Terramechanics

- Origin and nature of lunar soil
- Soil mechanics
- Rigid wheel mechanics
Notes about Revised Course Schedule

• No class next week (9/11 and 9/13)
• Makeup lectures to be announced
Lunar Regolith

- Broken down from larger pieces over time
- Major constituents
  - Rock fragments
  - Mineral fragments
  - Glassy particles
- Local environment
  - $10^{-12}$ torr (= $1.22 \times 10^{-10}$ Pa = $1.93 \times 10^{-14}$ psi)
  - Meteorites at velocities $>10^5$ m/sec
  - Galactic cosmic rays, solar particles
  - Temperature range $+250^\circ F$ – $-250^\circ F$
Regolith Creation Process

- Only “weathering” phenomenon on the moon is meteoritic impact!
- Weathering processes
  - Comminution: breaking rocks and minerals into smaller particles
  - Agglutination: welding fragments together with molten glass formed by impact energy
  - Solar wind spallation and implantation (miniscule)
  - Fire fountaining (dormant)
JSC-1 Simulant

• Ash vented from Merriam Crater in San Francisco volcano field near Flagstaff, AZ
• K-Ar dated at 150,000 years old ± 30,000
• Major constituents SiO$_2$, TiO$_2$, Al$_2$O$_3$, Fe$_2$O$_3$, FeO, MgO, CaO, Na$_2$O, other <1%
• Represents low-Ti regolith from lunar mare
• MLS-1 simulant (U.Minn.) preferred for simulation of highland material
Wheel-Soil Interaction

Wheel rolling over soil does work
- Compression
- “Bulldozing”

from Gibbesch and Schafer, “Advanced and Simulation Methods of Planetary Rover Mobility on Soft Terrain” 8th ESA Workshop on Advanced Space Technologies for Robotics and Automation, Noordwijk, The Netherlands, November, 2004
Soil Testing Apparatus

Bevameter (force vs. displacement)

Internal friction angle $\varphi$

Shear deformation modulus $K$
Soil Characterization – Direct Shear

\[ \tau = \frac{T}{A} \]

\[ \sigma = \frac{W}{A} \]

Shear Stress $\tau = \frac{T}{A}$

Normal Stress $\sigma = \frac{W}{A}$

$c = \text{Cohesion}$

$\phi = \text{angle of internal friction}$
Modeling Soil Reaction to a Wheel

Assume soil reaction is like a (nonlinear) spring

\[ P = k z^n \]

- \( P \) = applied pressure
- \( z \) = compression depth
- \( k, n \) = heuristic parameters
Effects of Soil Mechanics

Soil Penetration Depth $z$

Scaled Pressure $\frac{P}{k_c}$
Wheel-Soil Interactions

Distance rolled $d$

Wheel width $b$

Area of compressed soil $A = bd$

Displacement Energy

$$\frac{E}{A} = \int \frac{F}{A} dz = \int P dz$$

$$\frac{E}{A} = \int_{0}^{z_0} P dz = \int_{0}^{z_0} k z^n dz = k \frac{z_0^{n+1}}{n + 1}$$
Rolling Resistance

Total Energy \( \frac{E}{A} A = \frac{E}{A} bd = k \frac{z_n^{n+1}}{n+1} bd \)

Given a force resisting rolling \( \equiv R \),
the energy required to roll a distance \( d \) is
\[ E_{roll} = Rd \]
\[ E_{roll} = E_{displacement} \implies Rd = \frac{E}{A} bd \]
Rolling Resistance

For \( n = 1 \):
\[
P = k z; \quad \frac{E}{A} = k \frac{z_o^2}{2}; \quad R = \frac{1}{2} k b z_o^2
\]

For \( n = \frac{1}{2} \):
\[
P^2 = k^2 z; \quad \frac{E}{A} = \frac{2}{3} k z_o^{\frac{3}{2}}; \quad R = \frac{2}{3} k b z_o^{\frac{3}{2}}
\]

For \( n = 0 \):
\[
P = k z; \quad \frac{E}{A} = k z_o; \quad R = k b z_o
\]

Generic case:
\[
P = k z^n; \quad \frac{E}{A} = k \frac{z_o^{n+1}}{n+1}; \quad R = k b \frac{z_o^{n+1}}{n+1}
\]
Soil Displacement Calculations

\[ R - \int_{0}^{\theta_o} dF \sin \theta = 0 \]
\[ -W + \int_{0}^{\theta_o} dF \cos \theta = 0 \]
\[ dF = Pb \]
\[ dF \cos \theta = -Pb \, dx \]
\[ dF \sin \theta = Pb \, dz \]

\[ R = \int_{0}^{\theta_o} Pb \, dz \]
\[ W = -\int_{0}^{\theta_o} Pb \, dx \]

In general, \( P = kx^n \)

\[ W = -\int_{0}^{z_o} bkz^n \, dx \]
Soil Displacement Calculations

wheel width \( b \)

\[
\begin{align*}
\bar{AB} &= \frac{D}{2} - (z_o - z) \\
x^2 &= \left( \frac{D}{2} \right)^2 - \bar{AB}^2 = \left( \frac{D}{2} \right)^2 - \left[ \frac{D}{2} - (z_o - z) \right]^2 \\
&= \left( \frac{D}{2} \right)^2 - \left( \frac{D}{2} \right)^2 + 2 \frac{D}{2} (z_o - z) - (z_o - z)^2 \\
x^2 &= [D - (z_o - z)](z_o - z)
\end{align*}
\]
Soil Compression Calculations

But $D \gg z_o - z$

$$x^2 \approx D(z_o - z) \Rightarrow 2x \, dx = -D \, dz$$

so from $W = -\int_0^{z_o} bk z^n \, dx$ we get $W = -\int_0^{z_o} bk z^n \frac{-D}{2x} \, dz$

$$W = -bk \int_0^{z_o} z^n \left( \frac{-D}{2\sqrt{D}\sqrt{z_o - z}} \right) \, dz$$

$$W = bk \int_0^{z_o} z^n \left( \frac{\sqrt{D} dz}{2\sqrt{z_o - z}} \right) \, dz$$
Soil Displacement Calculations

Define $z_0 - z \equiv t^2 \Rightarrow dz = -2tdt$

$$W = bk\sqrt{D} \int_0^{\sqrt{z_0}} (z_0 - t^2)^n dt$$

Taylor Series expansion $(z_0 - t^2)^n \approx z_0^n - n z_0^{n-1} t^2 + \cdots$

$$W \approx \frac{bk\sqrt{D}z_0}{3} z_0^n (3 - n)$$

for $n = 1 \Rightarrow W = \frac{2}{3} bk z_0 \sqrt{D z_0}$

for $n = \frac{1}{2} \Rightarrow W = \frac{5}{6} bk z_0 \sqrt{D}$

for $n = 0 \Rightarrow W = bk \sqrt{D z_0}$
Rolling Resistance as $f(W)$

for $n = 0 \Rightarrow W = bk \sqrt{Dz_o} \Rightarrow z_o = \left(\frac{W}{bk}\right)^2 \frac{1}{D}$

$$R = kbz_o \Rightarrow R = \frac{kb}{(kb)^2} \frac{W^2}{D} \Rightarrow R = \frac{W^2}{kbD}$$

for $n = \frac{1}{2} \Rightarrow W = \frac{5}{6} bkz_o \sqrt{D} \Rightarrow z_o = 6 \frac{W}{5 bk \sqrt{D}}$

$$R = \frac{2}{3} k z_o^{\frac{3}{2}} \Rightarrow R = \frac{2}{3} kb \left(\frac{6}{5} \frac{W}{kb \sqrt{D}}\right)^{\frac{3}{2}} = \frac{2}{3} \left(\frac{6}{5}\right)^{\frac{3}{2}} \frac{W^{\frac{3}{2}}}{\sqrt{kbD^{\frac{3}{4}}}}$$

$$R = 0.876 \frac{W^{\frac{3}{2}}}{\sqrt{kbD^{\frac{3}{4}}}}$$
Rolling Resistance as \( f(W) \)

For \( n = 1 \) \( \Rightarrow \) \( W = \frac{2}{3} bkz_o^3 \sqrt{D} \) \( \Rightarrow \) \( z_o^2 = \left( \frac{3W}{2kb\sqrt{D}} \right)^{\frac{4}{3}} \)

\[
R = \frac{1}{2} kbz_o^2 \Rightarrow R = \frac{1}{2} kb \left( \frac{3W}{2kb\sqrt{D}} \right)^{\frac{4}{3}} = \frac{1}{2} \left( \frac{3}{2} \right)^{\frac{4}{3}} \left( \frac{W^4}{kbD^2} \right)^{\frac{1}{3}}
\]

\[R = 0.859 \left( \frac{W^4}{kbD^2} \right)^{\frac{1}{3}}\]
Rolling Resistance as \( f(W) \) (Generic)

\[
W = \frac{bk\sqrt{D}z_o}{3} z_o^n (3 - n) = \frac{bk\sqrt{D}}{3} z_o^{n+\frac{1}{2}} (3 - n)
\]

\[
z_o^{n+\frac{1}{2}} = \frac{3}{(3 - n)} \frac{W}{bk\sqrt{D}}
\]

\[
z_o^{n+1} = \left( \frac{3}{3 - n} \frac{W}{bk\sqrt{D}} \right)^{\frac{n+1}{n+\frac{1}{2}}} = \left( \frac{3}{3 - n} \frac{W}{bk\sqrt{D}} \right)^{\frac{2(n+1)}{2n+1}}
\]

\[
R = \frac{bk}{n+1} z_o^{n+1} = \frac{bk}{n+1} \left( \frac{3}{3 - n} \frac{W}{bk\sqrt{D}} \right)^{\frac{2(n+1)}{2n+1}}
\]

\[
R = \frac{1}{n+1} \left( \frac{3}{3 - n} \frac{W}{\sqrt{D}} \right)^{\frac{2(n+1)}{2n+1}} \left( \frac{1}{bk} \right)^\frac{1}{2n+1}
\]
More Detailed Soil Compression Equation

\[ k = \frac{k_c}{b} + k_\phi \]

\( k_c \) = modulus of cohesion of soil deformation

\( k_c \) units \( \Rightarrow < N/m^{(n+1)} > \)

\( k_\phi \) = modulus of friction of soil deformation

\( k_\phi \) units \( \Rightarrow < N/m^{(n+2)} > \)

\( b \) = wheel width

\[ P = \left( \frac{k_c}{b} + k_\phi \right) z^n \]
## Soil Characteristics

<table>
<thead>
<tr>
<th>soil type</th>
<th>n</th>
<th>( k_c \langle \frac{N}{m^{n+1}} \rangle )</th>
<th>( k_\phi \langle \frac{N}{m^{n+2}} \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry Sand</td>
<td>1.1</td>
<td>990</td>
<td>1,528,000</td>
</tr>
<tr>
<td>Lunar Regolith</td>
<td>1.0</td>
<td>1400</td>
<td>820,000</td>
</tr>
<tr>
<td>Sandy Loam</td>
<td>0.7</td>
<td>5270</td>
<td>1,515,000</td>
</tr>
<tr>
<td>Sandy Loam (MER-B)</td>
<td>1.0</td>
<td>28,000</td>
<td>7,600,000</td>
</tr>
<tr>
<td>Slope Soil (MER-B)</td>
<td>0.8</td>
<td>6800</td>
<td>210,000</td>
</tr>
<tr>
<td>Clay (Earth)</td>
<td>0.5</td>
<td>13,190</td>
<td>692,200</td>
</tr>
</tbody>
</table>
Equations for Compression Resistance

\[ z = \left( \frac{3W_w}{(3 - n)bk\sqrt{d}} \right)^{\frac{2}{2n+1}} \]

\[ W_w = \text{weight on wheel} \]
\[ d = \text{wheel diameter} \]

\[ R_c = \left( \frac{bk}{n + 1} \right)^{z^{n+1}} \]

\[ R_c = \text{compression resistance (per wheel)} \]
Soil Compression – Reece Formulation

\[ P = \left( \frac{k_c}{b} + k_\phi \right) z^n \]

Problem is that \( k_c \) and \( k_\phi \) have variable dimensions, based on \( n \)

- \( k_c \) units \( \Rightarrow < N/m^{(n+1)} > \)
- \( k_\phi \) units \( \Rightarrow < N/m^{(n+2)} > \)

Reece Formulation: nondimensionalize by \( b \)
Compression Resistance (Lunar Soil)

\[ R_c = \frac{1}{n + 1} \left( k_c + bk_\phi \right)^{-1} \left( \frac{3W_w}{(3 - n)\sqrt{d}} \right)^{\frac{2(n+1)}{2n+1}} \]

\[ n = 1 \]

\[ k_c = 0.14 \text{ N/cm}^2 \]

\[ k_\phi = 0.827 \text{ N/cm}^3 \]
Apollo Lunar Roving Vehicle Example

\[ z = \left( \frac{3 \times 253}{2(0.14 + 17.4 \times 0.827)\sqrt{82}} \right)^{\frac{2}{3}} = 2.03 \text{ cm} \]

\[ R_c = \frac{1}{2} (0.14 + 17.4 \times 0.827)^{-\frac{1}{3}} \left( \frac{3 \times 253}{2\sqrt{82}} \right)^{\frac{4}{3}} = 29.8 \text{ N} \]

check units -

\[ \left( \frac{N^{-1/3}}{cm^{-2/3}} \right) \left( \frac{N^{4/3}}{cm^{2/3}} \right) = N \]
Rolling and Gravitation Resistance

- Rolling resistance (tires, bearings, etc.)
  \[ R_r = W_v c_f \]
  \[ W_v = \text{weight of vehicle} \]
  \[ c_f = \text{coefficient of friction (typ. 0.05)} \]

- Gravitational resistance
  \[ R_g = W_v \sin \theta_{\text{slope}} \]

- LRV examples (15° slope)
  \[ R_r = 51 \ N \]
  \[ R_g = 262 \ N \]
Bulldozing Resistance

\[ R_b = \frac{b \sin (\alpha + \phi)}{2 \sin \alpha \cos \phi} \left( 2zcK_c + \gamma z^2 K_\gamma \right) + \frac{\pi \ell_o^3 \gamma (90 - \phi)}{540} + \frac{c \pi \ell_o^2}{180} \tan \left( 45 + \frac{\phi}{2} \right) \]

\( \alpha = \text{angle of attack of wheel in soil} \equiv \cos^{-1} \left( 1 - \frac{2z}{D} \right) \)

\( \gamma = \text{density of soil} \langle \frac{\text{kg}}{\text{m}^3} \rangle \)

\( \ell_o = \text{length of soil rupture} \equiv z \tan^2 \left( 45 - \frac{\phi}{2} \right) \)
Bulldozing Resistance

• “Bulldozing” is the process of pushing soil up ahead of the wheel
• Ranges from a small factor to a huge one, depending on soil and wheel factors
• Will be covered in detail in a later lecture
Tractive Force per Wheel (No Grousers)

\[ H = \left[ AC_b + W_w \tan \phi_b \right] \left[ 1 - \frac{K}{\ell} \left( 1 - e^{-\frac{sl}{K}} \right) \right] \]

- \( A \) = area of contact
- \( C_b \) = coefficient of soil/wheel cohesion
- \( \phi_b \) = wheel/soil friction angle
- \( s \) = wheel slip ratio
- \( K \) = coefficient of soil slip
- \( \ell \) = length of contact patch
Tractive Force per Wheel (With Grousers)

\[ H = \left[ blC_b \left( 1 + \frac{2h}{b} \right) N_g + W \tan \phi_b \left( 1 + 0.64 \frac{h}{b} \arctan \frac{b}{h} \right) \right] \left[ 1 - \frac{K}{\ell} \left( 1 - e^{-\frac{s\ell}{K}} \right) \right] \]

- \( A = \) area of contact \( \cong bl \)
- \( C_b = \) soil/wheel cohesion \( = 0.017 \ N/cm^2 \)
- \( \phi_b = \) wheel/soil friction angle \( = 35^\circ \)
- \( s = \) wheel slip ratio (typ. 0.02-0.05)
- \( K = \) coefficient of soil slip \( = 1.8 \ cm \)
- \( \ell = \) length of contact patch \( = \frac{D}{2} \cos^{-1} \left( 1 - \frac{2z}{D} \right) \)
- \( h = \) height of grouser

All values typical for lunar soil
Effect of Soil Thrust Fraction

Soil Thrust Fraction \( \left[ 1 - \frac{K}{\ell} \left( 1 - e^{-\frac{s\ell}{K}} \right) \right] \)
Basic Equation of Vehicle Propulsion

\[ DP = H - (R_c + R_b + R_g + R_r) \]

- DP: Drawbar pull (residual drive force)
- H: Maximum tractive force of wheels
- \( R_c \): Compaction resistance
- \( R_b \): Bulldozing resistance
- \( R_g \): Gravitational resistance
- \( R_r \): Rolling resistance (internal)
Example: Wheelbarrow (Single) Wheel

\[ R = (k_c + k_{\phi}b) \left( \frac{1}{2n+1} \right) W \left( \frac{2(n+1)}{n+1} \right) \frac{3}{3-n} \left( \frac{2(n+1)}{2n+1} \right) D^{-\frac{(n+1)}{2n+1}} \]

Compaction Resistance (N)

Wheel Weight (N)

- **Dry Sand**
- **Sandy Loam**
- **Clay**
- **Lunar**
- **MER-B Sandy Loam**
- **MER-B Slope Soil**

\[ D = 0.3 \text{ m} \]

\[ b = 0.1 \text{ m} \]
Effects of Wheel Parameters

![Graph showing the relationship between compaction resistance and wheel diameter for different wheel parameters. The graph plots compaction resistance (N) against wheel diameter (m) with lines for wheel parameters b=0.1, b=0.25, and b=0.5. The compaction resistance decreases as the wheel diameter increases. The drawbar resistance is constant at 500 N.]
Effect of Soil “Spring Constant” on R/W

Wheel 1 m diameter x 0.2 m width

Soil "k" value (N/m)

Resistance/Weight
Soil Type and Wheel Load

![Graph showing compaction resistance for different soil types](image)

- **Clay**
- **Dry Sand**
- **Snow**

Wheel: 1 m diameter x 0.2 m width

- **Compaction Resistance (N)**
- **Weight on Wheel (N)**
Soil Type and Specific Resistance

Wheel 1 m diameter x 0.2 m width

Weight on Wheel (N) vs. Resistance/Weight for different soil types:
- Blue line: Clay
- Red line: Dry Sand
- Yellow line: Snow

The graph shows the relationship between the weight on the wheel and the specific resistance for each soil type.
Effect of Wheel Diameter and Width

Wheel Load = 1000 N

Compaction Resistance (N) vs. Wheel Width (m)

- D=1 m
- D=2 m
- D=4 m
- Dual
- Quad

Wheel Width (m)

Wheel Load = 1000 N
Effect of Slope

The graph illustrates the relationship between slope (degrees) and gravitational resistance (N). As the slope increases, the gravitational resistance also increases. The curve shows a positive correlation, indicating that the greater the slope, the higher the gravitational resistance.
Wheel Test Apparatus

• Wheel testing done at MIT Field and Space Robotics Laboratory

• Independent control of motion and wheel velocity provides controllable slip

\[ s = 1 - \frac{V}{\omega r} \]
Wheel Torque vs. Time

9 grousers

\( \varphi = 0.24 \)

18 grousers
Sinkage vs. Slip Ratio

- 18 grousers (○)
- 9 grousers (×)

Sinkage vs. Slip Ratio

Sinkage (m)

Slip Ratio
Drawbar Pull vs. Slip Ratio

Sensor Force $F_y$ vs. Slip Ratio

- $o = 18$ grousers
- $x = 9$ grousers
Motor Torque vs. Slip Ratio

**Graph:**
- **Motor Torque (Nm):** 3.5, 3, 2.5, 2, 1.5
- **Slip Ratio:** -0.2, 0, 0.2, 0.4, 0.6, 0.8

- **○ = 18 grousers**
- **× = 9 grousers**