} = \frac{130}{130 + 2080} = \boxed{0.05882}$$

(e) Stage inert mass fraction for the second stage ϵ_2

$$\epsilon_2 = \frac{m_{in,2}}{m_{in,2} + m_{pr,2}} = \frac{36}{36 + 450} = \boxed{0.07407}$$

(f) Predicted value for ϵ_1 based on the heuristic equation from lecture 3 for storable (i.e., non-LH_2) propellants

From page 19 in the lecture slides,

$$\epsilon_{\text{storables}} = 1.6062 \left(M_{\text{stage}} \langle kg \rangle\right)^{-0.275}$$

[1] Throughout this class, "mt" (or occasionally "MT") refers to "metric ton", or units of 1000 kg

$$\epsilon_1 = 1.6062\,(130,000 + 2,080,000)^{-0.275} = \boxed{0.02891}$$

Note that because this heuristic is in terms of kg, you have to express the masses in kg rather than in metric tons (as above) to get a meaningful answer. Same thing for the next question.

(g) Predicted value for ϵ_2 based on the heuristic equation from lecture 3 for cryogenic (LOX/LH2) propellants
From page 19 in the lecture slides,

$$\epsilon_{\text{storables}} = 0.987\,(M_{\text{stage}}\langle kg\rangle)^{-0.183}$$

$$\epsilon_2 = 0.987\,(36,000 + 450,000)^{-0.183} = \boxed{0.08988}$$

(h) Velocity change provided by the first stage Δv_1

$$m_{f,1} = m_{in,1} + m_{o,2} = 130 + (36 + 450 + 108)) = 130 + 594 = 724\ mt$$

$$\Delta v_1 = -g_o I_{sp} \ln \frac{m_{f,1}}{m_o} = -9.8(264) \ln \frac{724}{2804} = \boxed{3503 \text{ m/sec}}$$

(i) Velocity change provided by the second stage Δv_2

$$m_{f,2} = min, 2 + m_{pl} = 36 + 108 = 144\ MT$$

$$\Delta v_2 = -g_o I_{sp} \ln \frac{m_{f,2}}{m_{o,2}} = -9.8(450) \ln \frac{144}{594} = \boxed{6249 \text{ m/sec}}$$

(j) Total velocity change provided by the entire launch vehicle Δv_{total}

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 = 3503 + 6249 = \boxed{9752 \text{ m/sec}}$$

(k) Trade-off ratio for change in payload mass due to a change in first stage inert mass $\frac{\partial m_{pl}}{\partial m_{in,1}}$

$$\frac{\partial m_{pl}}{\partial m_{in,1}} = \frac{-gI_{sp,1}\left(\frac{1}{m_{o,1}} - \frac{1}{m_{f,1}}\right)}{gI_{sp,1}\left(\frac{1}{m_{o,1}} - \frac{1}{m_{f,1}}\right) + gI_{sp,2}\left(\frac{1}{m_{o,2}} - \frac{1}{m_{f,2}}\right)}$$

$m_{o,1} = m_o$*, and* g *appears in every term so we can divide that out (which we'll do for the next steps)*

$$\frac{\partial m_{pl}}{\partial m_{in,1}} = \frac{-264\left(\frac{1}{2804} - \frac{1}{724}\right)}{264\left(\frac{1}{2804} - \frac{1}{724}\right) + 450\left(\frac{1}{594} - \frac{1}{144}\right)} = \boxed{-0.1025}$$

Note: this is a dimensionless number, so it is metric tons of payload lost per metric ton of inert mass added, or kg of payload per kg of inert mass

(l) Trade-off ratio for change in payload mass due to a change in first stage propellant mass $\frac{\partial m_{pl}}{\partial m_{pr,1}}$

$$\frac{\partial m_{pl}}{\partial m_{pr,1}} = \frac{-I_{sp,1}\left(\frac{1}{m_{o,1}}\right)}{I_{sp,1}\left(\frac{1}{m_{o,1}} - \frac{1}{m_{f,1}}\right) + I_{sp,2}\left(\frac{1}{m_{o,2}} - \frac{1}{m_{f,2}}\right)}$$

$$\frac{\partial m_{pl}}{\partial m_{pr,1}} = \frac{-264\left(\frac{1}{2804}\right)}{264\left(\frac{1}{2804} - \frac{1}{724}\right) + 450\left(\frac{1}{594} - \frac{1}{144}\right)} = \boxed{0.03569}$$

(2) It would have been interesting to have the first stage of the Saturn V, like the SpaceX Falcon 9 first stage, be designed to be recovered and reused. Assume that, similar to the Falcon 9 first stage, you would need to reserve 3% of the first stage propellant for use in a landing burn.

(a) All other things being kept as originally specified, how does reserving the 3% landing propellant for the first stage change the Δv_{total} for the LEO launch for this case?
3% of the first stage propellant (=62.4 MT) will be reserved for stage recovery

$$\Delta v_1 = -g_o I_{sp,1} \ln \frac{m_{f,1}}{m_o} = -9.8(264) \ln \frac{724 + 62.4}{2804} = 3289\ m/sec$$

Second stage Δv is unchanged

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 = 3289 + 6249 = \boxed{9538 \text{ m/sec}}$$

(b) If you had to match the total Δv calculated in (1)(j), how much payload could you carry into orbit while reserving propellant for first stage recovery?
Using the Deltav equations from problems (2)(a) and (1)(i) and leaving m_{pl} as a variable, plug into Excel or Matlab and use a numerical solver to find that the new payload mass is $\boxed{99.9\ mt}$

(c) How much Δv is available to the first stage to be used for landing?

$$\Delta v_{landing} = -g_o I_{sp,1} \ln \frac{m_{in,1}}{m_{in,1} + m_{pr1,landing}} = -9.8(264) \ln \frac{130}{130 + 62.4} = \boxed{1014 \text{ m/sec}}$$

(3) **Extra credit** – To compare Saturn V and SLS on an "apples to apples" basis, we're going to strap a pair of stock Space Shuttle solid rocket boosters onto the Saturn V. Each SRB has a propellant mass of 502 mt and an inert mass of 88 mt. They each have a thrust of 11.79 MN, an I_{sp} of 267.3 sec, and a burning time of 123 sec. The first stage of the Saturn V burned for 171 sec at a total thrust of 34,500 MN. Assume that you do not reserve any propellant for first stage recovery and you have to achieve the Δv performance calculated in (1)(j). How much payload can this system carry to orbit? How does this compare to the 130 mt payload predicted for the Block 2 version of SLS?

$$\chi = \frac{t_{b,c} - t_{b,b}}{t_{b,c}} = \frac{171 - 123}{171} = 0.2807$$

$$\bar{v}_e = \frac{v_{e,b}\dot{m}_b + v_{e,c}\dot{m}_c}{\dot{m}_b + \dot{m}_c} = \frac{v_{e,b}m_{pr,b} + v_{e,c}(1-\chi)m_{pr,c}}{m_{pr,b} + (1-\chi)m_{pr,c}}$$

$$= \frac{9.8 \times 267.3(2 \times 502) + 9.8 \times 264(1 - 0.2807)2080}{2 \times 502 + (1 - 0.2807)2080} = 2600\ m/sec$$

$$\Delta v_o = -\bar{v}_e \ln\left(\frac{m_{\text{final}}}{m_{\text{initial}}}\right) = -\bar{v}_e \ln\left(\frac{m_{in,b} + m_{in,c} + \chi m_{pr,c} + m_{0,2}}{m_{in,b} + m_{pr,b} + m_{in,c} + m_{pr,c} + m_{0,2}}\right)$$

$$= -2600 \ln\left(\frac{2 \times 88 + 130 + 0.2807(2080) + (36 + 450) + m_{pl}}{2 \times (88 + 502) + 130 + 2080 + (36 + 450) + m_{pl}}\right)$$

$$\Delta V_1 = -V_{e,c} \ln\left(\frac{m_{in,c} + m_{0,2}}{m_{in,c} + \chi m_{pr,c} + m_{0,2}}\right) = -9.8*264 \ln\left(\frac{130 + 32 + 450 + m_{pl}}{130 + 0.2807 \times 2080 + (32 + 450) + m_{pl}}\right)$$

$$\Delta V_2 = -V_{e,2} \ln\left(\frac{m_{in,2} + m_{pl}}{m_{in,2} + \chi m_{pr,2} + m_{pl}}\right) = -9.8 * 450 \ln\left(\frac{32 + m_{pl}}{32 + 450 + m_{pl}}\right)$$

Solve numerically through Excel or Matlab to get $\Delta v_0 = 2540\ m/sec,\ \Delta v_1 = 1491\ m/sec,\ \Delta v_2 = 5751\ m/sec,$ *and* $\boxed{m_{pl} = 133.2\ mt}$