ENAE 483/788D LECTURE #03 (SYSTEMS ANALYSIS) – FALL, 2024

We're going to again use some of the rocket performance calculations from Lecture 2, but this time you're going to do trade studies on the best design configuration.

Your goal is to design a launch vehicle optimized for delivering a payload to low Earth orbit (required $\Delta v = 9500$ m/sec). The first stage will use LOX/RP-1 with an exhaust velocity of 3300 m/sec. The second stage will use LOX/LH_2 with an exhaust velocity of 4500 m/sec. The design payload will be 100,000 kg. *Note: You will not be able to solve these problems analytically. You should set up the equations in Matlab or Excel and let their numerical solvers find the solution.*

(1) Assume that the inert mass fractions are $\delta_1 = 0.05$, and $\delta_2 = 0.08$. If the Δ v is distributed evenly between the stages, find m_0, $m_{pr,1}$, $m_{in,1}$, $m_{pr,2}$, and $m_{in,2}$

$$r_1 = e^{-\frac{\Delta v}{v_{e,1}}} = e^{-\frac{4750}{3300}} = 0.2371$$
$$r_2 = e^{-\frac{\Delta v}{v_{e,2}}} = e^{-\frac{4750}{4500}} = 0.3480$$
$$\lambda_2 = r_2 - \delta_2 = 0.3480 - 0.08 = 0.2680 \Longrightarrow m_{o,2} = \frac{m_{pl}}{\lambda_2} = \frac{100000}{0.2680} = \boxed{373{,}100 \text{ kg}}$$
$$m_{in,2} = \delta_2 m_{o,2} = 0.08(373100) = \boxed{29{,}850 \text{ kg}};\ m_{pr,2} = (1-r_2)m_{o,2} = (1-0.3480)373100 = \boxed{243{,}300 \text{ kg}}$$
$$\lambda_1 = r_1 - \delta_1 = 0.2371 - 0.05 = 0.1871 \Longrightarrow m_{o,1} = \frac{m_{o,2}}{\lambda_1} = \frac{373100}{0.1871} = \boxed{1{,}995{,}000 \text{ kg}}$$
$$m_{in,1} = \delta_1 m_{o,1} = 0.05(1995000) = \boxed{99{,}730 \text{ kg}};\ m_{pr,1} = (1-r_1)m_{o,1} = (1-0.2371)1995000 = \boxed{1{,}522{,}000 \text{ kg}}$$

(2) Continue to assume that the inert mass fractions are $\delta_1 = 0.05$, and $\delta_2 = 0.08$. Find the Δv distribution between the stages that minimizes the vehicle gross mass at liftoff m_0. (For the answer, list Δv_1, Δv_2, m_0, $m_{pr,1}$, $m_{in,1}$, $m_{pr,2}$, and $m_{in,2}$)

Let Δv_1 be a variable, so $\Delta v_2 = 9500 - \Delta v_1$. Set up all the equations above in Excel or Matlab and command it to find the value of Δv_1 that minimizes m_o $(= m_{o,1})$. You will get

$$\Delta v_2 = \boxed{6679 \text{ m/sec}} \quad m_{o,2} = \boxed{681{,}700 \text{ kg}} \quad m_{in,2} = \boxed{54{,}500 \text{ kg}} \quad m_{pr,2} = \boxed{527{,}100 \text{ kg}}$$
$$\Delta v_1 = \boxed{2822 \text{ m/sec}} \quad m_{o,1} = \boxed{1{,}816{,}000 \text{ kg}} \quad m_{in,1} = \boxed{90{,}800 \text{ kg}} \quad m_{pr,2} = \boxed{1{,}044{,}000 \text{ kg}}$$

(3) The heuristic equations for stage inert mass fraction ϵ $\left(\text{equals } \frac{m_{in}}{m_{in}+m_{pr}}\right)$ should be more accurate than assumed values for δ_1 and δ_2. Using the data you calculated in (2) as your initial estimate, calculate the predicted values for ϵ_1 and ϵ_2 using the equations from the Lecture 2 slide set. (The first stage will use the equation labeled "storables"; the second stage will obviously use the equation for LOX/LH_2.)

First calculate total stage masses $m_{st,2} = m_{in,2} + m_{pr,2}$=54500+527100=581,700 kg. Likewise, $m_{st,1} = m_{in,1} + m_{pr,1}$=90800+1044000=1,135,000 kg. Just for the fun of it, the calculated values of ϵ in this case are

$$\epsilon_{2,calc} = \frac{m_{in,2}}{m_{stage,2}} = \frac{54,500}{581,700} = 0.0938$$

$$\epsilon_{1,calc} = \frac{m_{in,1}}{m_{stage,1}} = \frac{90,800}{1,135,000} = 0.0800$$

From page 19 of the lecture slides,

$$\epsilon_{2,predicted} = 0.987 m_{st,2}^{-0.183} = 0.987(581,700)^{-0.183} = \boxed{0.0870}$$

$$\epsilon_{1,predicted} = 1.6062 m_{st,1}^{-0.275} = 1.6062(1,135,000)^{-0.275} = \boxed{0.0347}$$

(4) Using ϵ_1 and ϵ_2 from the previous problem, and keeping the total masses of each of the stages constant, calculate the new values for inert and propellant masses, and then calculate the $\Delta\ v$ for each stage. List Δv_{total}, Δv_1, Δv_2, m_0, $m_{pr,1}$, $m_{in,1}$, $m_{pr,2}$, and $m_{in,2}$

$$m_{in,2} = \epsilon_2 m_{st,2} = 0.0870(581,700) = \boxed{50{,}600};\ m_{pr,2} = m_{st,2} - m_{in,2} = 581,700 - 50,600 = \boxed{531{,}000 \text{ kg}}$$

$$m_{in,1} = \epsilon_1 m_{st,1} = 0.0347(1,135,000) = \boxed{39{,}400};\ m_{pr,1} = m_{st,1} - m_{in,1} = 1,135,000 - 39,400 = \boxed{1{,}095{,}000 \text{ kg}}$$

It was specified that the overall stage masses do no change, so

$$r_2 = \frac{m_{final}}{m_{initial}} = \frac{m_{in,2} + m_{pl}}{m_{o,2}} = \frac{m_{in,2} + m_{pl}}{m_{st,2} + m_{pl}} = \frac{50,600 + 100,000}{581,700 + 100,00} = 0.2209$$

$$r_1 = \frac{m_{final}}{m_{initial}} = \frac{m_{in,1} + m_{o,2}}{m_{o,1}} = \frac{m_{in,1} + m_{st,2} + m_{pl}}{m_{st,1} + m_{st,2} + m_{pl}} = \frac{39,400 + 581,700 + 100,000}{1,135,000 + 581,700 + 100,000} = 0.3970$$

$$\Delta v_1 = -v_{e,1} \ln r_1 = -3300 \ln 0.3970 = \boxed{3049\ \tfrac{m}{sec}};\ \Delta v_2 = -v_{e,2} \ln r_2 = -4500 \ln 0.2209 = \boxed{6795\ \tfrac{m}{sec}}$$

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 = 3049 + 6795 = \boxed{9844\ \tfrac{m}{sec}}$$

Out of curiosity, use these new parameters to calculate the values of δ_1 and δ_2 that correspond to these predicted values of ϵ_1 and ϵ_2 to find $\delta_{2,new} = 0.0742$ and $\delta_{1,new} = 0.0217$

(5) You will find that the total Δv is changed somewhat due to the revised mass numbers. Using the vehicle configuration you derived in (4), what is the new payload capacity to low Earth orbit at $\Delta v_{total} = 9500$ m/sec?

Using the new mass numbers derived above, use a similar set-up as in (2) to solve for the unknown payload mass that results in a Δv_{total} of 9,500 m/sec. This turns out to be $\boxed{\textit{113,900 kg}}$

Be aware that when you changed from using δ to ϵ in (3), you were no longer working with an optimal solution. The next step after (5) would be to iterate the entire process to find the new optimum Δv distribution with the new mass values. This will be considerably more work, and I'm not going to ask you to do it in this homework. You will, however, be doing it as teams in the first term project.