ENAE 483/788D LECTURE #05 (RELIABILITY) PROBLEMS – FALL, 2022

(1) Falcon 9 first stages equipped with the Merlin 1D ("M1D") rocket engines have flown 184 times to date. Each stage has 9 engines, so this record represents 1656 missions for this engine. If there were no failures, what reliability could SpaceX claim at 80% confidence?

$$\textit{For no failures, } R^N + C = 1 \ \Rightarrow \ R = (1-C)^{1/N} = 0.2^{1/1656} = \boxed{0.99903}$$

(2) In reality, there have been two M1D failures during that time, although due to redundancy in the Falcon design all payloads were successfully delivered to orbit. Given the two failures, what is the actual reliability of the M1D engine at 80% confidence?

For two failures, we have to account for the outcome we saw (2 failures) and all better outcomes (1 or 0 failures)

$$R^N + NR^{N-1}(1-R) + \frac{N(N-1)}{2}R^{N-2}(1-R)^2 + C = 1$$

For $N = 1656$ *and* $C = 0.8$*, use Excel or Matlab to find the correct value of R numerically* $\Longrightarrow R = \boxed{0.9974}$

(3) The current generation of the Falcon 9 ("Full Thrust") has attempted 122 recoveries of the first stage with 117 successes. What reliability can SpaceX claim for first stage recovery at 80% confidence?

$$\textit{For J failures over N trials, } R^N + \sum_{i=1}^{J} \frac{N!i!}{(N-i)!} R^{N-i}(1-R)^i = 1 - C$$

For $N = 122$ *and* $J = 5$*, solve numerically to get* $R = \boxed{0.9359}$

(4) For the following questions, assume the reliability of a single Merlin 1D engine is 0.9975.

(a) Each first stage contains nine M1D engines. If all of the engines must work for a successful launch, what is the reliability of the first stage propulsion system?

$$P = R^N = 0.9975^9 = \boxed{0.9777}$$

(b) If the stage can complete a mission successfully after the loss of one engine, what is the new reliability of the first stage?

$$P = R^N + NR^{N-1}(1-R) = 0.9975^9 + 9(0.9975)^8(1-0.9975) = \boxed{0.99978}$$

(c) How does your answer to (b) change if the engines have a 20% intercorrelation rate?

$$P = R^N + (1-f)NR^{N-1}(1-R) = 0.9975^9 + (1-0.2)9(0.9975)^8(1-0.9975) = \boxed{0.99537}$$

(d) The Falcon Heavy consists of three Falcon 9 first stages mounted side-by-side. How does your answer to (a) change with 27 M1D engines in the first stage?

$$P = R^N = 0.9975^{27} = \boxed{0.93465}$$

(e) How does your answer to (b) change with 27 engines in the first stage?

$$P = R^N + NR^{N-1}(1-R) = 0.9975^{27} + 27(0.9975)^{26}(1-0.9975) = \boxed{0.99790}$$

(f) How does your answer to (c) change with 27 engines in the first stage?

$$P = R^N + (1-f)NR^{N-1}(1-R) = 0.9975^{27} + (1-0.2)27(0.9975)^{26}(1-0.9975) = \boxed{0.98525}$$

(g) If the Falcon Heavy can have a successful mission with two engine failures, what is the overall system reliability?

$$P = R^N + NR^{N-1}(1-R) + \frac{N(N-1)}{2}R^{N-2}(1-R)^2$$
$$= 0.9975^{27} + 27(0.9975)^{26}(1-0.9975) + \frac{27(26)}{2}0.9975^{25}(1-0.9975)^2 = \boxed{0.99996}$$

(h) How does your answer to (g) change if you can have two failures, but they must be in separate cores (i.e., you can only lose one engine out of nine on any single booster core, but you can lose one engine in two out of three booster cores)?
Since you can only accept one failure per module, but two failed modules still produces a successful flight, think about this as the likelihood of module failure rather than single engine failure. For each of the three first-stage modules, the reliability is

$$\text{No engine failures in a module: } R_{mod} = R^N = 0.9975^9 = 0.97772$$

So for a three-module first stage,

$$\text{No module failures: } P_0 = R_{mod}^N = 0.97772^3 = 0.93465$$
$$\text{One module failure: } P_1 = NR_{mod}^{N-1}(1-R_{mod}) = 3(0.97772)^2(1-0.97772) = 0.063885$$
$$\text{Two module failures: } P_2 = \frac{N(N-1)}{2}R_{mod}^{N-2}(1-R_{mod}^2) = 3(0.97772)(1-0.97772)^2 = 1.87 \times 10^{-5}$$
$$P_{tot} = P_0 + P_1 + P_2 = \boxed{0.99855}$$