Mass Estimating Relations

- Lecture #8 - September 19, 2024
- Review of iterative design approach
- Mass Estimating Relations (MERs)
- Sample vehicle design analysis

Akin's Laws of Spacecraft Design - #3

Design is an iterative process. The necessary number of iterations is one more than the number you have currently done. This is true at any point in time.

Overview of the Design Process

Program Objectives
System Requirements

Vehicle-level Estimation
(based on a few parameters from prior art)

System-level Estimation
(system parameters based on prior experience)

System-level Design
(based on discipline-oriented analysis)

Basic Axiom: Relative rankings between competing systems will remain consistent from level to level

Increasing complexity

Increasing accuracy

Decreasing ability to comprehend the "big picture"

Vehicle-Level Prelim Design - 1st Pass

- Single Stage to Orbit (SSTO) vehicle
- ΔV=9200 m/sec
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec (Ve=4214 m/sec)
  - δ=0.08

$$r = e^{-\frac{\Delta V}{V_e}} = 0.1127$$

$$\lambda = r - \delta = 0.0327$$

$$M_o = \frac{M_\ell}{\lambda} = 153,000\ kg$$

$$M_i = \delta M_o = 12,240\ kg$$

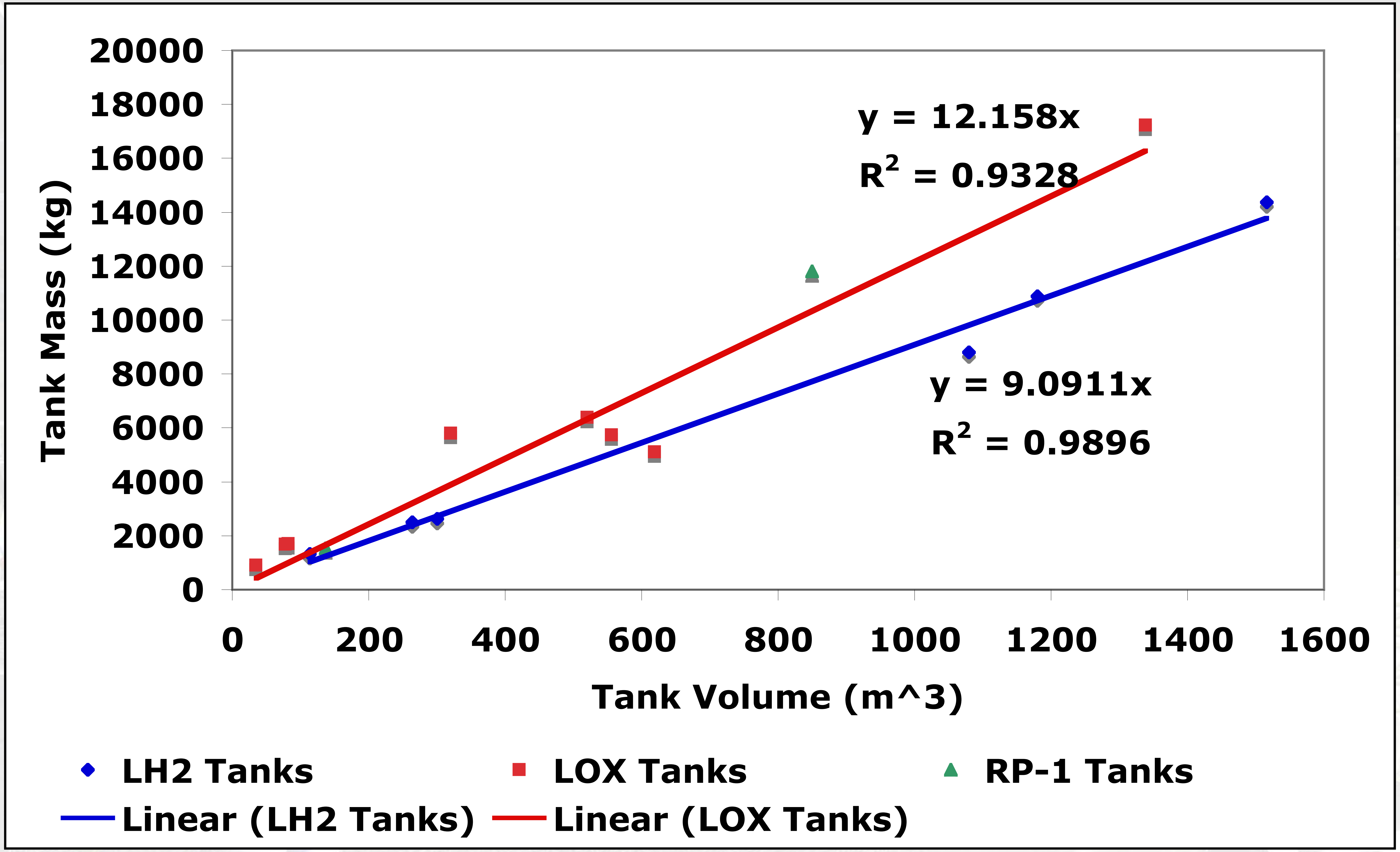
$$M_p = M_o(1 - r) = 135,800\ kg$$

System-Level Estimation

- Start with propellant tanks (biggest part)
- LOX/LH2 engines generally run at mixture ratio of 6:1 (by weight)
  - LH2: 19,390 kg
  - LOX: 116,400 kg
- Propellant densities

$$\rho_{LOX} = 1140 \ \frac{kg}{m^3} \qquad \rho_{LH_2} = 71 \ \frac{kg}{m^3}$$

Propellant Tank Regression Data



Propellant Tank MERs (Volume)

- LH_2 tanks

$$M_{LH_2\ Tank}\langle kg\rangle = 9.09V_{LH_2}\langle m^3\rangle$$

- All other tanks (e.g., LOX, RP-1, LCH_4, hypergols)

$$M_{Tank}\langle kg\rangle = 12.16V_{prop}\langle m^3\rangle$$

Propellant Tank MERs (Mass)

- LH_2 tanks

$$\rho_{LH_2} = 71 \frac{kg}{m^3} \Longrightarrow M_{LH_2\ Tank}\langle kg \rangle = 0.128 M_{LH_2}\langle kg \rangle$$

- LOX tanks

$$\rho_{LOX} = 1140 \frac{kg}{m^3} \Longrightarrow M_{LOX\ Tank}\langle kg \rangle = 0.0107 M_{LOX}\langle kg \rangle$$

- RP-1 tanks

$$\rho_{RP1} = 820 \frac{kg}{m^3} \Longrightarrow M_{RP1\ Tank}\langle kg \rangle = 0.0148 M_{RP1}\langle kg \rangle$$

- LCH_4 tanks

$$\rho_{LCH_4} = 423 \frac{kg}{m^3} \Longrightarrow M_{LCH_4\ Tank}\langle kg \rangle = 0.0287\ M_{LCH_4}\langle kg \rangle$$

Cryogenic Insulation MERs

$$M_{LH_2\ Insulation}\langle kg\rangle = 2.88 A_{tank}\langle\frac{kg}{m^2}\rangle$$

$$M_{LOX/LCH_4\ Insulation}\langle kg\rangle = 1.123\ A_{tank}\ \langle\frac{kg}{m^2}\rangle$$

LOX Tank Design

- Mass of LOX=116,400 kg

$$M_{LOX\ Tank} = 0.0107(116,400) = 1245\ kg$$

- Need area to find LOX tank insulation mass - assume a sphere

$$V_{LOX\ Tank} = \frac{M_{LOX}}{\rho_{LOX}} = 102.1\ m^3$$

$$r_{LOX\ Tank} = \left(\frac{V_{LOX}}{4\pi/3}\right)^{\frac{1}{3}} = 2.90\ m$$

$$A_{LOX\ Tank} = 4\pi r^2 = 105.6\ m^2$$

$$M_{LOX\ Insulation} = 1.123\langle\frac{kg}{m^2}\rangle(105.6\langle m^2\rangle) = 119\ kg$$

LH_2 Tank Design

- Mass of LH_2=19,390 kg

$$M_{LH_2\ Tank}\langle kg\rangle = 0.128(19,390) = 2482\ kg$$

- Again, assume LH_2 tank is spherical

$$V_{LH_2\ Tank} = \frac{M_{LH_2}}{\rho_{LH_2}} = 273.1\ m^3$$

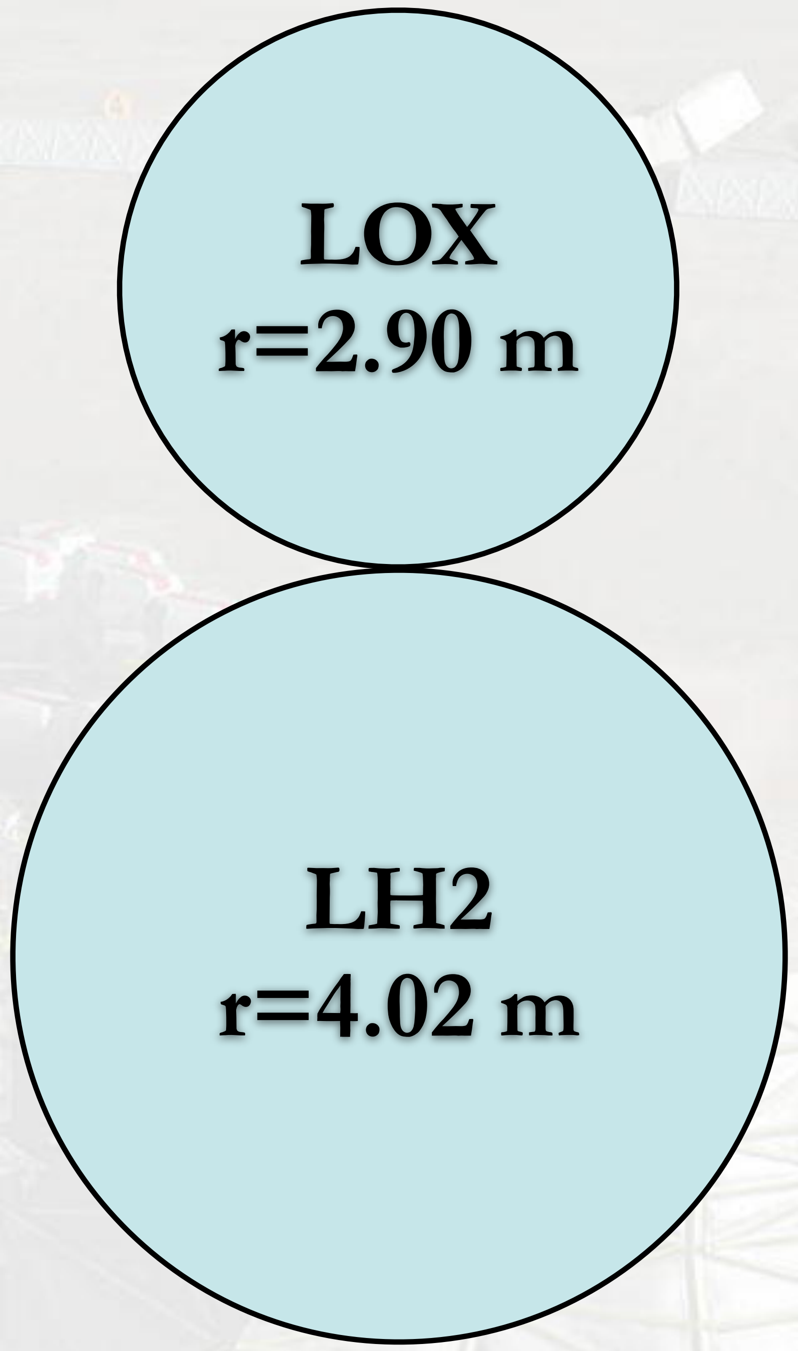
$$r_{LH_2\ Tank} = \left(\frac{V_{LH_2}}{4\pi/3}\right)^{\frac{1}{3}} = 4.02\ m$$

$$A_{LH_2\ Tank} = 4\pi r^2 = 203.6\ m^2$$

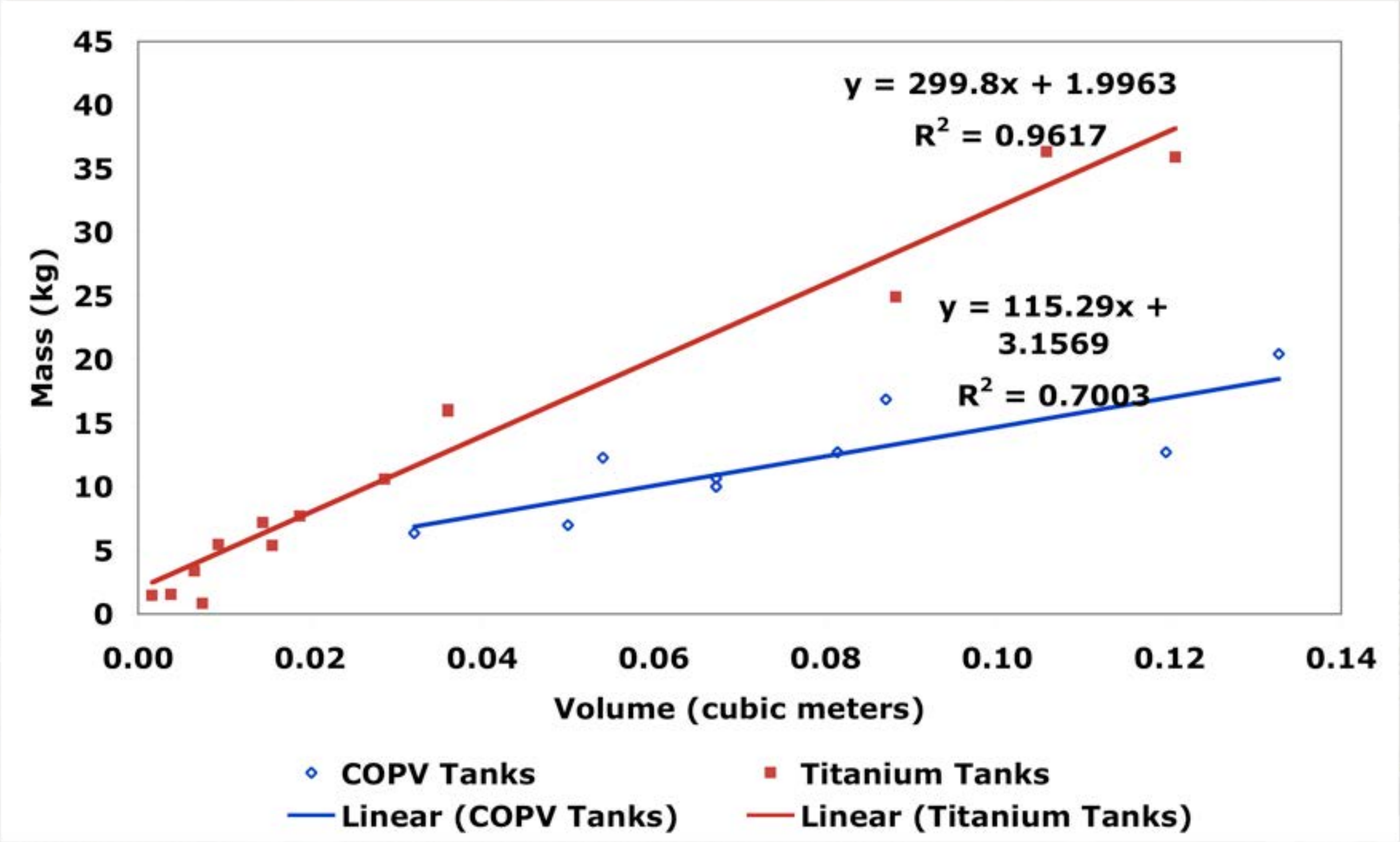
$$M_{LH_2\ Insulation} = 2.88\langle\frac{kg}{m^2}\rangle(203.6\langle m^2\rangle) = 586\ kg$$

Current Design Sketch

- Masses
  - LOX Tank 1245 kg
  - LOX Tank Insulation 119 kg
  - LH_2 Tank 2482 kg
  - LH_2 Tank Insulation 586 kg



High-Pressure Gas Tanks



Pressurized Gas Tank MERs

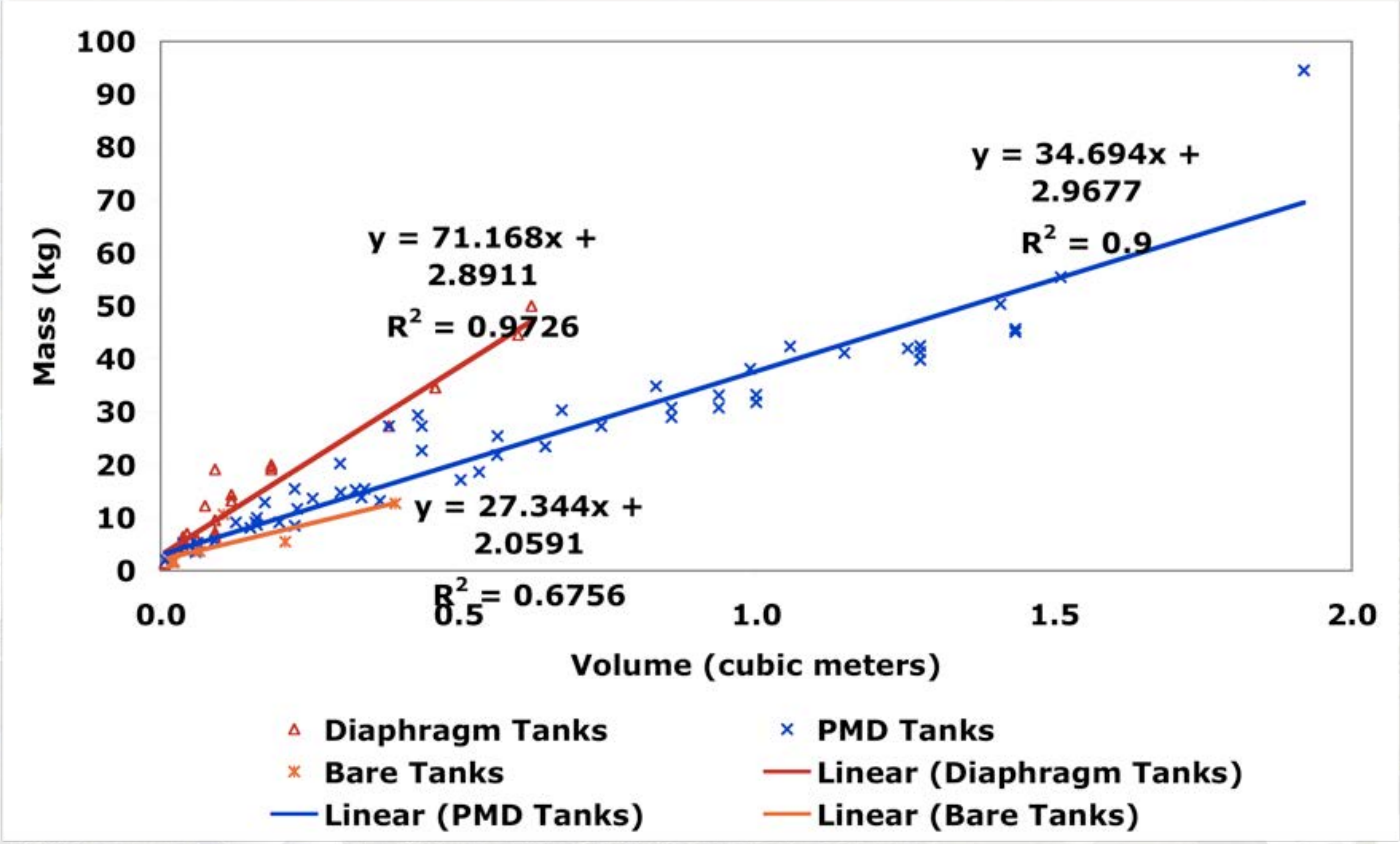
- COPV (Composite Overwrapped Pressure Vessel)

$$M_{COPV\ Tank}(kg) = 115.3\ V_{contents}(m^3) + 3$$

- Titanium tank

$$M_{Ti\ Tank}(kg) = 299.8\ V_{contents}(m^3) + 2$$

Smaller Storable Liquids Tanks



Small Liquid Tankage MERs

- Bare metal tanks

$$M_{Bare\ Tank}(kg) = 27.34\ V_{contents}(m^3) + 2$$

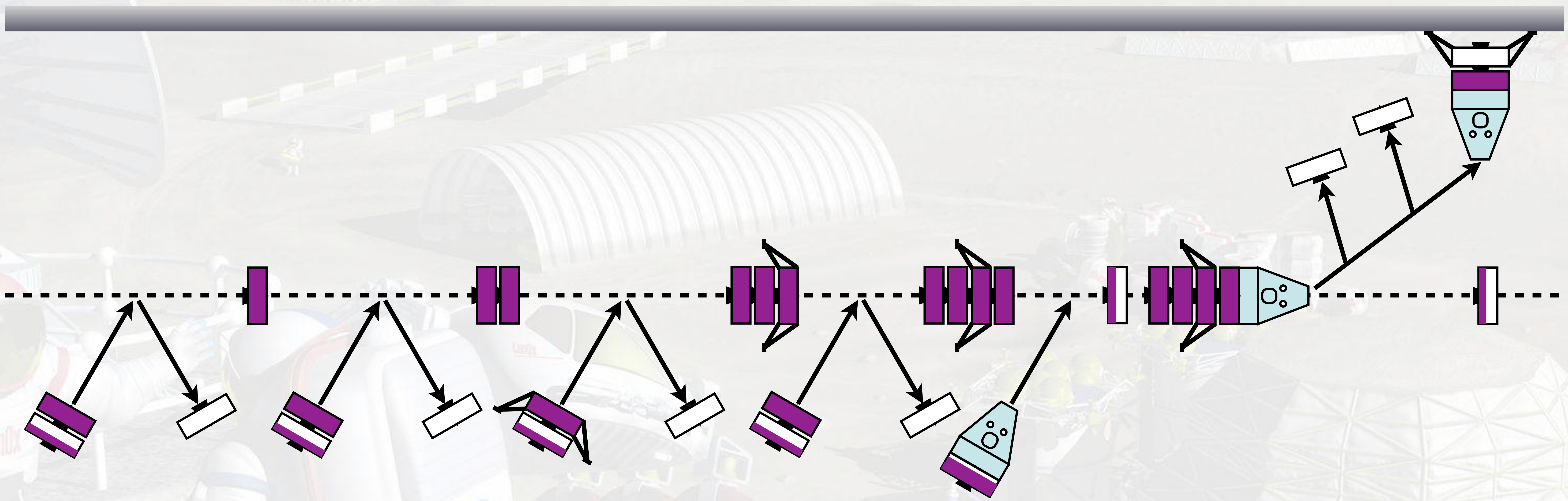
- Tanks with propellant management devices

$$M_{PMD\ Tank}(kg) = 34.69\ V_{contents}(m^3) + 3$$

- Titanium tanks with positive expulsion bladders

$$M_{Diaphragm\ Tank}(kg) = 71.17\ V_{contents}(m^3) + 3$$

Minimum Cost Lunar Architecture



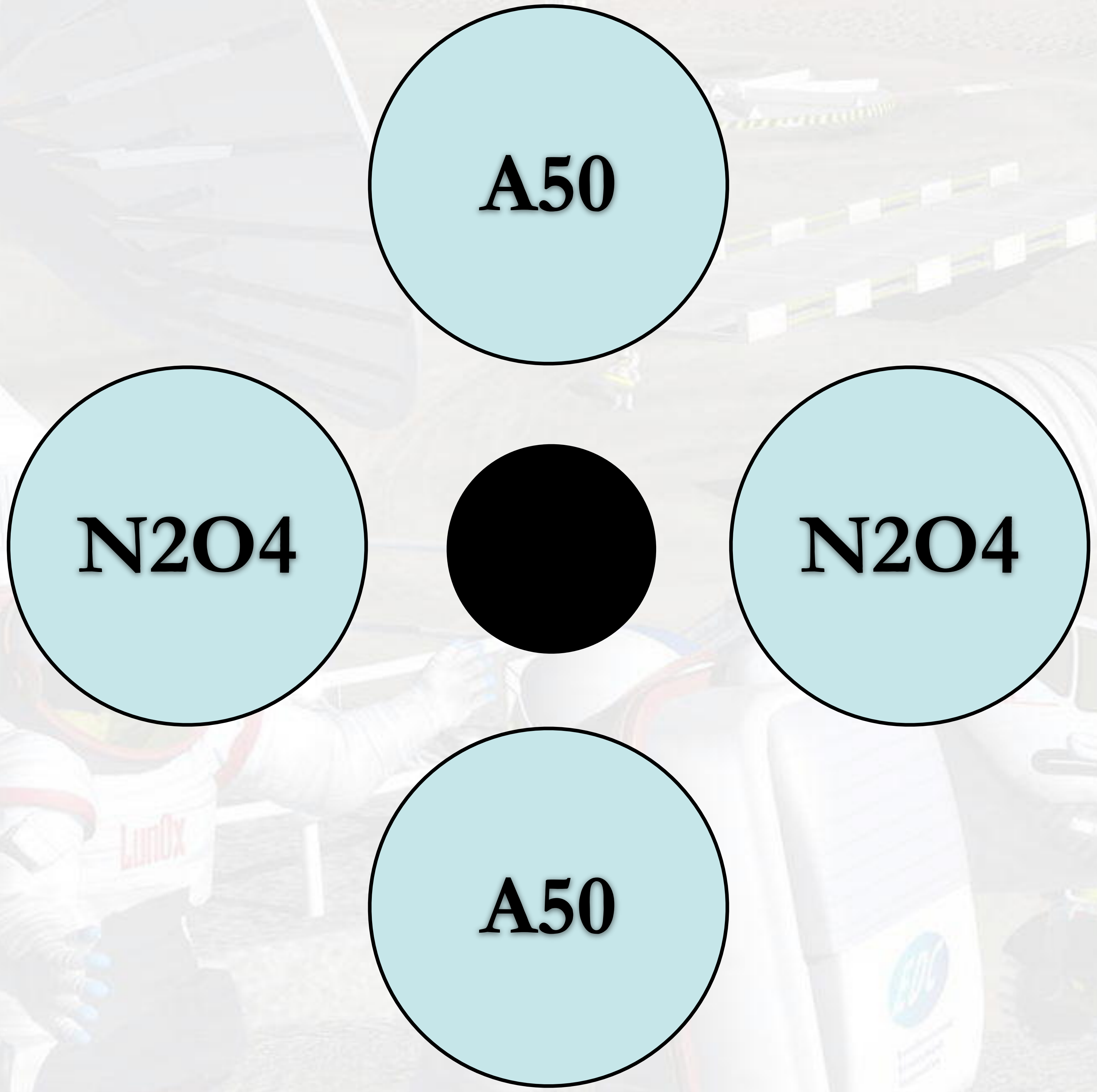
Orbital Maneuvering Stage (OMS)

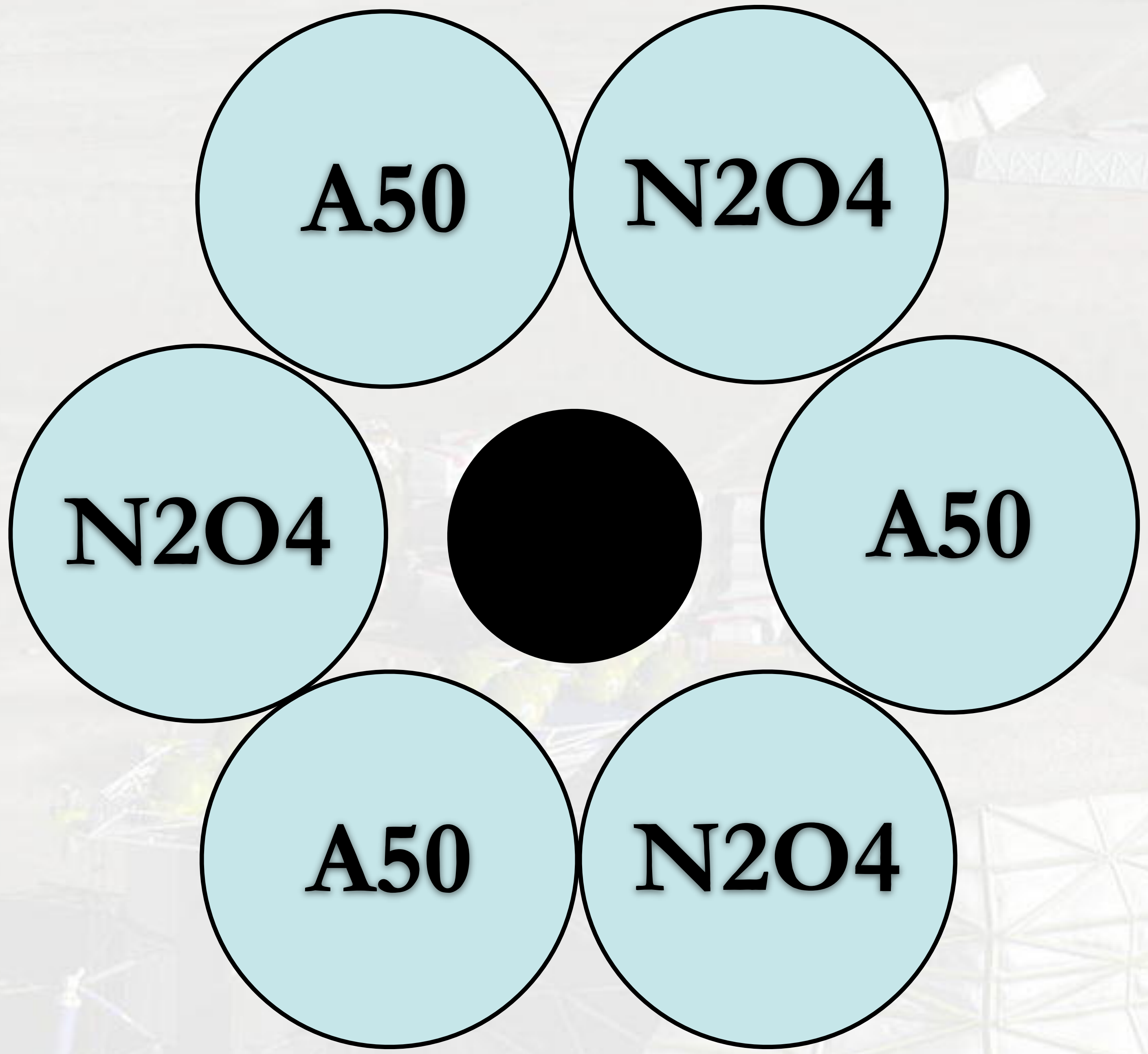
- Gross mass 6950 kg
  - Inert mass 695 kg
  - Propellant mass 6255 kg
  - Mixture ratio N_2O_4/UDMH = 2.0 (by mass)
- N_2O_4 tank
  - Mass = 4170 kg
  - Density = 1450 kg/m^3
  - Volume = 2.876 m^3
- UDMH tank
  - Mass = 2085 kg
  - Density = 793 kg/m^3
  - Volume = 2.629 m^3

N_2O_4 Tank Sizing

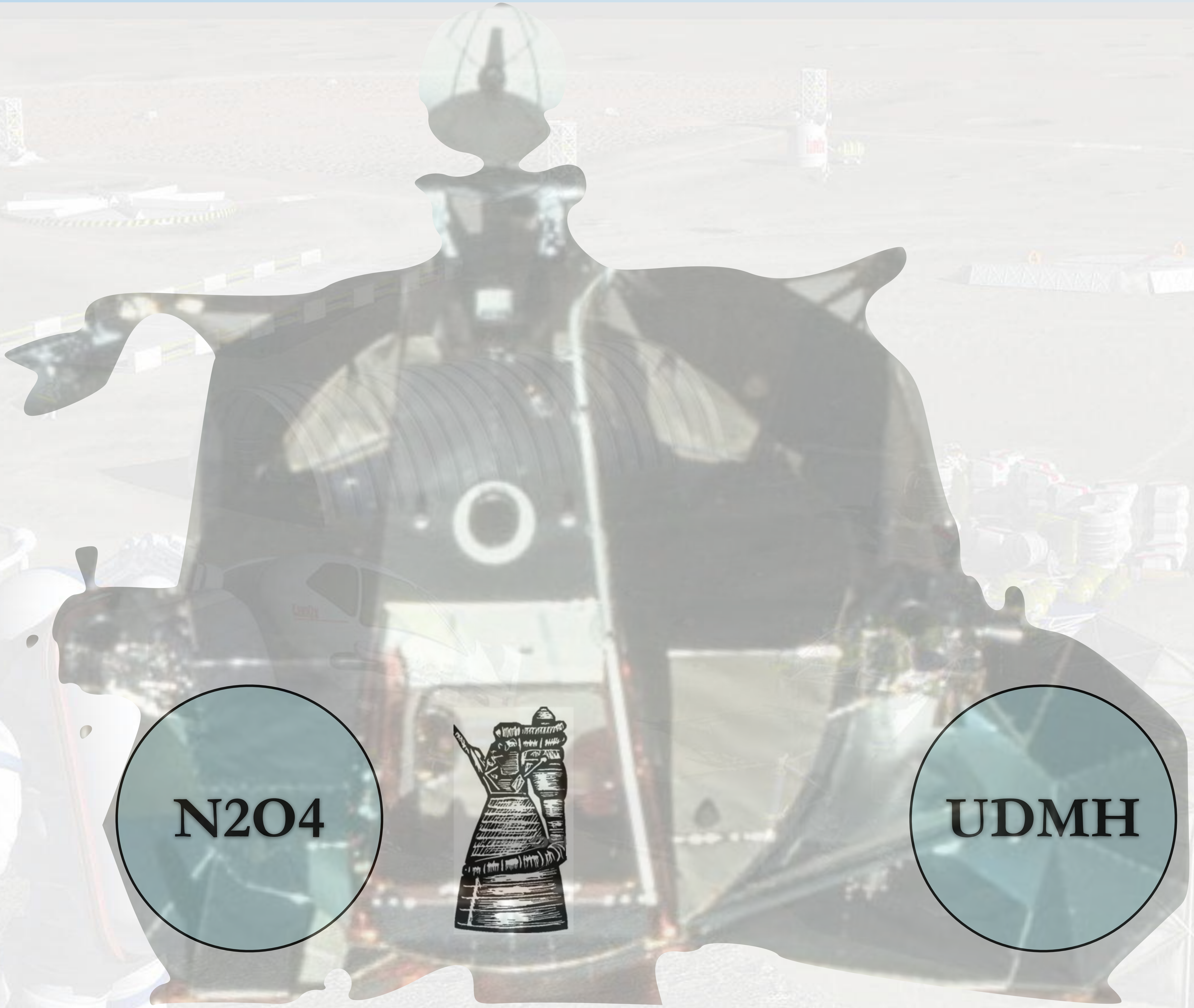
- Need total N_2O_4 volume = 2.876 m^3
- Single PMD tank
  - Radius = 0.882 m
  - Mass = 102.8 kg
- Dual PMD tanks
  - Radius = 0.700 m
  - Mass = 52.9 kg (x2 = 105.8 kg)
- Triple PMD tanks
  - Radius = 0.612 m
  - Mass = 36.3 kg (x3 = 108.9 kg)

Apollo LM Ascent Tank Configurations





LM Ascent Stage Final Tank Configuration



Other Structural MERs

- Fairings and shrouds

$$M_{fairing}\langle kg\rangle = 4.95\left(A_{fairing}\langle m^2\rangle\right)^{1.15}$$

- Avionics

$$M_{avionics}\langle kg\rangle = 10\left(M_o\langle kg\rangle\right)^{0.361}$$

- Wiring

$$M_{wiring}\langle kg\rangle = 1.058\sqrt{M_o\langle kg\rangle}\,\ell^{0.25}$$

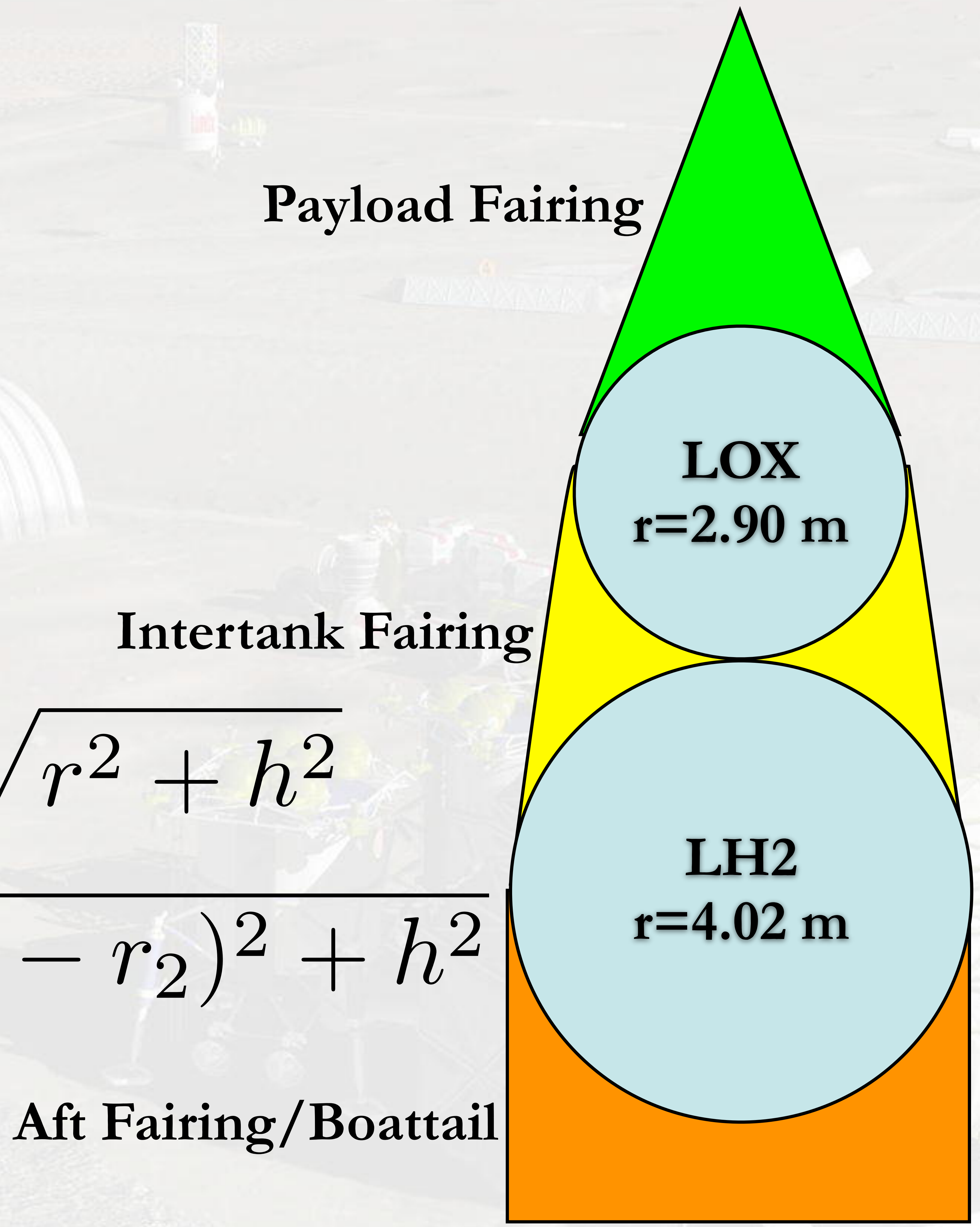
External Fairings - First Cut

- Masses
  - LOX Tank 1245 kg
  - LOX Tank Insulation 119 kg
  - LH_2 Tank 2482 kg
  - LH_2 Tank Insulation 586 kg

$$A_{cone} = \pi r \sqrt{r^2 + h^2}$$

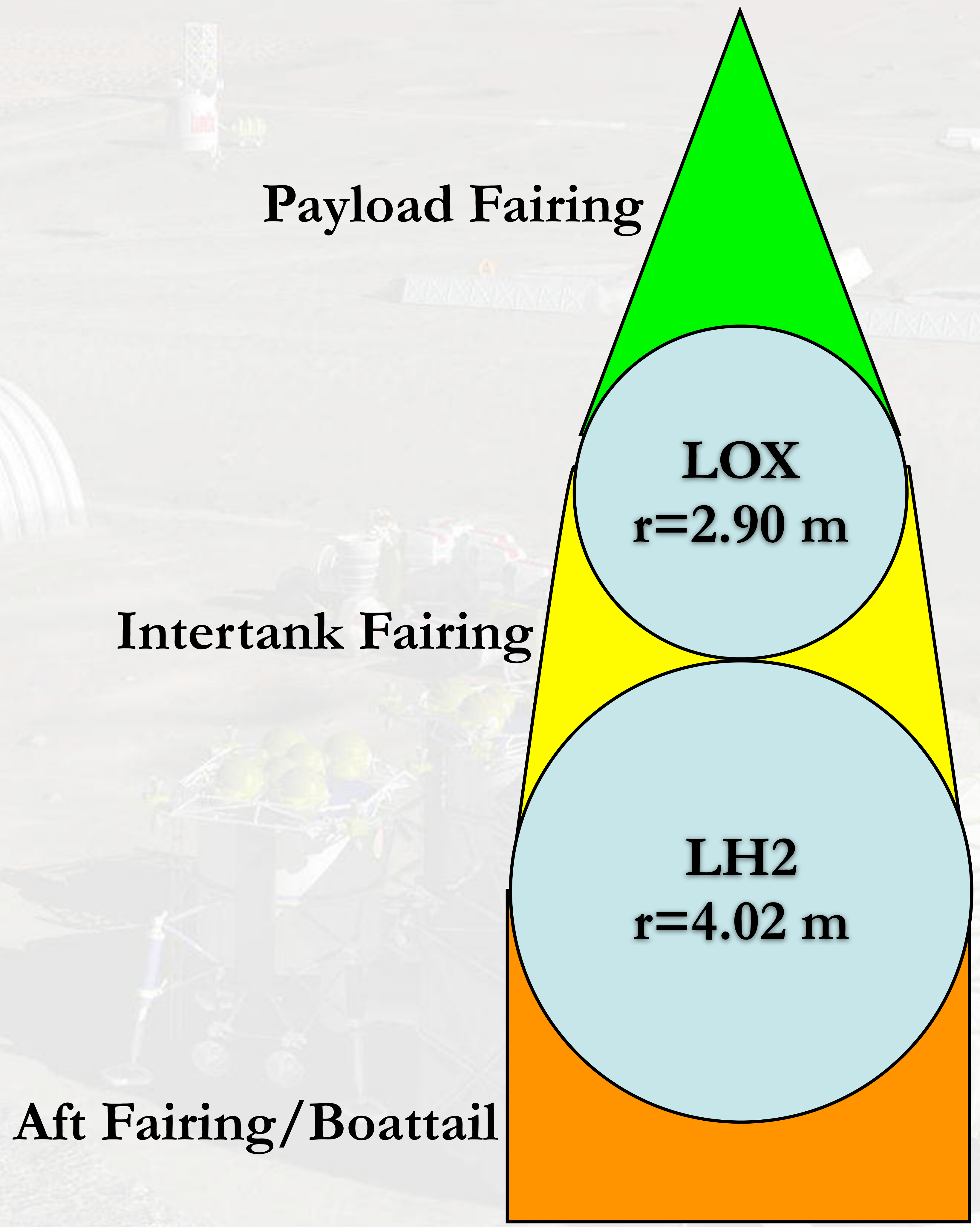
$$A_{frustrum} = \pi (r_1 + r_2) \sqrt{(r_1 - r_2)^2 + h^2}$$

$$A_{cylinder} = 2\pi r h$$



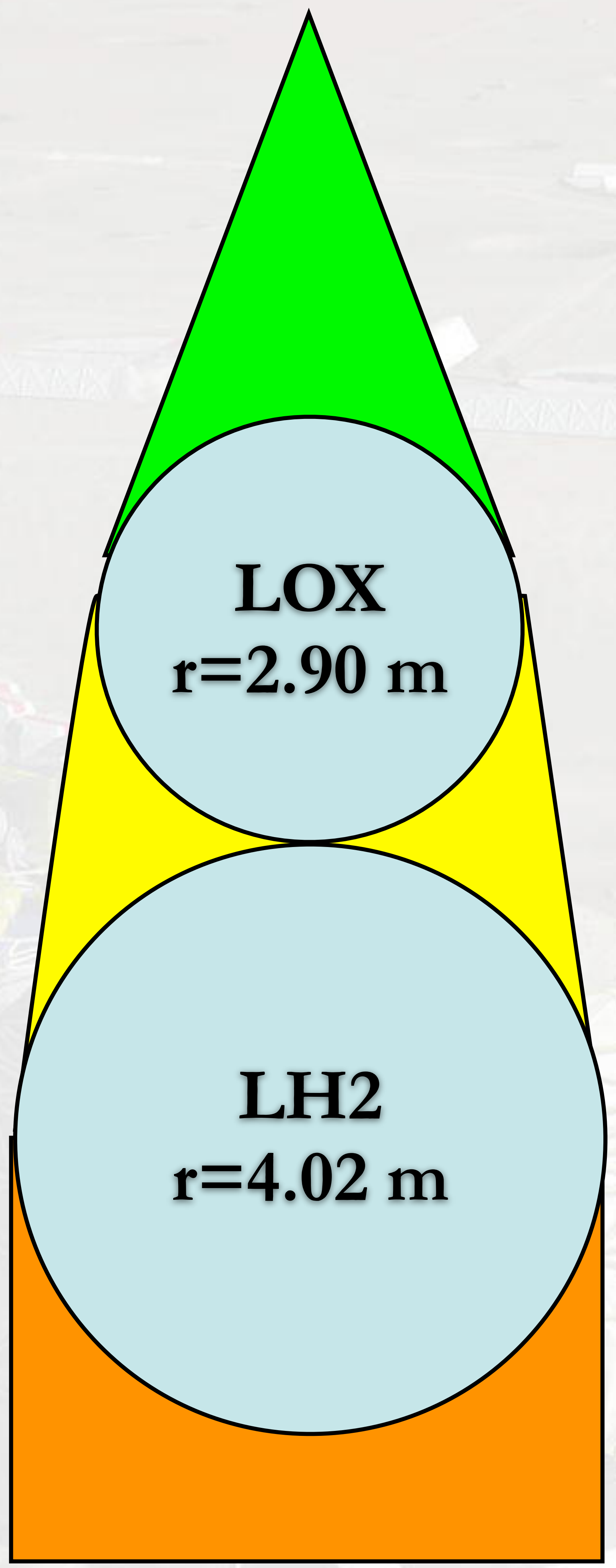
External Fairings - First Cut

- Assumptions
  - P/L fairing h 7 m
  - P/L fairing r 2.9 m
  - I/T fairing h 7 m
  - I/T fairing r_1 4.02 m
  - I/T fairing r_2 2.9 m
  - Aft fairing h 7 m
  - Aft fairing r 4.02 m



Fairing Analysis

- Payload Fairing
  - Area 69.03 m^2
  - Mass 645 kg
- Intertank Fairing
  - Area 154.1 m^2
  - Mass 1624 kg
- Aft Fairing
  - Area 176.8 m^2
  - Mass 1902 kg



Avionics and Wiring Masses

- Avionics

$$M_{avionics}\langle kg\rangle = 10\left(153,000\right)^{0.361} = 744\ kg$$

- Wiring

$$M_{wiring}\langle kg\rangle = 1.058\sqrt{153,000}(21\ m)^{0.25} = 886\ kg$$

Propulsion MERs

- Liquid Pump-Fed Rocket Engine Mass

$$M_{Rocket\ Engine}(kg) = 7.81 \times 10^{-4} T(N) + 3.37 \times 10^{-5} T(N) \sqrt{\frac{A_e}{A_t}} + 59$$

- Solid Rocket Motor

$$M_{Motor\ Casing} = 0.135 M_{propellants}$$

- Thrust Structure Mass

$$M_{Thrust\ Structure}(kg) = 2.55 \times 10^{-4} T(N)$$

Propulsion MERs (continued)

- Gimbal Mass

$$M_{Gimbals}(kg) = 237.8\left[\frac{T(N)}{P_0(Pa)}\right]^{.9375}$$

- Gimbal Torque

$$\tau_{Gimbals}(N \cdot m) = 990{,}000\left[\frac{T(N)}{P_0(Pa)}\right]^{1.25}$$

Propulsion System Assumptions

- Initial T/mg ratio = 1.3
  - Keeps final acceleration low with reasonable throttling
- Number of engines = 6
  - Positive acceleration worst-case after engine out

$$\frac{5}{6}(1.3) = 1.083 > 1$$

- Chamber pressure = 1000 psi = 6897 kPa
  - Typical for high-performance LOX/LH2 engines
- Expansion ratio A_e/A_t=30
  - Compromise ratio with good vacuum performance

Propulsion Mass Estimates

- Rocket Engine Thrust (each)

$$T(N) = \frac{m_0 g (T/W)_0}{n_{engines}} = 324{,}900\ N$$

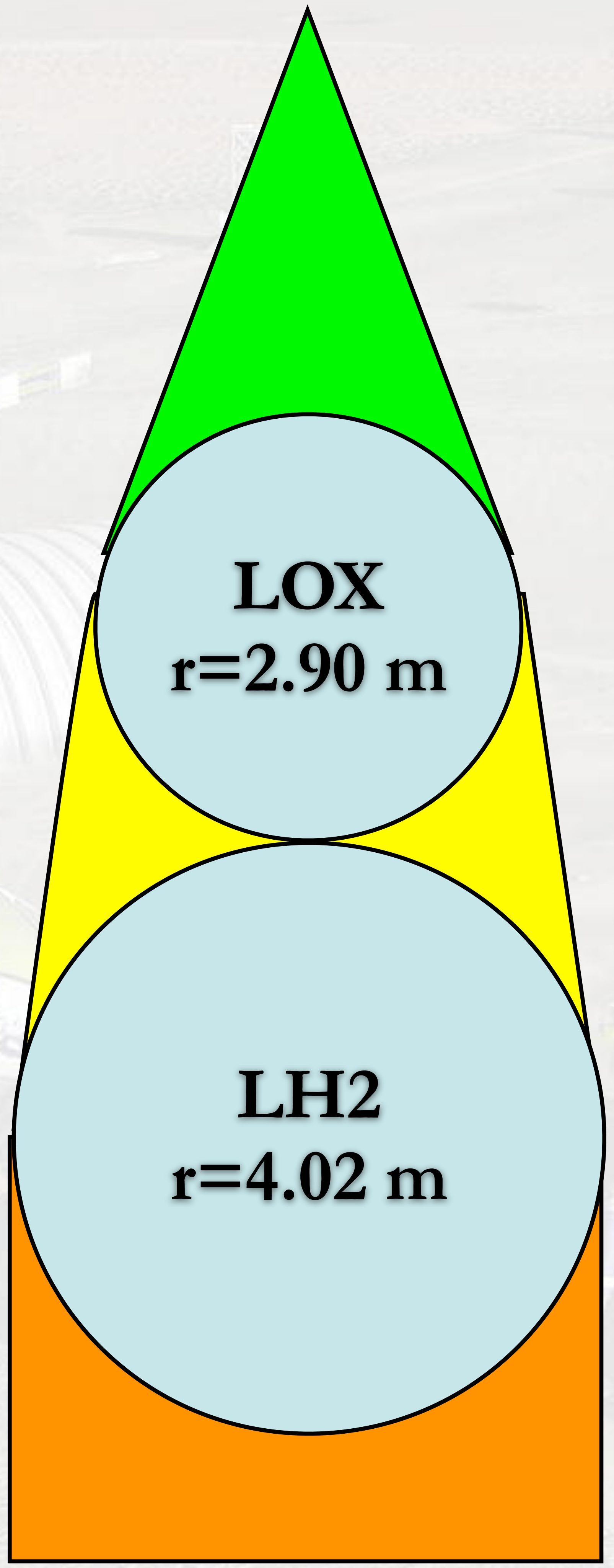
- Rocket Engine Mass (each)

$$M_{Rocket\ Engine}(kg) = 7.81 \times 10^{-4}(324{,}900) + 3.37 \times 10^{-5}(324{,}900)\sqrt{30} + 59 = 373\ kg$$

- Thrust Structure Mass

$$M_{Thrust\ Structure}(kg) = 2.55 \times 10^{-4}(324{,}900) = 82.8\ kg$$

First Pass Vehicle Configuration

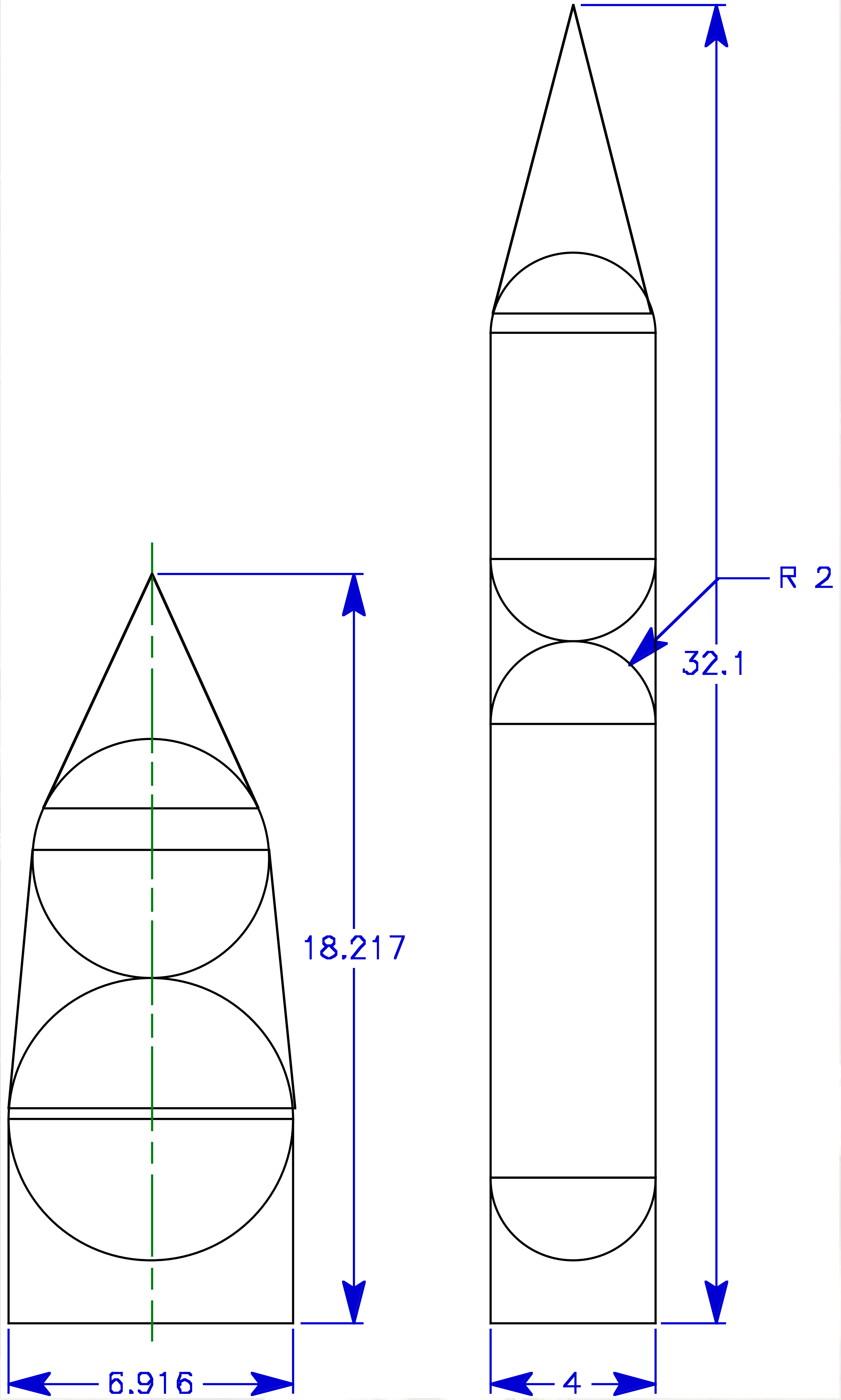


Mass Summary - First Pass

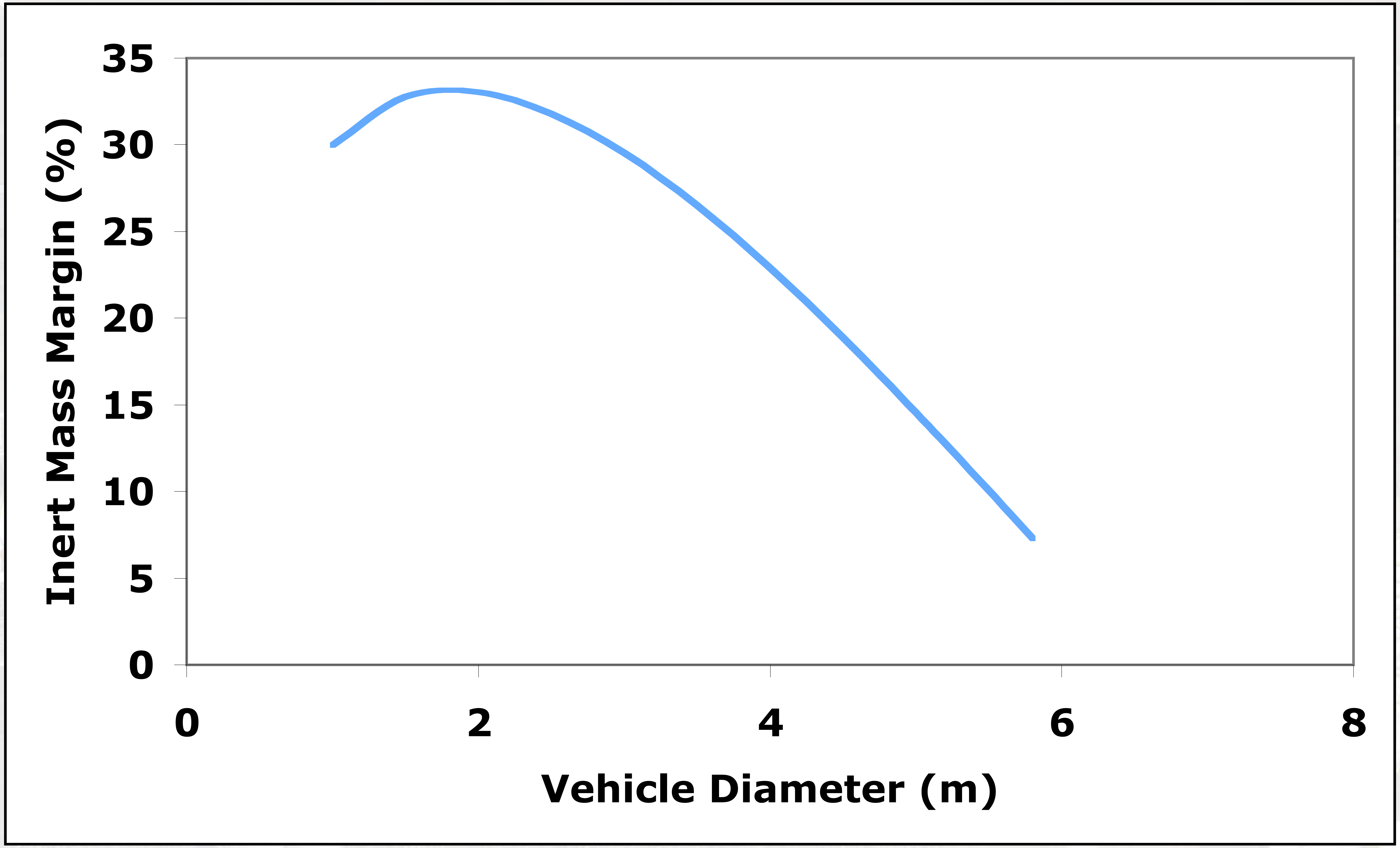
Initial Inert Mass Estimate	**12,240 kg**
LOX Tank	1245 kg
LH2 Tank	2482 kg
LOX Insulation	119 kg
LH2 Insulation	586 kg
Payload Fairing	645 kg
Intertank Fairing	1626 kg
Aft Fairing	1905 kg
Engines	2236 kg
Thrust Structure	497 kg
Gimbals	81 kg
Avionics	744 kg
Wiring	886 kg
Reserve	-
Total Inert Mass	**13,052 kg**

Modifications for Second Pass

- Keep all initial vehicle sizing parameters constant
- Pick vehicle diameter and make tanks cylindrical to fit
- Redo MER analysis



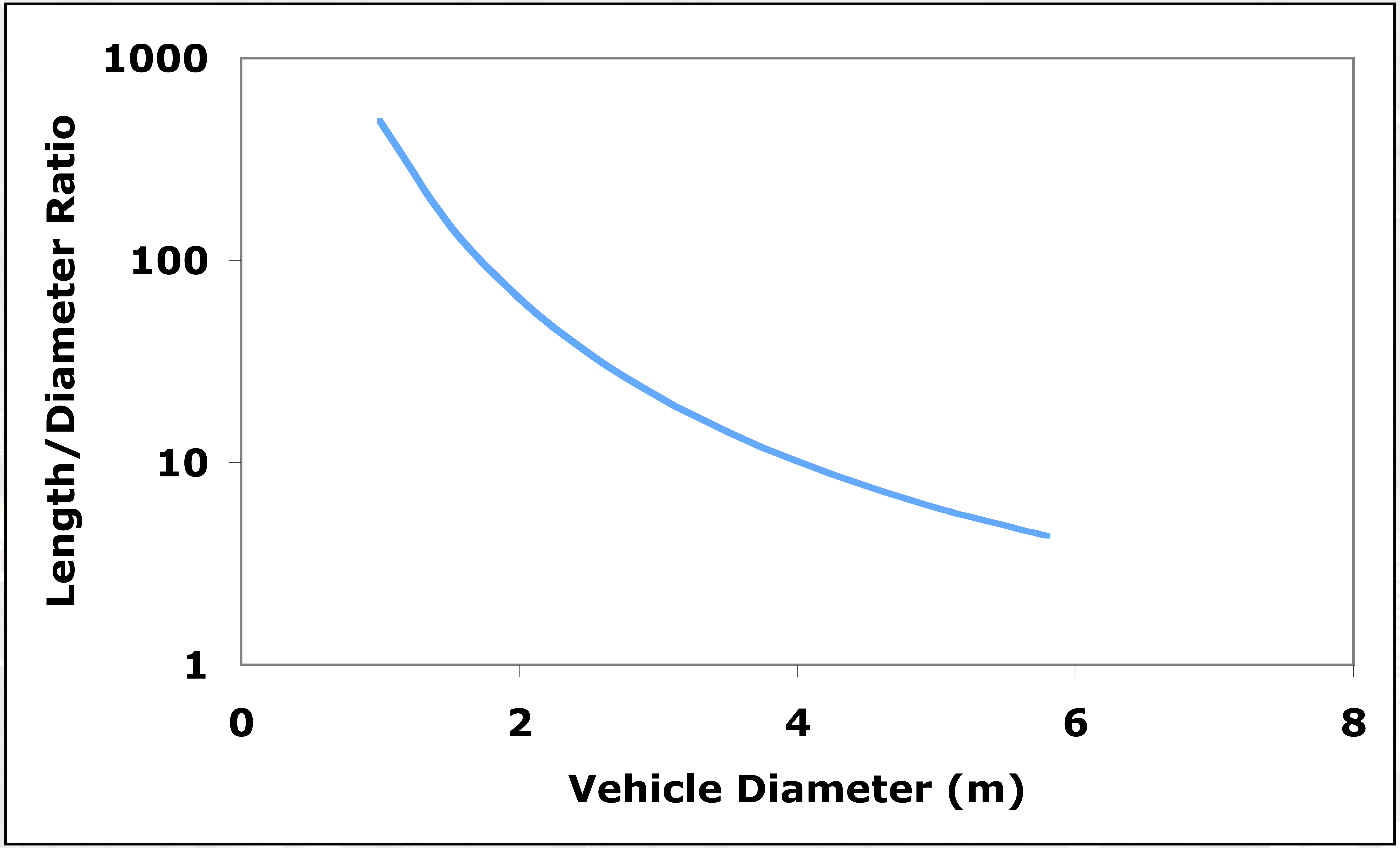
Effect of Vehicle Diameter on Mass Margin



Effect of Mass-Optimal Diameter Choice

- Mass-optimal vehicle has diameter=1.814 m
- Mass margin goes from -6.22% to +33.1%
- Vehicle length=155 m
- Length/diameter ratio=86 – approximately equivalent to piece of spaghetti
- No volume for six rocket engines in aft fairing
- Infeasible configuration

Effect of Diameter on Vehicle L/D



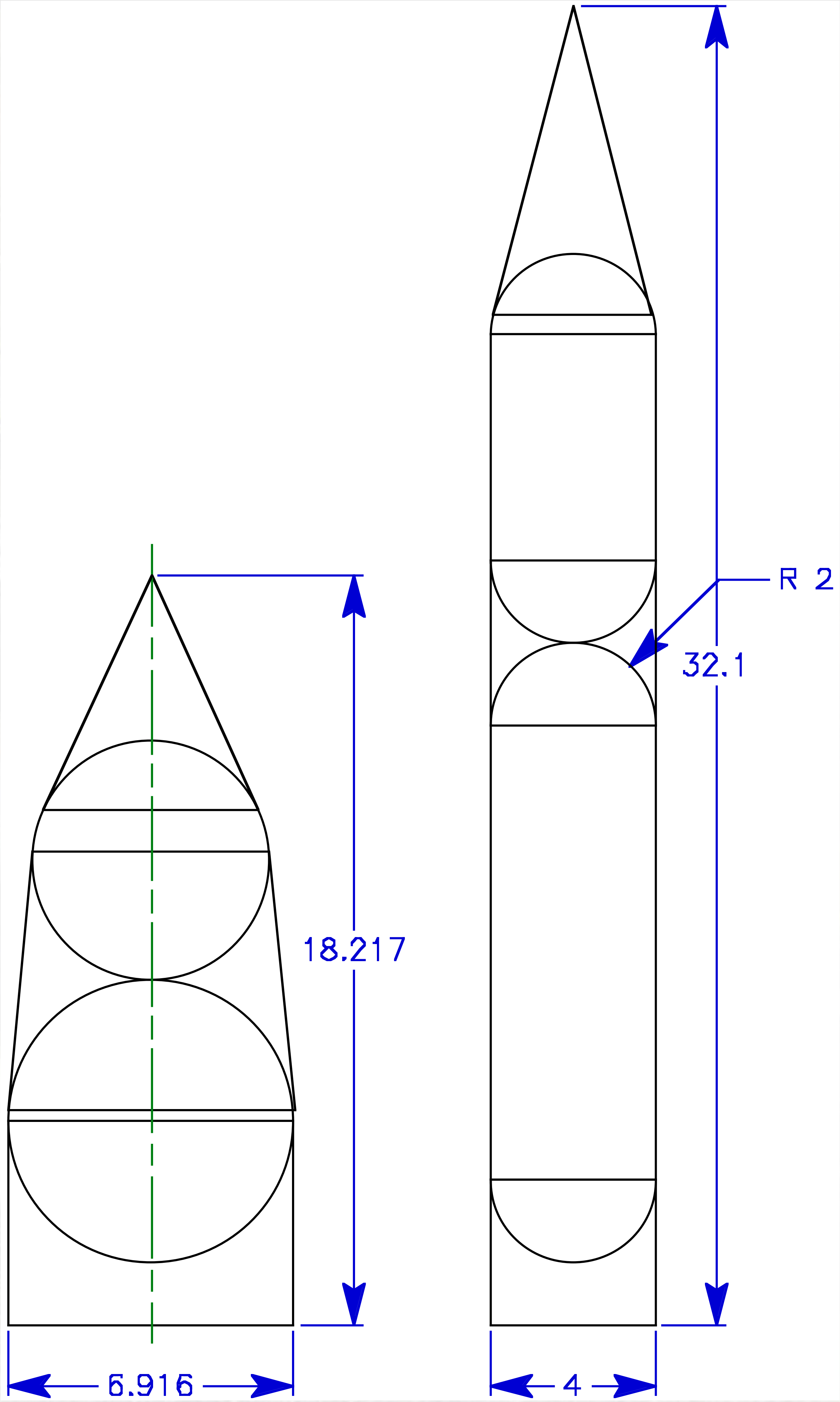
S-IC Barge Delivery (10m diameter)

S-IVB Air Transport (7m diameter)

Atlas/Delta Delivery System (4-5m diam)

SpaceX Falcon 9 Delivery (3.7m diam)

Second Pass Vehicle Configuration

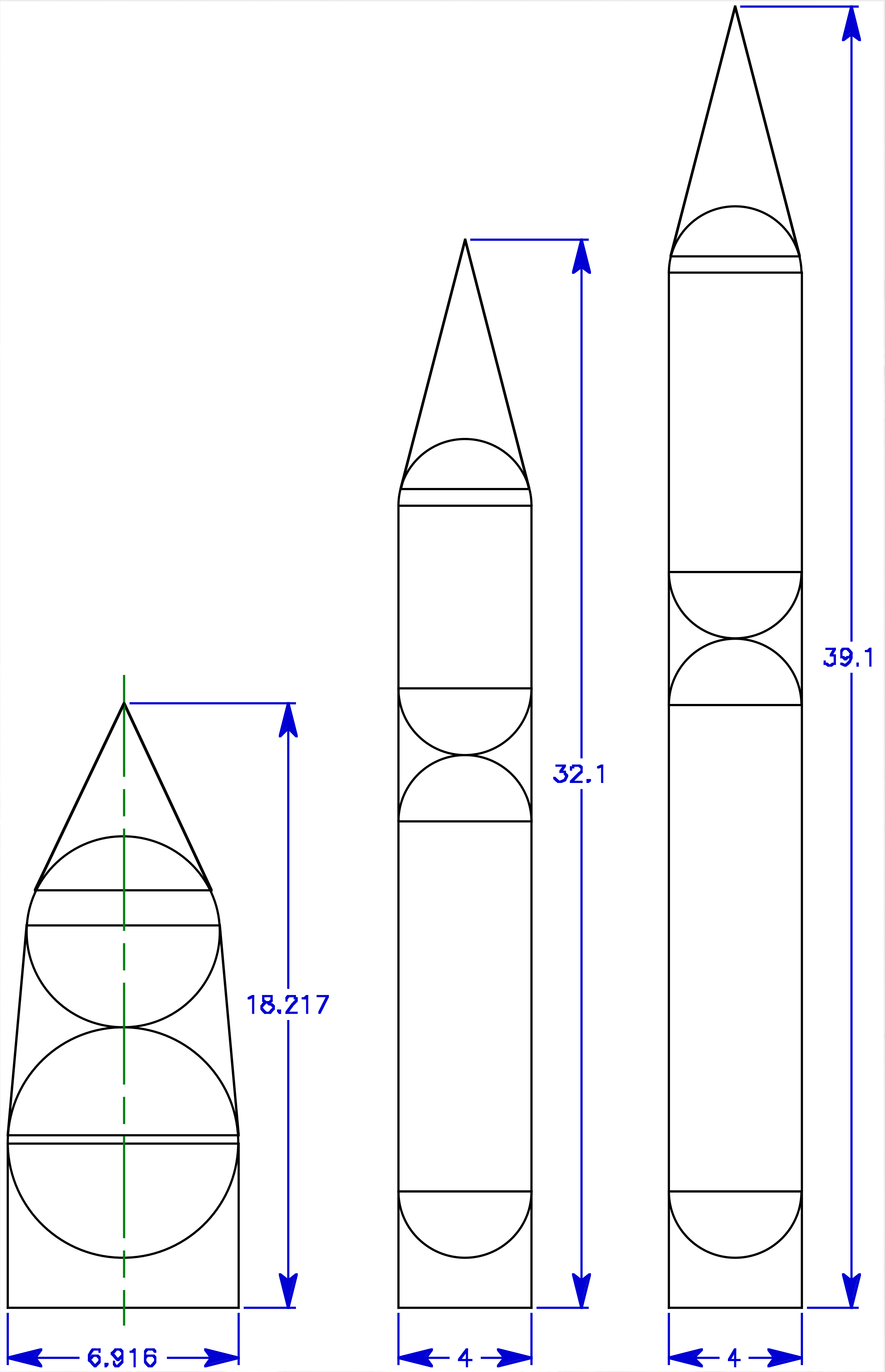


Mass Summary - Second Pass

Initial Inert Mass Estimate	**12,240 kg**	**12,240 kg**
LOX Tank	1245 kg	1245 kg
LH2 Tank	2482 kg	2482 kg
LOX Insulation	119 kg	56 kg
LH2 Insulation	586 kg	145 kg
Payload Fairing	645 kg	402 kg
Intertank Fairing	1626 kg	448 kg
Aft Fairing	1905 kg	579 kg
Engines	2236 kg	2236 kg
Thrust Structure	497 kg	497 kg
Gimbals	81 kg	81 kg
Avionics	744 kg	744 kg
Wiring	886 kg	1044 kg
Reserve	-	-
Total Inert Mass	**13,052 kg**	**9960 kg**
Design Margin	**-6.22 %**	**+22.9 %**

Modifications for Iteration 3

- Keep 4 m tank diameter
- Change initial assumption of δ iteratively, with resulting changes in m_0 and m_i, to reach 30% mass margin
- Modify diameter to keep L/D≤10 and iterate again for optimal initial mass estimate



Vehicle-Level Prelim Design - 3rd Pass

- Single Stage to Orbit (SSTO) vehicle
- ΔV=9200 m/sec
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec (Ve=4214 m/sec)
  - δ=0.08323
- Diameter=4.2 m
- L/D=9.7

$$r = e^{-\frac{\Delta V}{V_e}} = 0.1127$$

$$\lambda = r - \delta = 0.0294$$

$$M_o = \frac{M_\ell}{\lambda} = 169,800\ kg$$

$$M_i = \delta M_o = 14,130\ kg$$

$$M_p = M_o(1 - r) = 150,700\ kg$$

Mass Summary - Third Pass

Initial Inert Mass Estimate	12,240 kg	12,240 kg	14,130 kg
LOX Tank	1245 kg	1245 kg	1382 kg
LH2 Tank	2482 kg	2482 kg	2755 kg
LOX Insulation	119 kg	56 kg	62 kg
LH2 Insulation	586 kg	145 kg	160 kg
Payload Fairing	645 kg	402 kg	427 kg
Intertank Fairing	1626 kg	448 kg	501 kg
Aft Fairing	1905 kg	579 kg	626 kg
Engines	2236 kg	2236 kg	2443 kg
Thrust Structure	497 kg	497 kg	552 kg
Gimbals	81 kg	81 kg	90 kg
Avionics	744 kg	744 kg	773 kg
Wiring	886 kg	1044 kg	1101 kg
Reserve	-	-	–
Total Inert Mass	13,052 kg	9960 kg	10,870 kg
Design Margin	-6.22 %	+22.9 %	+30.0 %

Mass Budgeting

	Estimates	Budgeted	Margins
Initial Inert Mass Estimate	14,131 kg	14,131 kg	
LOX Tank	1382 kg	1589 kg	207 kg
LH2 Tank	2755 kg	3168 kg	413 kg
LOX Insulation	62 kg	72 kg	9 kg
LH2 Insulation	160 kg	184 kg	24 kg
Payload Fairing	427 kg	491 kg	64 kg
Intertank Fairing	501 kg	576 kg	75 kg
Aft Fairing	626 kg	720 kg	94 kg
Engines	2443 kg	2809 kg	366 kg
Thrust Structure	552 kg	634 kg	83 kg
Gimbals	90 kg	103 kg	13 kg
Avionics	773 kg	889 kg	116 kg
Wiring	1101 kg	1267 kg	165 kg
Reserve	–	1630 kg	
Total Inert Mass	10,870 kg	12,500 kg	

Masses of Pressurized Systems

Spacecraft/Stations/Habitats

- Gross mass

$$m_G < kg >= 460V < m^3 >^{0.76}$$

- Pressure hull mass

$$m_{hull} < kg >= 91.03V < m^3 >^{0.83}$$

- Internal systems mass

$$m_{sys} < kg >= 366.3V < m^3 >^{0.74}$$

Today's Tools

- Heuristic equations for estimating mass of vehicles at a component level
- Concept of mass margin as a design driver
- Budgeting of margin for future levels of design detail

References

- C. R. Glatt, WAATS - *A Computer Program for Weights Analysis of Advanced Transportation Systems* NASA CR-2420, September 1974.
- I. O. MacConochie and P. J. Klich, *Techniques for the Determination of Mass Properties of Earth-to-Orbit Transportation Systems* NASA TM-78661, June 1978.
- Willie Heineman, Jr., *Fundamental Techniques of Weight Estimating and Forecasting for Advanced Manned Spacecraft and Space Stations* NASA TN-D-6349, May 1971
- Willie Heineman, Jr., *Mass Estimation and Forecasting for Aerospace Vehicles Based on Historical Data* NASA JSC-26098, November 1994