ENAE 483/788D LECTURE #06 (RELIABILITY) PROBLEMS – FALL, 2024

(1) Falcon 9 flew successfully 18 times prior to its first failure. What was its reliability prior to the failure at an 80% confidence?

Here we can assign our values to the individual variables. In this case we have a confidence of 80% so $C = 0.80,\ N = 18$, *we can use the equation on page 18 of the slides:*

$$R^N + C = 1 \Longrightarrow R = (1-C)^{\frac{1}{N}} = (1-0.80)^{\frac{1}{18}} = (0.2)^{\frac{1}{18}} = \boxed{0.9145}$$

(2) What was its reliability after the failure at the same confidence level?

To calculate the reliability after the failure, inclusive of the 80% confidence level, the general form of the probability/confidence relationship is described as in equation 1.

$$P(ObservedAndAllBetterOutcomes) + C = 1 \tag{1}$$

In this scenario, the "Observed" case is that of one failure in the 19 total flights. The probability of "All Better Outcomes" would be the probability of 19 successful flights. The probability for each of these scenarios can be calculated using the general form of the probability function as defined on slide 22 (equation 2):

$$P(K \mid N) = \frac{N!}{K!\,(N-K)!} R^K (1-R)^{N-K} \tag{2}$$

This equation utilizes the reliability, R, to calculate the probability of a given scenario. The pair of probabilities as described above can be written:

$$P(ObservedAndAllBetterOutcomes) = P(19 \mid 19) + P(18 \mid 19)$$

$$P(19 \mid 19) = \frac{19!}{19!\,(19-19)!} R^{19} (1-R)^{19-19} = R^{19}$$

$$P(18 \mid 19) = \frac{19!}{18!\,(19-18)!} R^{19} (1-R)^{19-18} = 19R^{18}(1-R)^1$$

Summarizing these two, and preparing to solve for R provides the following equation:

$$R^{19} + 19R^{18}(1-R)^1 + 0.8 = 1 \rightarrow R^{19} + 19R^{18}(1-R)^1 - 0.2 = 0$$

Solve numerically in Excel or Matlab to get that the Falcon 9 reliability at 80% confidence is $\boxed{0.8504}$

(3) Falcon 9 had a second in-flight failure on mission 354. If you assert that its reliability is 99%, what is the confidence in this estimate prior to the failure?

The definition for a system's confidence is shown in equation 1. In this scenario, the "observed" outcome is the single failure (P(352)) and the "all better outcomes" is the outcome that all flights occurred without failure (P(353)).

The total probability estimate for the first 353 missions can be determined using equation 2.

$$P(ObservedAndAllBetterOutcomes) = P(353) + P(352)$$

$$P(353) = (0.99)^{353} = 0.0287$$

$$P(352) = \frac{353!}{352!\,(1)!} R^{352}(1-R)^{1} = 0.1027$$

$$0.0287 + 0.1027 + C = 1$$

$$0.1314 + C = 1 \Longrightarrow C = \boxed{0.8686}$$

(4) What is the confidence in this estimate after the failure?

In this case, the same method can be used, but now including the two failure observed case.

$$P(ObservedAndAllBetterOutcomes) = P(354) + P(353) + P(352)$$

$$P(354) = \frac{354!}{354!\,(1)!} R^{354} = (0.99)^{354} = 0.0285$$

$$P(353) = \frac{354!}{353!\,(1)!} R^{353}(1-R)^{1} = 0.1019$$

$$P(352) = \frac{354!}{352!\,(1)!} R^{352}(1-R)^{2} = 0.1817$$

$$C = 1 - (P(354) + P(353) + P(352)) = 1 - 0.3121 = \boxed{0.6879}$$

(5) Falcon 9 Block 5 first-stage boosters have landed successfully 326 times on 331 attempts. At an 80% confidence level, what is the reliability of first stage landing?

We need to now work backwards to the previous problems, to find an R value given a C, still using equation 1. In this case, C = 0.80, requiring that $P(ObservedAndAllBetterOutcomes) = 0.2$.

$$P(ObservedAndAllBetterOutcomes) = \sum_{K=326}^{331} \frac{N!}{K!\,(N-K)!} R^{K}(1-R)^{N-K}$$

which is equivalent to:

$$P(ObservedAndAllBetterOutcomes) = \sum_{K=326}^{331} \left[\left(\prod_{z=0}^{(N-K)} \frac{(N-z)}{z+1} \right) R^{(K)}(1-R)^{(N-K)} \right]$$

Expanded out, this equation expands to, with N=331:

$$R^N + NR^{N-1}(1-R) + \frac{N(N-1)}{2}R^{N-2}(1-R)^2$$
$$+ \frac{N(N-1)(N-2)}{2(3)}R^{N-3}(1-R)^3 + \frac{N(N-1)(N-2)(N-3)}{2(3)(4)}R^{N-4}(1-R)^4$$
$$+ \frac{N(N-1)(N-2)(N-3)(N-4)}{2(3)(4)(5)}R^{N-5}(1-R)^5$$
$$= R^{331} + 331R^{330}\cdot(1-R) + 54,615R^{329}\,(1-R)^2 + 5,989,445R^{328}\,(1-R)^3$$
$$+ 491,134,490R^{327}\,(1-R)^4 + 32,120,195,646R^{326}\,(1-R)^5$$
$$= 0.2$$

Solve in Matlab or Excel to get $\boxed{R=0.9762}$

(6) There are 33 Raptor rocket engines on the first stage of the Starship first stage. If each engine is 99% reliable, what is the probability of no failures during a launch?

The probability of 0 failures is the same probability as all 33 Raptor engines operating successfully:

$$P(33) = P^{33} = (0.99)^{33} = \boxed{0.7177}$$

(7) If Starship can survive two Raptor failures during the first stage burn, what is the probability of a successful first stage launch?

The requirement for a successful first stage launch are broken down into 3 cases: 1) All 33 Raptor engines work (P(33)), 2) Exactly 32 engines work and 1 fails(P(32)) , and 3) Exactly 31 engines work and 2 fail (P(31)). The total probability can be expressed as the sum of each of these probabilities. Equation 2 is used to ensure all possibilities within each case are accounted for.

$$P(32) = P(32 \mid 33) = \frac{33!}{32!\,(1)!}R^{32}(1-R)^1 = 0.2392$$
$$P(31) = P(31 \mid 33) = \frac{33!}{31!\,(2)!}R^{31}(1-R)^2 = 0.0386$$

$$P_{Total} = P(33) + P(32) + P(31) = 0.7177 + 0.2392 + 0.0386 = \boxed{0.9956}$$

(8) How does your answer to the previous question change if there is a 20% intercorrelation rate in Raptor failures?

From Lecture 7, Slide 31 the intercorrelated failure rate is the "probability f that the failure causes a total system failure" - in this case: f = 0.2. For each of the probabilities calculated in problem 7, the probability that each failure could cause an intercorrelated failure needs to be included. With f = 0.2, the probability that the failures do not cause failure is 0.8.

$$P_{Total} = P(33) + P_{safe}(32) + P_{safe}(31)$$

Where P_{safe} is the probability that each scenario with a failure does not cause an intercorrelated failure where $(1-f)^n$ is the chance that n failures do not result in intercorrelated failure.

$$P_{safe}(32) = (1-f)P(32) = 0.8 * 0.2392 = 0.1914$$

$$P_{safe}(31) = (1-f)^2 P(31) = 0.8^2 * 0.0386 = 0.0247$$

$$P_{Total} = 0.7177 + 0.1914 + 0.0247 = \boxed{0.9338}$$

(9) In this phase of the program, Starship should nominally fly every 4 weeks. Due to regulatory issues, it is down for 16 weeks following a failure. In a surge condition, it can fly every 3 weeks. To be resilient with no loss of payloads in the manifest, what is the minimum number of successful launches expected on average between failures?

We must convert the inputs into proper units for the resiliency formula: $r = 13$ *flts/yr,* $d = 0.3077$ *yrs,* $S = 17.33/13 = 1.333$*, and no loss of payloads means that k=1. Now we solve for m:*

$$\frac{Srkd}{S-1} \le m \Longrightarrow \frac{(1.333)*(13)*(1)*(0.3077)}{(1.333)-1} = 16 \le m$$

$$m \ge \boxed{\text{16 flights expected between failures for resiliency}}$$

(10) Sending a Starship to Mars and landing will require onboard systems to operate continuously for 300 days. For a single unit, what would the required mean time between failure (MTBF) be, in hours, for a 99% reliability on the mission to Mars?

300 days = 7,200 hours

$$R = e^{-\frac{t}{MTBF}} \Longrightarrow MTBF = -\frac{t}{lnR} = -\frac{7,200}{ln(0.99)} = \boxed{\text{716,400 hrs}}$$

(11) There are 10 LED lighting units in the Starship habitat for the trip to Mars, each of which is 99% reliable over that mission duration. How many spares should they carry to have a 99% chance of having 10 functional units when you arrive at Mars?

First of all, I apologize for not thinking this through, I really didn't cover spares enough in lecture for this problem. (I won't ask a spares analysis question on the exam.) But, since you should learn how to do it, ...

Probability of all 10 working $= R^{10} = 0.99^{10} = 0.9044$

Probability of 1 failure (1-0.9044) and all 10 working after the replacement (0.9044) =(0.0956)(0.9044)=0.0865

Probability that all 10 work or that 10/11 work=0.9044+0.0865=0.9909 so $\boxed{\textit{1 spare is necessary}}$