ENAE 483/788D LECTURE #04
(ORBITAL MECHANICS) PROBLEMS – FALL, 2022

The primary Russian launch site is located at 51.2° north latitude. They launch due East and inject a spacecraft into a circular low earth orbit at an altitude of 300 km. The spacecraft needs to maneuver to wind up in a geostationary orbit at a constant radial distance of 42,240 km at 0° inclination. Calculate

(1) Δv_1, Δv_2, and Δv_{total} assuming a coplanar Hohmann transfer

$$r_1 = h_1 + r_E = 300 + 6378 = 6678\ km;\ r_2 = 42240\ km$$

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right] = \sqrt{\frac{398604}{6678}}\left[\sqrt{\frac{2(42240)}{6678+42240}} - 1\right] = \boxed{2.427\ \frac{km}{sec}}$$

$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}}\left[1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right] = \sqrt{\frac{398604}{42240}}\left[1 - \sqrt{\frac{2(6678)}{6678+42240}}\right] = \boxed{1.467\ \frac{km}{sec}}$$

$$\Delta v_{tot} = \Delta v_1 + \Delta v_2 = \boxed{3.894\ \frac{km}{sec}}$$

(2) Δv_1, Δv_2, and Δv_{total} assuming a Hohmann transfer with all of the plane change accomplished at apogee

$$\Delta v_1\ unchanged = \boxed{2.427\ \frac{km}{sec}}$$

$$v_{c2} = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{398604}{42240}} = 3.072\ \frac{km}{sec}$$

$$v_a = v_{c2}\sqrt{\frac{2r_1}{r_1+r_2}} = 3.072\sqrt{\frac{2(6678)}{6678+42240}} = 1.605\ \frac{km}{sec}$$

$$\Delta v_2 = \sqrt{v_{c2}^2 + v_a^2 - 2v_{c2}v_a\cos\Delta i} = \sqrt{3.072^2 + 1.605^2 - 2(3.072)(1.605)\cos 51.2^\circ} = \boxed{2.415\ \frac{km}{sec}}$$

$$\Delta v_{tot} = \Delta v_1 + \Delta v_2 = \boxed{4.842\ \frac{km}{sec}}$$

(3) Δv_1, Δv_2, and Δv_{total} assuming a Hohmann transfer with the plane change split between the perigee and apogee maneuvers to minimize Δv_{total}. What is the optimum Δi split?

$$v_{c1} = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{398604}{6678}} = 7.726\ \frac{km}{sec}$$

$$v_p = v_{c1}\sqrt{\frac{2r_2}{r_1+r_2}} = 7.793\sqrt{\frac{2(42240)}{6678+43340}} = 10.153\ \frac{km}{sec}$$

$$\Delta v_1 = \sqrt{v_{c1}^2 + v_p^2 - 2v_{c1}v_p\cos\Delta i_1}$$

$$\Delta v_2 = \sqrt{v_{c2}^2 + v_a^2 - 2v_{c2}v_a\cos(51.2^\circ - \Delta i_1)}$$

Load equations into your favorite solver and find the value of Δi_1 to minimize total Δv_{tot} and find

$$\boxed{\Delta i_1 = 2.84^\circ;\ \Delta i_2 = 48.36^\circ;\ \Delta v_1 = 2.466\ \frac{km}{sec};\ \Delta v_2 = 2.337\ \frac{km}{sec};\ \Delta v_{tot} = 4.803\ \frac{km}{sec}}$$

(4) Generally speaking, Δv_1 is performed by the upper stage of the launch vehicle, placing the spacecraft into a geostationary transfer orbit (GTO). The spacecraft has to perform all subsequent maneuvers to reach geostationary orbit (GEO) using its own propulsion system. If the launch vehicle has additional capability, it will insert the spacecraft into an orbit with a higher apogee than GEO altitude to reduce the requirements on the spacecraft. Assume for this problem we are adopting the approach of problem (2) - all plane change is performed at apogee.

(a) The launch vehicle injects the spacecraft into a superGTO orbit with an apogee radius of 100,000 km. What is the new Δv_1 for the insertion into superGTO orbit? (Yes, I know, "superGTO orbit" is redundant. I just wanted to make it very clear.)

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left[\sqrt{\frac{2r_2}{r_1+r_2}}-1\right] = \sqrt{\frac{398604}{6678}}\left[\sqrt{\frac{2(100000)}{6678+100000}}-1\right] = \boxed{2.853\ \frac{km}{sec}}$$

(b) What is the Δv for the maneuver at apogee that performs the plane change and raises the perigee to GEO distance?

$$v_{ap,1} = \sqrt{\frac{\mu}{r_2}}\sqrt{\frac{2r_1}{r_1+r_2}} = \sqrt{\frac{398604}{100000}}\sqrt{\frac{2(6678)}{6678+100000}} = 0.706\ \frac{km}{sec}$$

$$v_{ap,2} = \sqrt{\frac{\mu}{r_3}}\sqrt{\frac{2r_3}{r_1+r_2}} = \sqrt{\frac{398604}{50000}}\sqrt{\frac{2(42240)}{42240+50000}} = 1.539\ \frac{km}{sec}$$

$$\Delta v_2 = \sqrt{v_{ap,1}^2+v_{ap,2}^2-2v_{ap,1}v_{ap,2}\cos\Delta i} = \boxed{1.226\ \frac{km}{sec}}$$

(c) What is the Δv to circularize at GEO?

$$\Delta v_3 = \sqrt{\frac{\mu}{r_3}}\left[\sqrt{\frac{2r_2}{r_3+r_2}}-1\right] = \sqrt{\frac{398604}{42240}}\left[\sqrt{\frac{2(100000)}{42240+100000}}-1\right] = \boxed{0.571\ \frac{km}{sec}}$$

(d) What is the Δv_{total} for the spacecraft in this case? How does it compare with the spacecraft Δv from question (2)?

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 + \Delta v_3 = 2.853 + 1.226 + 0.571 = \boxed{4.650\ \frac{km}{sec}}$$

Actually, it's a little savings (about 192 m/sec) compared to doing all plane change at apogee, and 153 m/sec less than an optimal plane change

(e) How long would it take from LEO departure to GEO arrival using this bielliptical transfer?

$$a_1 = \frac{1}{2}(r_1+r_2) = \frac{1}{2}(6678+100000) = 53,339\ km$$

$$t_1 = \pi\sqrt{\frac{a^3}{\mu}} = \pi\sqrt{\frac{53339^3}{398604}} = 61,299\ sec$$

$$a_2 = \frac{1}{2}(r_2+r_3) = \frac{1}{2}(100000+42240) = 71,120\ km$$

$$t_2 = \pi\sqrt{\frac{a^3}{\mu}} = \pi\sqrt{\frac{71120^3}{398604}} = 94,377\ sec$$

$$t_{tot} = t_1 + t_2 = 61299 + 94377 = \boxed{\text{155,675 sec=43h14m35s}}$$