ENAE 483/788D LECTURE #10
(SPACE ENVIRONMENT) PROBLEMS – FALL, 2022

NASA is very interested in inflatable habitats, which will provide more internal volume for living and working in space. For this problem, consider a cylindrical inflatable space habitat with a diameter of 8 meters, a cylindrical length of 16 m, and *only for the purposes of keeping this problem set simpler*, flat endcaps for problems (1) and (2).

(1) At an Earth orbital altitude of 700 km, what is the drag force on the habitat with the cylindrical axis perpendicular to the velocity vector?

The planform area perpendicular to the flow would be length × diameter, or 128 m^2. From the notes (Lecture #10 pg. 20) the c_D for a cylinder in this orientation is 8/3. The orbital velocity is

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398604}{6378+700}} = 7.504\ km/sec$$

From page 13,

$$\rho = 3.875 \times 10^{-9} e^{-\frac{h}{59.06}} = 3.875 \times 10^{-9} e^{-\frac{700}{59.06}} = 2.760 \times 10^{-14}\ kg/m^3$$

$$D = \frac{1}{2}\rho v^2 A c_D = \frac{1}{2} 2.760 \times 10^{-14} (7504)^2 128(2.667) = \boxed{2.653{\times}10^{-4}\ \mathrm{N}}$$

Note that the velocity had to be converted to m/sec to make the units work out correctly.

(2) In the same conditions, what is the drag force in the case of the cylindrical axis aligned with the velocity vector?

From page 17, c_D for a flat plate perpendicular to the flow is 4.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} 8^2 = 50.26\ m^2$$

$$D = \frac{1}{2}\rho v^2 A c_D = \frac{1}{2} 2.760 \times 10^{-14} (7504)^2 50.26(4) = \boxed{1.562{\times}10^{-4}\ \mathrm{N}}$$

(3) The habitat could remain in Earth orbit for fifty years. Over that lifetime, what is the largest MMOD particle would you have to design for (on average)? For the purposes of this and the remaining questions, assume the habitat has a diameter of 8 m, a cylindrical length of 8 meters, and hemispherical endcaps on each end (more realistic model than the flat-ended cylinder above).

$$A = \pi d^2 + \pi \ell d = \pi d (d + \ell) = 8\pi (8+8) = 402.1\ m^2$$

$$Flux = \frac{1\ hit/m^2 - yr}{(402.1\ m^2)(50\ yrs)} = 4.974 \times 10^{-5}\ hits/m^2 - yr$$

From the chart on page 34 of the lecture slides, you would have to design for orbital debris particles of $\boxed{5\ mm}$ *(red lines on attached chart)*

(4) If you relocated the habitat far enough away from the Earth that orbital debris were no longer an issue, what is the largest micrometeoroid hit you would expect over the same time period?

Looking at the blue line where the flux intercepts the micrometeoroid line, the largest particle would be around $\boxed{3 \text{ mm}}$.

(5) Over that same time (and back in Earth orbit), how many MMOD hits would you expect to have from particles with a diameter of 5 mm?

Okay, this illustrates (again!) the point that I should double-check the numbers before posting the problem. Since the answer to (3) was 5 mm, we know that on average you could expect a hit of this size $\boxed{1}$ *time over the 50-year lifetime.*