ENAE 483/788D TERM PROJECT 1
LAUNCH VEHICLE TRADE STUDY – FALL, 2024

Project Overview

This term project has a group goal that is supported by individual members each performing portions of a trade study. The group will work together to collaboratively create well-commented code for analysis tools so that each group member can then use that tool to analyze the effect of propellant mix on launch vehicle design. The group is responsible for analyzing a 5x5 matrix of possible propellant mixes on a 2-stage rocket. After a vehicle-level staging trade study has been conducted, the group will also collaborate on a systems-level evaluation of top designs using the mass estimating relations from lecture. Finally, the two top designs will be selected by the group (supported by numbers), and CAD models will be created collaboratively based on MER sizing.

There are both individual and group portions to this project. Constant discussion and support from your group is necessary to perform well in this project, and all group members must contribute equally. This project is intended to emulate the work structure of ENAE484, where individuals each work on subparts of the greater design trades while also coming together as a group to discuss results and make decisions that benefit the overall mission success.

There will be a teammate-review that will be taken into account for the final grade to ensure this is true. A form will be distributed before each submission to allow for this review process.

THROUGHOUT this project, use proper significant figures and units. Label axes and legends with large enough font to be read from a distance on a bad projector.

Following the procedures illustrated in the lecture notes, perform a vehicle- and system-level trade study for a two-stage launch vehicle while following the given mission requirements:

Mission Requirements & Parameters

Requirement	Requirement Description
M0	Mission Requirements
M1	Destination of LEO requires $\Delta V = 9.8$ km/s
M2	Payload mass to destination is 63,000 kg
M3	Payload volume must be at least 5.2m diameter by 13m height
M4	Program uses 10 flights
M5	Unit production learning curve rate = 85%
M6	Do not exceed $L/D = 12$.
M7	Initial thrust:weight ratio must be ≥ 1.3 for stage 1 and ≥ 0.76 for subsequent stages
M8	The launch vehicle must have two stages.

Table 1. Requirements Table

Use the following propellant parameters in the analysis:

Propellants	LOX/LCH_4	LOX/LH_2	LOX/RP1	Solid	N_2O_4:UDMH (Storables)
Oxidizer:Fuel mixture ratio	3.6:1	6.03:1	2.72:1	N/A	2.67:1
Specific impulse sea-level (sec)	327	366	311	269	285
Thrust 1^{st} stage (MN)	2.26	1.86	1.92	4.5	1.75
Thrust 2^{nd} stage (MN)	0.745	0.099	0.061	2.94	0.067
Engine exhaust diameter 1^{st} stage [A_e] (m)	2.4	2.4	3.7	6.6	1.5
Engine exhaust diameter 2^{nd} stage [A_e] (m)	1.5	2.15	0.92	2.34	1.13
Chamber pressure 1^{st} stage (MPa)	35.16	20.64	25.8	10.5	15.7
Chamber pressure 2^{nd} stage (MPa)	10.1	4.2	6.77	5	14.7
Nozzle area ratio sea-level 1^{st} stage	34.34	78	37	16	26.2
Nozzle area ratio sea-level 2^{nd} stage	45	84	14.5	56	81.3

Table 2. Propellant parameters

Propellant	Density[ρ] (kg/m^3)
LH_2	71
LOX	1140
RP-1	820
LCH_4	423
APCP (Solid)	1680
N_2O_4	1442
UDMH	791

Table 3. Densities of propellant components

1. Vehicle-level Staging Trade Study

Collaborating equally in your team, create code in MATLAB or equivalent (JuPyter, SciPy, etc.) for this assignment, as you will be varying parameters and reusing mass estimating relations. You *MUST ALL AGREE* on what analysis tool to use as everyone is expected to participate. Make sure that you have meaningful comments for your code as you will be submitting your code with this project, and all team members must be able to understand all parts of the code to use it properly [Code Documentation Example] https://blog.codacy.com/code-documentation. Read through the project document and decide as a team how the group work will be broken up across the group members (perhaps with individual functions). In the comments, list the member responsible for that particular section of code. This can be more than one individual as you are expected to work as a team.

1.1. **Create a vehicle-level analysis tool (Group).** We would like to estimate the mass of a launch vehicle that follows the mission requirements provided above. Use the relations discussed in the rocket performance lecture, and **assume** $\delta_1 = 0.05$ and $\delta_2 = 0.08$ to perform a first-pass mass estimation for each stage.

The following analyses will require the ΔV supplied by each stage to vary while still totaling the required ΔV to reach LEO: First stage ΔV_{tot} fraction (X) + second stage ΔV_{tot} fraction (1-X) = 1, such that $\Delta V_{tot} = 9.8$ km/s. Therefore for any given X fraction: $\Delta V_1 = X * \Delta V_{tot}$ and $\Delta V_2 = (1 - X) * \Delta V_{tot}$. Now, apply the formulae from the rocket performance lecture to determine the corresponding stage masses m_1, m_2, $m_{pr,1}$, $m_{pr,2}$, and the total launch vehicle mass m_0 for any given ΔV from your split. Make sure it is possible to vary specific impulse (isp) based on propellant mix, since you will be evaluating several different mixes.

1.2. **Optimize Launch Vehicle Mass.** First, we would like each individual teammate to make sure the code is working as intended while being aware of the design trends that this vehicle-level trade study is highlighting. The group is responsible for analyzing the full 5x5 matrix of possible propellant mixes on a 2-stage rocket (T.4). Each group member will be responsible for 5 of the combinations in the full design matrix. Note: if your group has less than 5 members you will have to divide up the full matrix amongst yourselves.

First stage prop. (columns) - Second stage prop. (rows)	LOX/LCH_4	LOX/LH_2	$LOX/RP1$	Solid	Storables
LOX/LCH_4					
LOX/LH_2					
$LOX/RP1$					
Solid					
Storables					

TABLE 4. Full design matrix, to be split up equally amongst the group. For the final slide deck, label which combination was analyzed by which teammate. Each first/second stage propellant mix will have two designs: one minimum mass, and one minimum cost.

1.2.a. *Sample Mass Trends (Individual).* Choose one of the propellant combination from your individual subset of the design matrix. Present a graph with X/Y axes being the first stage ΔV fraction and mass, respectively. Title the graph with your first and second stage propellant mixes. Vary the first stage ΔV fraction and plot lines for 1) the corresponding first stage mass, 2) the corresponding second stage mass, and 3) the sum for the corresponding gross vehicle mass. Don't forget to include the payload in the gross mass. Lastly, find the minimum on the gross launch vehicle mass curve and plot the point. The units for mass on this graph should be metric tons (t).

1.2.b. *Automate Finding the Minimum Mass Solution (Group).* In your code, make a function that returns the minimum overall gross mass of a single launch vehicle (m_0) for a given propellant mixture, like each of you showed graphically above. This is the tool that you will use in section 1.5.

1.3. **Optimize Launch Vehicle Cost.** We would like to estimate the cost of launch vehicles in addition to their mass. As a group, write code to estimate the cost of producing the number of launch vehicles stated in the mission requirements with the provided learning curve rate (T.1).

Using the launch vehicle stage SVLCM costing model (discussed in the cost estimation lecture and homework), introduce code to estimate the NRE and production costs of any launch vehicle

Second stage prop. First stage prop.	LOX/LCH_4 LOX/LCH_4	LOX/LCH_4 LOX/LH_2	LOX/LCH_4 $LOX/RP1$	LOX/LCH_4 Solid	LOX/LCH_4 Storables
Minimum LV gross mass soln. (t)					
Min. LV mass soln. stage 1 ΔV-fraction					
Min. LV mass soln. program cost ($B2024)					
Minimum program cost soln. ($B2024)					
Min. program cost soln. stage 1 ΔV-fraction					
Min. program cost soln. LV gross mass (t)					

Table 5. Example design matrix subset for an individual group member (looking at 5 out of the possible 25 stage-wise propellant mixes, and finding both the minimum-mass (shaded) and the minimum-cost (unshaded) solutions)

stage that you estimate the mass of. Be sure to use each individual stage mass in units of *kilogram* as the input to the cost functions. Exclude the payload mass from your costing, as the launch provider does not pay for payload. Adjust for inflation and use millions of 2024 dollars ($M2024), and sum the costs of each stage to output the total launch vehicle NRE costs and production costs throughout the program flights. Do not include the payload in the cost, since the launch vehicle provider does not pay for it. Be sure to discuss the learning curve application and the method of adjusting for inflation in the SVLCM stage costing function as a group, since this may cause mistakes.

1.3.a. *Sample Cost Trends (Individual).* Use the same first and second stage propellant mixes you used in the mass trends (section 1.2.a). This time, present a graph with X/Y axes being the first stage ΔV fraction and overall program cost, respectively. Title the graph with your first and second stage propellant mixes. Vary the first stage ΔV fraction and plot lines for 1) the corresponding first stage cost (NRE+production of all flights), 2) the corresponding second stage cost, and 3) the sum for the corresponding combined launch vehicle cost. Lastly, find the minimum on the launch vehicle cost curve and plot the point.

1.3.b. *Automate Finding the Minimum Cost Solution (Group).* In your group code, make a function that returns the minimum overall cost solution for a given propellant mixture, like each of you showed graphically above. This is the tool that you will use in section 1.5.

1.4. **Sample Comparison (Individual).** Create a table that displays the following information about each of the two graphed solutions *graphed* in the previous problems (sections 1.2.a & 1.3.a):

Write a few sentences comparing and contrasting the two solutions. Which quantity (mass or cost) might you want to optimize for and why, given that this design focuses on a launch vehicle? If we were designing a payload instead, what might we want to optimize differently? These will most likely not be the global minimum solutions.

	Minimum Mass Soln.	Minimum Cost Soln.
ΔV fraction in Stage 1	-	-
Overall LV Mass (t)	-	-
Overall LV Cost ($M2024)	-	-

TABLE 6. Example trend comparison table (S.1.4)

1.5. **Conduct a Vehicle-Level Trade Study Comparing Propellant Mixes (Group).** Now that you have each seen the trends that your code is calculating, and you have a way of selecting the optimal-mass/cost ΔV-split for a two stage launch vehicle of a given propellant mix, we will evaluate all the other potential propellant mixes. The full design matrix that your group is investigating is shown in table 4. You should have already split this design matrix equally across your team, so each group member will now run the analysis tool on their subset of propellant mixtures. Combine the results as a group.

Each propellant mix will have two designs generated by your analysis tools: one design with the stage ΔV-split optimized for minimum mass, and one design with the stage ΔV-split optimized for minimum program cost. Determine as a group how your results might be best represented - Can you plot the results in a meaningful way? Will a table work best? Etc. Generate 2 graphics to display this dataset for your group slides, with one of the graphics highlighting the top designs. Create two additional slides that discuss these trade study results so far and how the group came to these conclusions as a team.

2. Systems-level Mass Estimation

2.1. **Create a systems-level analysis tool (Group).** Using a first stage ΔV fraction as an input variable and the mass estimating relations from class, create a script that returns the masses for each sub-system in a stage. This will sum to the mass for each stage. You will be finding the two total stage masses, summing those to the total gross mass for the launch vehicle, and comparing this number to the solutions found in the vehicle-level design section. *Again, the group must meaningfully collaborate on this tool.*

Include mass estimates for each of the following sub-systems:

- Propellant
- Propellant tanks
- Propellant tank insulation (0 unless cryogenic)
- Engines (except for solid)
- Thrust structure
- Casing (only if solid)
- Gimbals
- Avionics
- Wiring
- Payload fairing (make it aerodynamic!)
- Inter-tank fairing
- Inter-stage fairing
- Aft fairing (*Leave sufficient engine fairing below the propellant tanks of each stage to accommodate the engines)
- Lastly, add a 30% mass margin on top of your total for design safety - this is only a preliminary design and is missing the masses of numerous other small components.

Follow along the process for including the mass estimating relations from lecture using the $m_0, m_1, m_2, m_{pr,1}, m_{pr,2}$, etc. from the "ideal" stage ΔV-split that you generated in the earlier section. Calculate thrust and engine number required at each stage with your vehicle weight first-pass estimate and the thrust to weight ratio provided in the requirements. Don't forget the payload! The values provided in the propellant table will help you estimate your engine/thrust structure/gimbal masses. Your estimated stage mass will feed back into the lifting requirements for your engines, again increasing your mass. This should be repeated until the values converge.

The deliverable for this section is a clean copy of your published code attached to your submission.

2.2. **System-Level Mass and Total Cost Summary (Individual).** Each individual ran an equal number of vehicle-level designs in section 1.5. From those analyses, each individual should use the minimum-mass and minimum-cost designs as a basis for this section. Now, run your group's system-level mass estimation code for the propellant mixture and stage ΔV-split for your two designs. Create a table of masses for each of the sub-systems considered in this system-level mass estimation, and include the 1st stage, 2nd stage, and total LV mass sums at the bottom of the table. Lastly, calculate the overall (NRE+production) costs of the launch vehicle program based on these *new system-level* estimation results. Present this total cost in the same table as the masses.

How do the mass totals estimated as part of the systems-level design compare to the mass totals estimated as part of your previous vehicle-level designs? What % difference is there between the mass estimated? Are the inert mass (δ) values different? Are these still the lowest-cost and lowest-mass launch vehicle designs compared to the vehicle-level matrix you presented in section 1.5?

2.3. **Optimal Designs and Reasoning (Group).** As a group, compile the 2 system-level results from each teammate from section 2.2. As you did in section 1.5, determine a good way to represent the total masses and costs of these results and produce two slides on this (graph/table/etc.).

Decide on which of the system-level designs presented by each group member are 1) the overall lowest-mass design and 2) the overall lowest-cost design. Produce one slide with *numerically-supported* reasoning for your selection. Remember, engineering is done with numbers. Analysis without numbers is only an opinion.

Finally, also create one slide with a discussion of which of these two (minimum-cost and minimum-mass) solutions is the overall optimal design that you would select for further development, and why.

3. Optimal Design CAD (Group)

Congratulations, you have conducted an exhaustive trade study for creating a two-stage launch vehicle with the given propellant options and mission requirements! Make CAD assembly models of your group's minimum-mass and minimum-cost launch vehicles.

3.1. **Computer-Aided Design.** Now we can use that trade study to produce a preliminary CAD model that accurately reflects the systems-level design step, since dimensions are known for all the fuel tanks, oxidizer tanks, engines, fairings, and structure. Construct a sub-assembly of each rocket stage with tank parts fitted into a structural fairing, then create an overall assembly of the various stacked stages and the payload fairing with a payload on top. Include the following components:

- Propellant (fuel/oxidizer) tanks, each as unique CAD parts.
- Stage fairing structure (fore/aft/inter-tank fairing), or the hollow cylinder your tanks will live in.
- Engines with number and size corresponding to your system-level analysis.
- Payload fairing. We don't fly flat cones or cylinders in real life.
- Sub-assembly of each stage with its propellant tanks inside its structural fairing.
- Overall launch vehicle assembly with stacked stages and payload fairing.

Split up the responsibilities for CAD part creation equally. For a five person team, please do as follows:

(1) First stage components and subassembly of the minimum-mass LV

(2) Second stage components and subassembly of the minimum-mass LV
(3) First stage components and subassembly of the minimum-cost LV
(4) Second stage components and subassembly of the minimum-cost LV
(5) Payload model (get creative) and payload fairing subassembly for both LV

For a four-person team, share the payload and fairing components reasonably. The final CAD assembly should be as easy as constraining 3 subassemblies together per rocket (1st, 2nd stage, payload). Both vehicles should be included in the same, single overall assembly side-by-side.

3.2. **Rendering.** Change the appearance of your parts to make them look nice. In Fusion, this is done by selecting a part, right clicking, and selecting appearance. Choose from the materials and colors Fusion provides. [Fusion Rendering Tutorial] https://www.autodesk.com/products/fusion-360/blog/create-realistic-fusion-360-renderings-tutorial/

(1) Make a photo-realistic, ray-traced, high-resolution render of the exterior of your rockets.
(2) Hide the structural shell parts of your rockets. Color your internal propellant tanks differently and create an internal component render to show off the fuel and oxidizer tank models. Label the tanks (outside of Fusion) to indicate what propellant they contain. **Also label who modelled which stage/payload.**
(3) Create one more unique and interesting image/diagram/animation of your choice to show off your creations, as demonstrated in the CAD lecture.

For some examples, see Fig. 1.

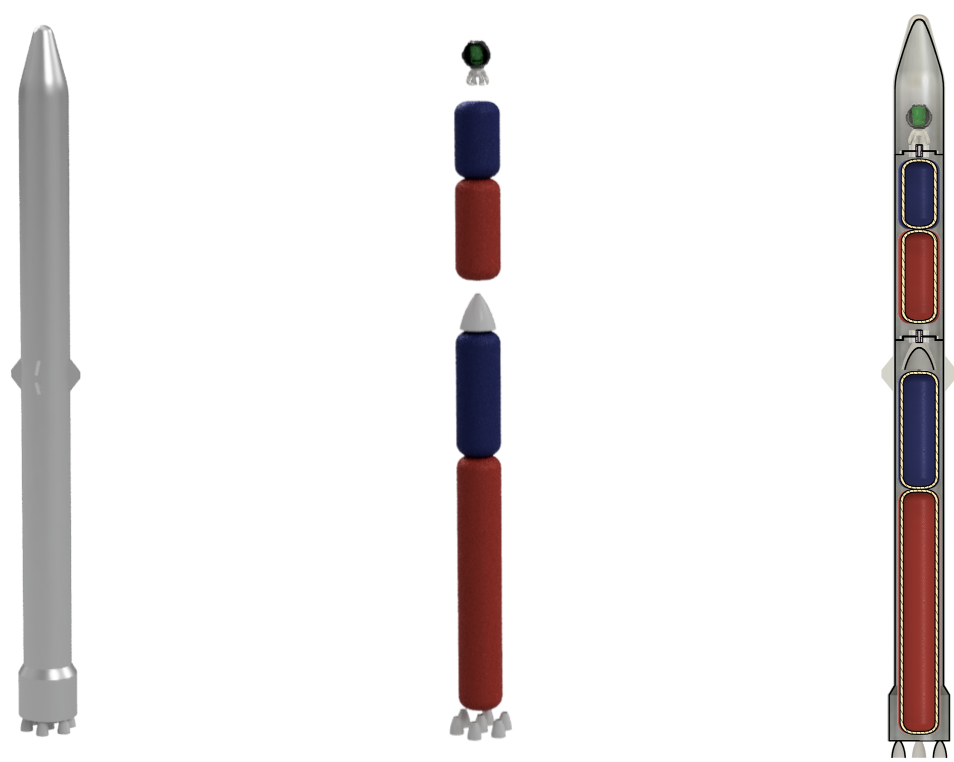


FIGURE 1. Examples for low-detail CAD renders and images - in ENAE484 you'll want a bit more detail! From left to right: full exterior ray-traced render, internal components ray-traced render, rocket cross-sectional image (in Fusion the only good way to render this is to cut all your parts in half). Note there are no flat endcaps on tanks!

3.3. **Engineering Drawing.** Through your CAD program, produce a dimensioned 3-view engineering drawing of your final designs. Be sure to include the following dimensions at least once across your 3 views:

- Overall launch vehicle height and widths
- Overall dimensions for each stage
- Dimensions for each propellant tank
- Dimensions of your engines (just dimension one of each unique size)

Now assemble your results into the group slide deck!

4. Peer Review

(Due later) Review the work of 2 other groups through the ELMS peer-review system. In a paragraph, leave a comment that discusses two aspects of the trade studies, results, and/or presentation that the group handled well compared to your own project, and also discuss one aspect that could be improved *and how you might improve it.* The goal of this is for students to experience analyzing other classmate's work while providing and accepting constructive criticism, which is a large part of ENAE484.

Appendix A. Submission Items/Rubric

ALL FILES WILL BE SUBMITTED AS A PDF FILES

Every person will submit their individual submission as a single pdf file.
Each group will submit their group submission as a single pdf file.

A.1. **File Naming Convention.** Each submission should be named using the following naming convention:

A.1.a. *Group File Naming Convention.*

Team#_P1_Submission#_Year.pdf
(*e.g.* Team∞_P1_Sub1_2024.pdf)

A.1.b. *Individual File Naming Convention.*

LastnameFirstname_P1_Submission#_Year.pdf
(*e.g.* LightyearBuzz_P1_Sub1_2024.pdf)

A.2. **Individual Items (Every Person).**

(I.1) Analysis of your chosen stage fuel combination (S.1.2.a & S.1.3.a)
- Chosen mass combination graph
- Chosen cost combination graph

(I.2) Table and comparison discussion of your two previous solutions (S.1.4, T.6)

(I.3) Full analysis of your assigned 5 stage fuel combinations. This will be a table that is contains the full set of combinations and optimzations as shown in the example table T.5. This table will be made using the group developed tools in sections S.1.2.b and S.1.3.b and should contain the full set of cost optimized and mass optimized combinations that you individually analyzed. (S.1.5)

(I.4) System-level mass and cost summary table (S.2.2)

(I.5) Comparison and discussion of system-level and vehicle-level findings (S.2.2)

(I.6) Copy of CAD render slide from group project (S.3.2)

(I.7) Engineering drawings developed in section S.3.3

(I.8) Peer Reviews (S.4)

A.3. **Group Items.**

(G.1) Slide deck of global optimal design (1.1)

1 Title slide with Team # and team member names
2 Full design matrix (T.4) labelled/coded with the responsibilities of each teammate
3 Vehicle-level trade study results graphic 1 (S.1.5)
4 Vehicle-level trade study results graphic 2 (S.1.5)
5 Vehicle-level results discussion 1 (S.1.5)
6 Vehicle-level results discussion 2 (S.1.5)
7 System-level results 1 (S.2.3)
8 System-level results 2 (S.2.3)
9 Overall lowest-mass and overall lowest-cost results (S.2.3)
10 Optimal design discussion (S.2.3)
11 Photo-realistic, ray-traced render of LVs exterior (S.3.2)
12 Labelled render of internal components (S.3.2)
13 Additional unique graphic (S.3.2)
14 Engineering drawing of LVs (S.3.3)

(G.2) Vehicle-level analysis tool (1.1)
- Code exported to .pdf. This should be well organized!
- Code Flow Chart exported to .pdf.

- Code Documentation with team member names in comments

(G.3) System-level analysis tool (2)
- Code exported to .pdf. This should be well organized!
- Code Flow Chart exported to .pdf.
- Code Documentation with team member names in comments

NOTE: The tables presented in this document are examples showing what is expected of a full data set. If your team comes up with a better way to present the data, please feel free to do so as long as all of the requested information is presented.

A.4. **Due Dates.** The table (T.7) below shows what items from sections A.2 and A.3 are due as part of each submission. Late submissions will be only accepted for 3 days following the due date to allow for both extensions and late submissions. After the 3 days it will be counted as if solutions have been posted. Teams will be accountable for making sure that their submission is complete and on time!

Each submission will include the relevant portions listed in Sections A.2 and A.3. *i.e.* for submission 1 the list of deliverables will only contain the parts completed for section 1. Submission 2 will contain all parts with submission 1 being copied into submission 2.

	What is due	Due date
Submission 1	Section 1	10/20/24 11:59pm
Submission 2	Section 2 & 3	11/3/24 11:59pm
Peer Review	Section 4	11/14/24 11:59pm

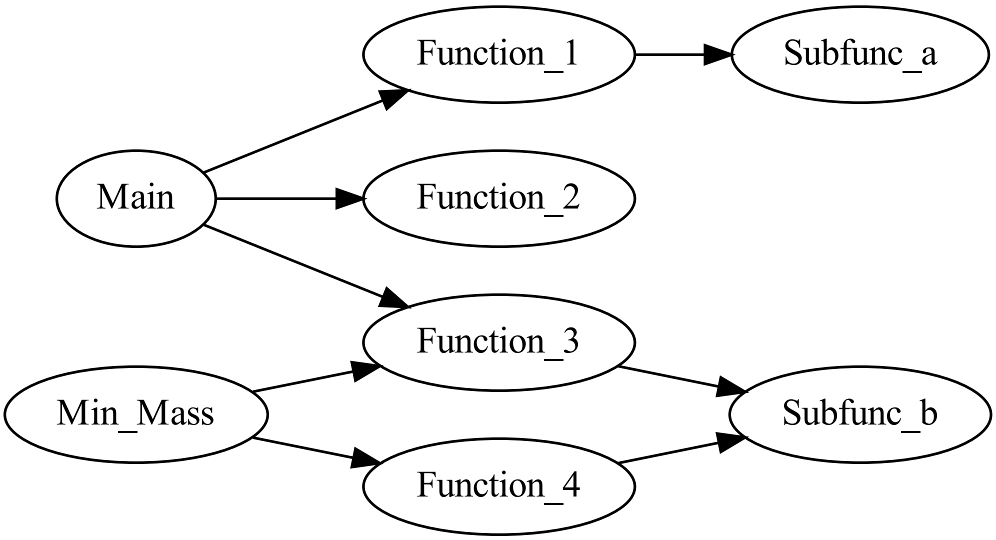
TABLE 7. Submission dates for each section

A.5. **Code/Analysis Tools.**

A.5.a. *Code Submission.* Your code should be submitted as a PDF file that contains each of your code files compiled into a single PDF. The code is expected to be well documented. The pdf file containing all of your team's code should be compiled into the group submission pdf at the end of the submitted slide deck.

The code submitted at the end of the slide deck should not be formatted as slides but as a "print-out" of the code files appended to the end of the slide deck pdf file

A.5.b. *Code Flowchart.* A code flowchart is due with every code submission. No one else can read your mind so this is a way for other users of your code to be able to see how the code is interconnected and which are the top level functions. You can use any tool you would like to use. Some suggested tools are draw.io, graphviz, Visio, LibreOffice Draw, and Doxygen for Matlab.



Example code flow chart diagramming which functions are called by each piece of code. These functions and sub functions should only be the code that you have written and not the built-in functions of your chosen tool.

A.6. **Slide Decks.** You will be submitting both an individual slide deck and a group slide deck.

A.6.a. *Group Slide Deck.* The group slide deck will be the main slide deck and will be where everything is turned in to tell the story of this trade study and how each team has come to the group conclusion that is presented in the slide deck. This slide deck will be slightly different than if you were presenting it to an audience as you will have to include your reasoning for the design decisions as concise text in the slide deck. You would *not* normally include this reasoning as text in a presentation that you are giving as you would be giving that reasoning verbally.

A.6.b. *Individual slide deck.* Each individual will submit the work that they contributed to the group slide deck as the first part of you individual submission. This will both show that you participated in coming to your own conclusions and then integrated and communicated them effectively to the team. The second portion will be a copy of the code portion contained within the group slide deck.

Appendix B. Useful Equations

Throat area of rocket engine is defined as the following equation (1):

(1)
$$A_t = \frac{\dot{m}C^*}{p_o}$$

Thrust equation (2):

(2)
$$T = \dot{m}v_e + (p_e - p_{amb})\, A_e$$

Exhaust velocity (3):

(3)
$$v_e = \sqrt{\frac{2\gamma}{\gamma - 1}\frac{\Re T_o}{\bar{M}}\left[1 - \left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Effective exhaust velocity (4):

(4)
$$T = \dot{m}c \Longrightarrow c = v_e + (p_e - p_{amb})\,\frac{A_e}{\dot{m}}, \quad \left(I_{sp} = \frac{c}{g_o}\right)$$

Expansion ratio (5):

(5)
$$\epsilon = \frac{A_t}{A_e} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}\left(\frac{p_e}{p_o}\right)^{\frac{1}{\gamma}}\sqrt{\frac{\gamma+1}{\gamma-1}\left[1 - \left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Delta V (6):

(6)
$$\Delta v = v_{\mathrm{e}} \ln\left(\frac{m_0}{m_f}\right) = I_{sp}(g_0)\ln\left(\frac{m_0}{m_f}\right)$$

Mass flow rate ($\frac{kg}{s}$):

(7)
$$\dot{m} = \frac{A_t v_t}{V_t} = A_t p_1 k \frac{\sqrt{[2/(k+1)]^{(k+1)/(k-1)}}}{\sqrt{kRT_1}}$$

A_t	Area of the throat (m^2)
A_e	Area of the exhaust (m^2)
C^*	Characteristic velocity ($\frac{m}{s}$)
g_o	Local gravitational constant ($\frac{m}{s^2}$)
p_e	Exhaust pressure (Pa)
p_o	Chamber pressure (Pa)
p_{amb}	Ambient pressure (Pa)
$\dot{m}$	Mass flow rate of engine ($\frac{kg}{s}$)
$\bar{M}$	Average molecular weight of exhaust
m_0	Mass initial (kg)
m_f	Mass final (kg)
$\Re$	Universal gas constant (8.3143 $\frac{Joules}{mole*Kelvin}$)
T_o	Chamber Temperature (Kelvin)
γ	Ratio of specific heat