ENAE 483/788D LECTURE #09
(COST ESTIMATING RELATIONS) PROBLEMS – FALL, 2022

Earlier this week, Firefly Aerospace had the first successful launch of their Alpha launch vehicle. We are going to use the NASA SVLCM costing model (in the lecture notes) to estimate the costs of Alpha. It should be noted that this costing model predicts what the cost would be *if it were produced using traditional NASA processes.* Use the following parameters in your analysis:

- The nominal payload mass to a 500 km low Earth orbit is 745 kg
- The inert mass for the first stage is 2895 kg
- The inert mass for the second stage is 909 kg

(1) Convert the SVLCM launch vehicle heuristic equations from 2018$ to 2022$ on page 7 of the lecture slides using the NASA "2020 NASA New Start Inflation Index" spreadsheet – the link to download it is on page 19 of the slides.

From the table, the conversion factor going from 2018 to 2021 is 1.143. (It's cell BN68 if you're not sure about the layout.) These heuristic equations convert vehicle inert mass (in kg) to estimates of nonrecurring and first-unit production costs (in $M2022).

$$c_{NR} = 12.23 m_{in}^{0.55}$$

$$c_{R1} = 0.2904 m_{in}^{0.662}$$

(2) Calculate the nonrecurring costs for each of the two stages.

$$c_{NR,1} = 12.23(2895)^{0.55} = \boxed{\$980.2\text{ M}}$$

$$c_{NR,2} = 12.23(909)^{0.55} = \boxed{\$518.4\text{ M}}$$

(3) Calculate the first unit production costs for each of the two stages.

$$c_{R1,1} = 0.2904(2895)^{0.662} = \boxed{\$56.83\text{ M}}$$

$$c_{R1,2} = 0.2904(909)^{0.662} = \boxed{\$26.40\text{ M}}$$

(4) Both of the stages of Alpha are currently expendable. Assume that vehicle production follows an 83% learning curve. Create a table showing the production costs for each stage for each of the first ten missions.

For an 83% learning curve, $p = \frac{\ln 0.83}{\ln 2} = -0.2688$

$$c_{Ri} = c_{R1} i^p \Rightarrow c_{R2,1} = c_{R1,1} \times 2^p = 980.2M \times 2^{-0.2688} = \$813.6M$$

For the units of each stage fabricated, the recurring costs are

Unit #	1st stage	2nd stage
1	56.83	26.40
2	47.17	21.91
3	42.30	19.65
4	39.15	18.19
5	36.87	17.13
6	35.11	16.31
7	33.68	15.65
8	32.50	15.09
9	31.48	14.62
10	30.60	14.22
Total	385.7	179.2

(5) What is the total cost for all 10 launches, including nonrecurring costs?

$$c_{tot} = c_{NR,1} + c_{NR,2} + \sum_{i=1}^{10} (c_{Ri,1} + c_{ri,2}) = 980.2 + 518.4 + 385.7 + 179.2 = \boxed{\$2063\text{M}}$$

(6) What is the cost per kilogram of payload for break-even in this program?

$$\textit{Cost per kg payload} = \frac{\$2063}{10(745)} = \boxed{\$277{,}000 \text{ kg}}$$

(7) The development of Alpha took six years, from 2017 to 2022. Find the year-by-year distribution of the nonrecurring cost using a beta function with $c_f = 0.50$ and $Pk = 0.50$.

$$c_f \geq 0.5: \ A = \frac{P(c_f - 0.8125) + (c_f - 0.1875)}{0.625} = \frac{0.50(0.50 - 0.8125) + (0.50 - 0.1875)}{0.625} = 0.25$$

$$B = P\frac{0.8125 - c_f}{0.3125} = 0.50\frac{0.8125 - 0.50}{0.3125} = 0.50$$

$$C(\tau) = 10\tau^2(1-\tau)^2(A + B\tau) + \tau^4(5 - 4\tau)$$

Year	τ	$C(\tau)$	ΔC_{NR}, yearly (\$M)	NFV (\$M2024)
2017	0.1667	0.0676	101.4	163.3
2018	0.3333	0.2510	274.8	402.4
2019	0.5000	0.5000	373.1	496.6
2020	0.6667	0.7490	373.1	451.5
2021	0.8333	0.9324	274.8	302.3
2021	1	1.000	101.4	1093

(8) Based on your results from (7), find the net future value in 2022 of the nonrecurring costs spread from 2017-2022 assuming a discount rate of 10%.

Since we're calculating NFV based on 2022, the cost in that year will be unchanged. All subsequent years will be calculated by

$$NFV_i(c_n) = c_n(1 + r)^{i-n}$$

In this case we're calculating NFV_{2022}, *and* $r = 0.10$. *Being lazy I included this set of annual calculations in the right column of the table in the previous problem, but if you add it up the total net future value of the nonrecurring costs in the year 2022 would be* $\boxed{\textit{\$1917M}}$.