Terramechanics 2

- Effects of grousers
- Example solutions for rolling resistance
- Discussion of first steps in term project

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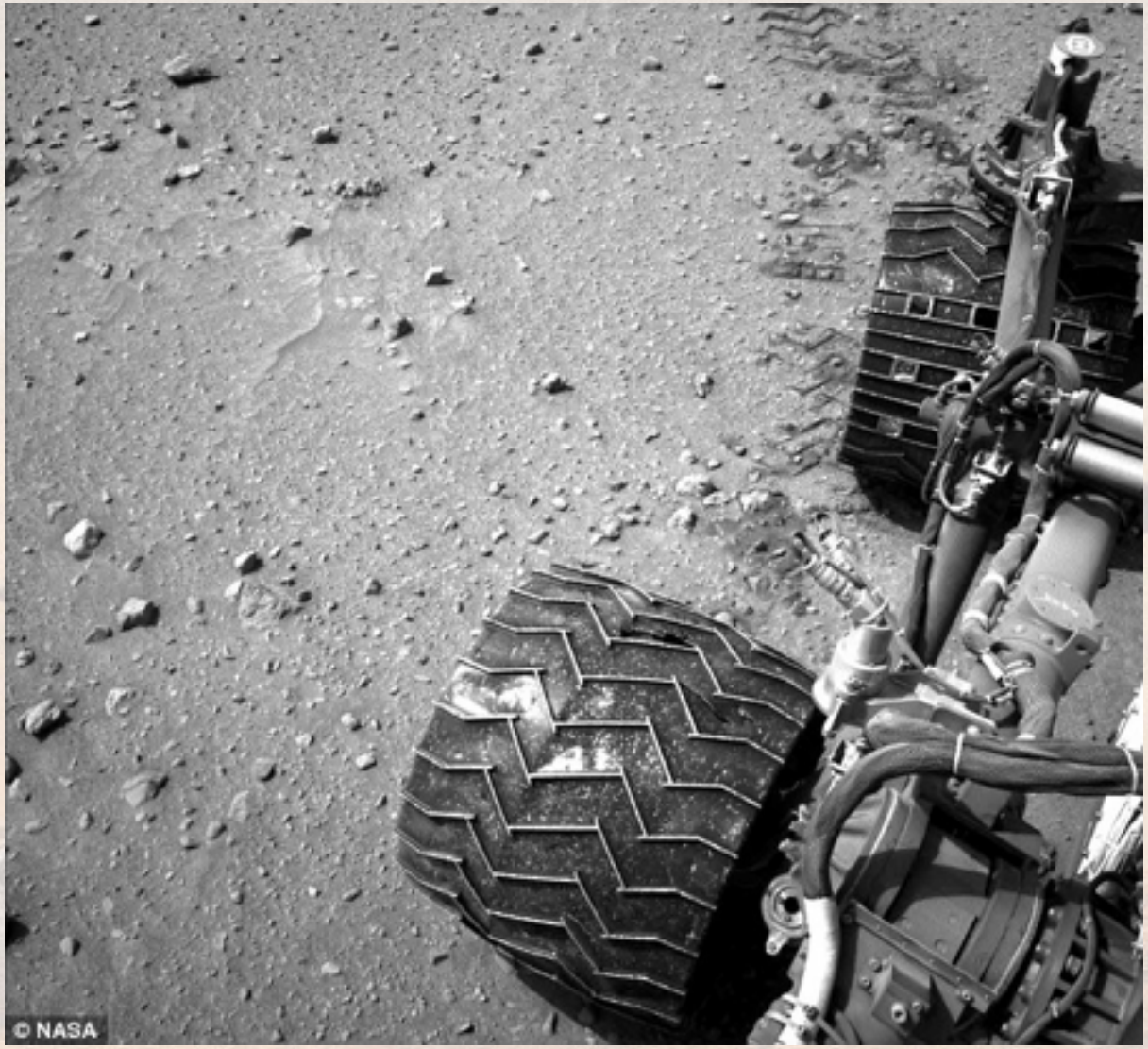
Grousers (on Tracked Vehicle)

Grousers (MSL Wheels)



Mars Rover Wheels

Tractive Force per Wheel (No Grousers)

$$H = [AC_b + W_w \tan \phi_b] \left[1 - \frac{K}{\ell} \left(1 - e^{\frac{-s\ell}{K}} \right) \right]$$

A = area of contact

C_b = coefficient of soil/wheel cohesion

ϕ_b = wheel/soil friction angle

s = wheel slip ratio

$$s = 1 - \frac{V}{\omega r}$$

K = coefficient of soil slip

ℓ = length of contact patch

Tractive Force per Wheel (With Grousers)

$$H = \left[b\ell C_b \left(1 + \frac{2h}{b}\right) N_g + W \tan\phi_b \left(1 + 0.64\frac{h}{b} \arctan\frac{b}{h}\right)\right]\left[1 - \frac{K}{\ell}\left(1 - e^{-\frac{s\ell}{K}}\right)\right]$$

A = area of contact $\cong b\ell$

C_b = soil/wheel cohesion $= 0.017\ N/cm^2$

ϕ_b = wheel/soil friction angle $= 35°$

s = wheel slip ratio (typ. 0.02-0.05)

K = coefficient of soil slip $= 1.8$ cm

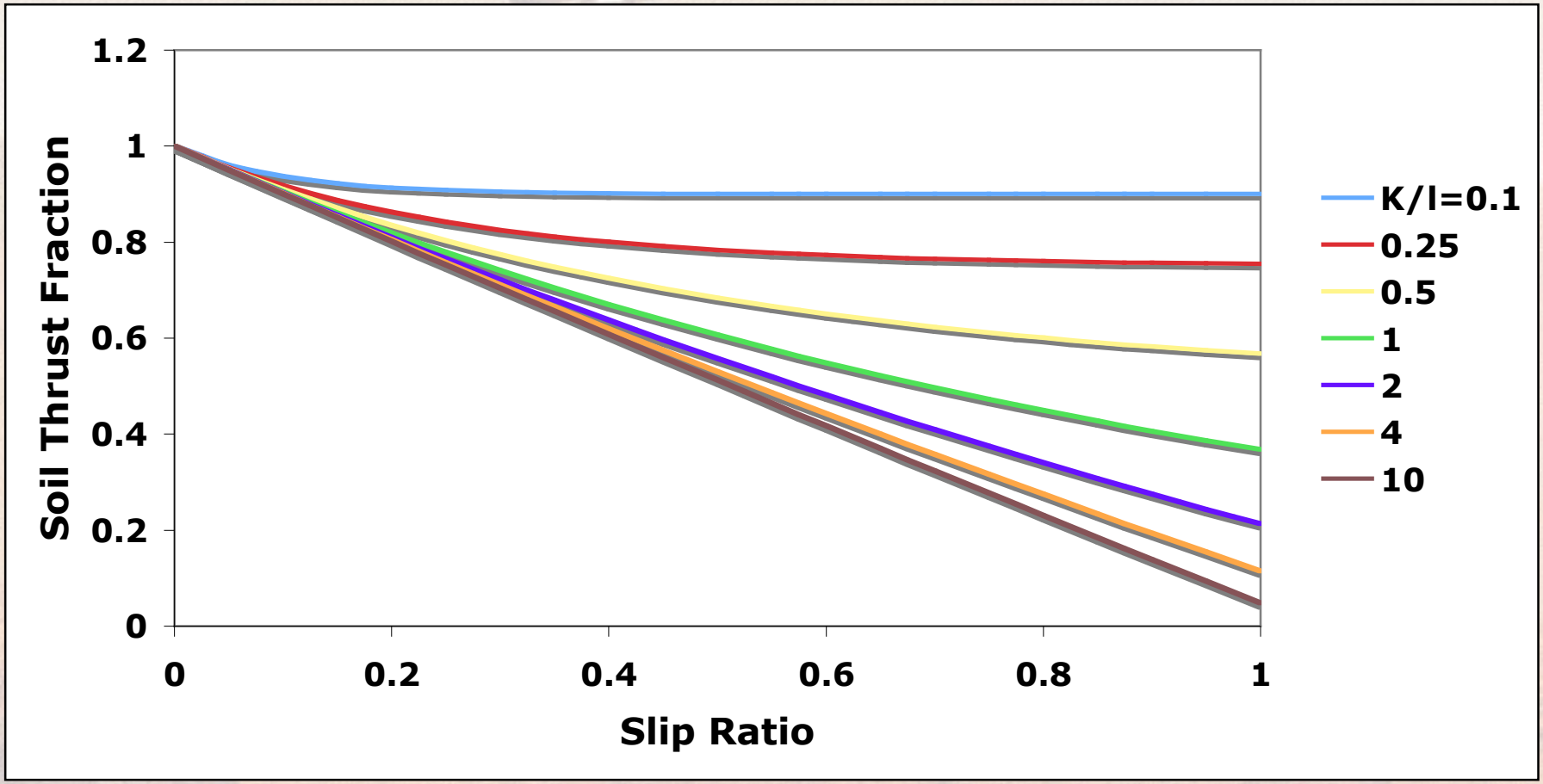
ℓ = length of contact patch $= \frac{D}{2}\cos^{-1}\left(1 - \frac{2z}{D}\right)$

h = height of grouser

All values typical for lunar soil

Effect of Soil Thrust Fraction

$$\text{Soil Thrust Fraction} \quad \left[1 - \frac{K}{\ell}\left(1 - e^{-\frac{s\ell}{K}}\right)\right]$$



Basic Equation of Vehicle Propulsion

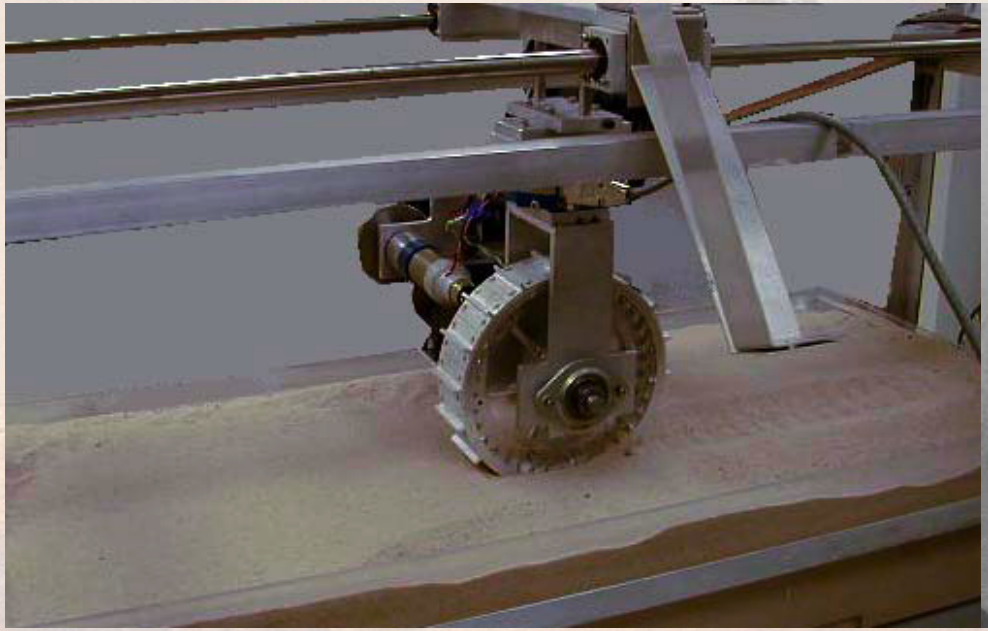
$$DP = H - (R_c + R_b + R_g + R_r)$$

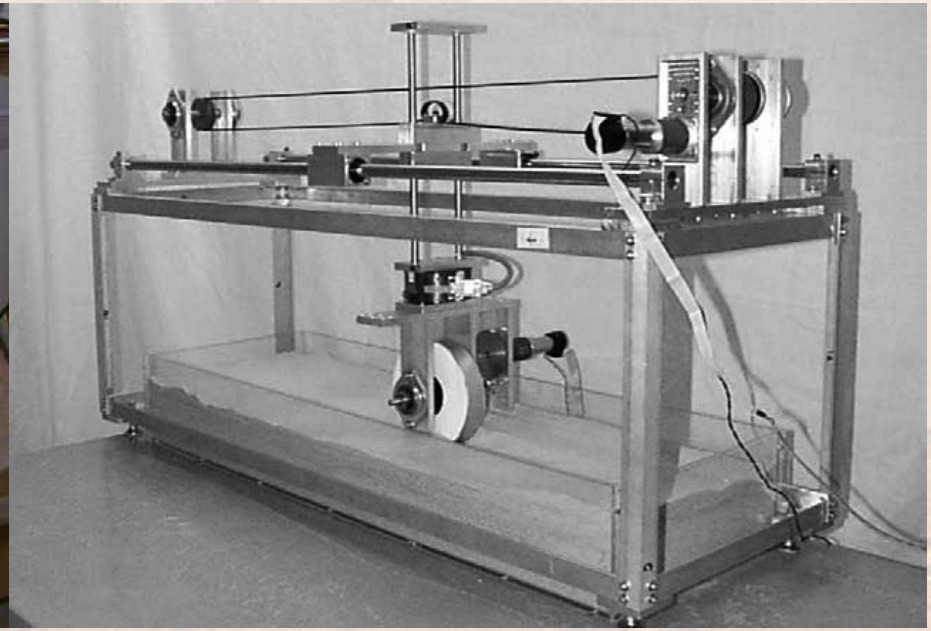
- DP: Drawbar pull (residual drive force)
- H: Maximum tractive force of wheels
- R_c: Compaction resistance
- R_b: Bulldozing resistance
- R_g: Gravitational resistance
- R_r: Rolling resistance (internal)

Wheel Test Apparatus

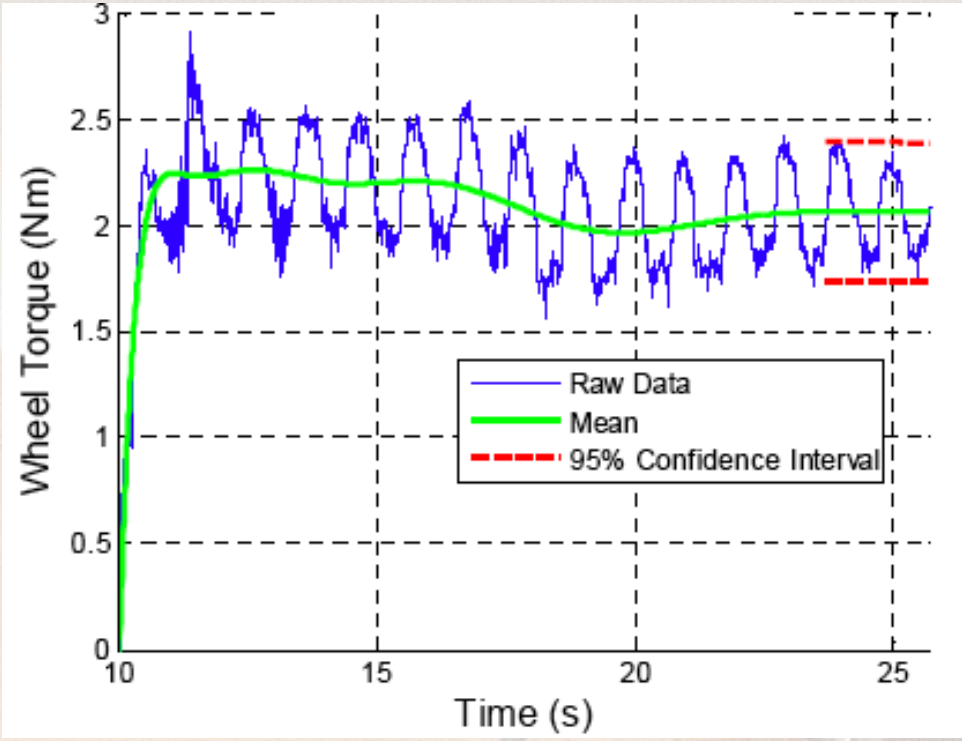
- Wheel testing done at MIT Field and Space Robotics Laboratory
- Independent control of motion and wheel velocity provides controllable slip

$$s = 1 - \frac{V}{\omega r}$$

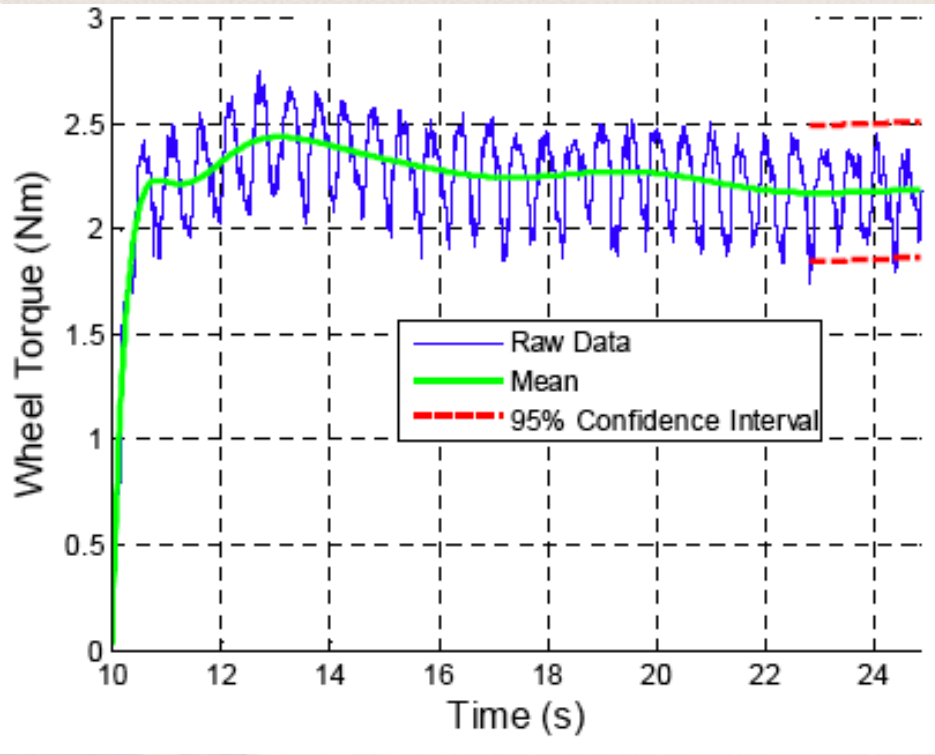




Wheel Torque vs. Time



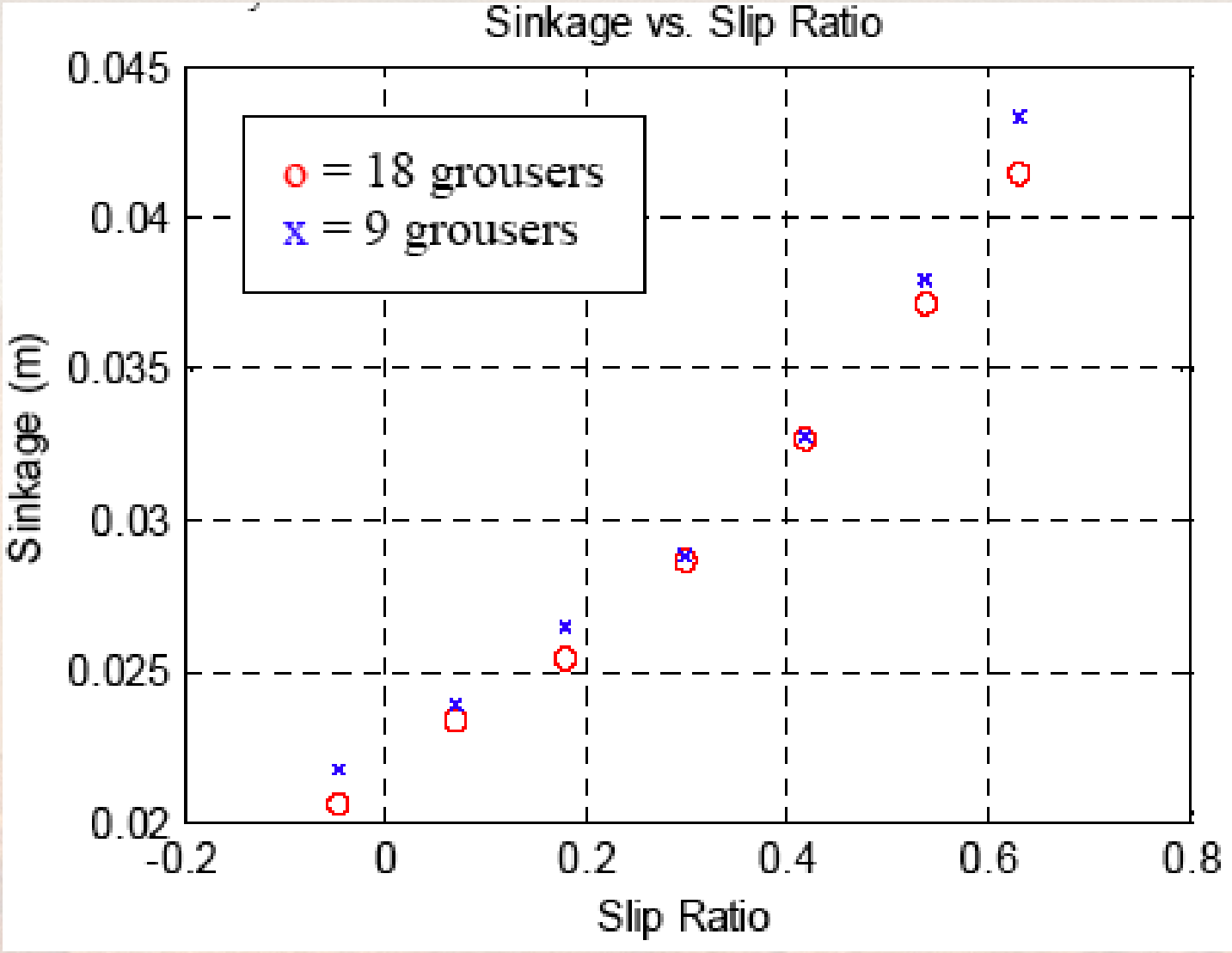
9 grousers



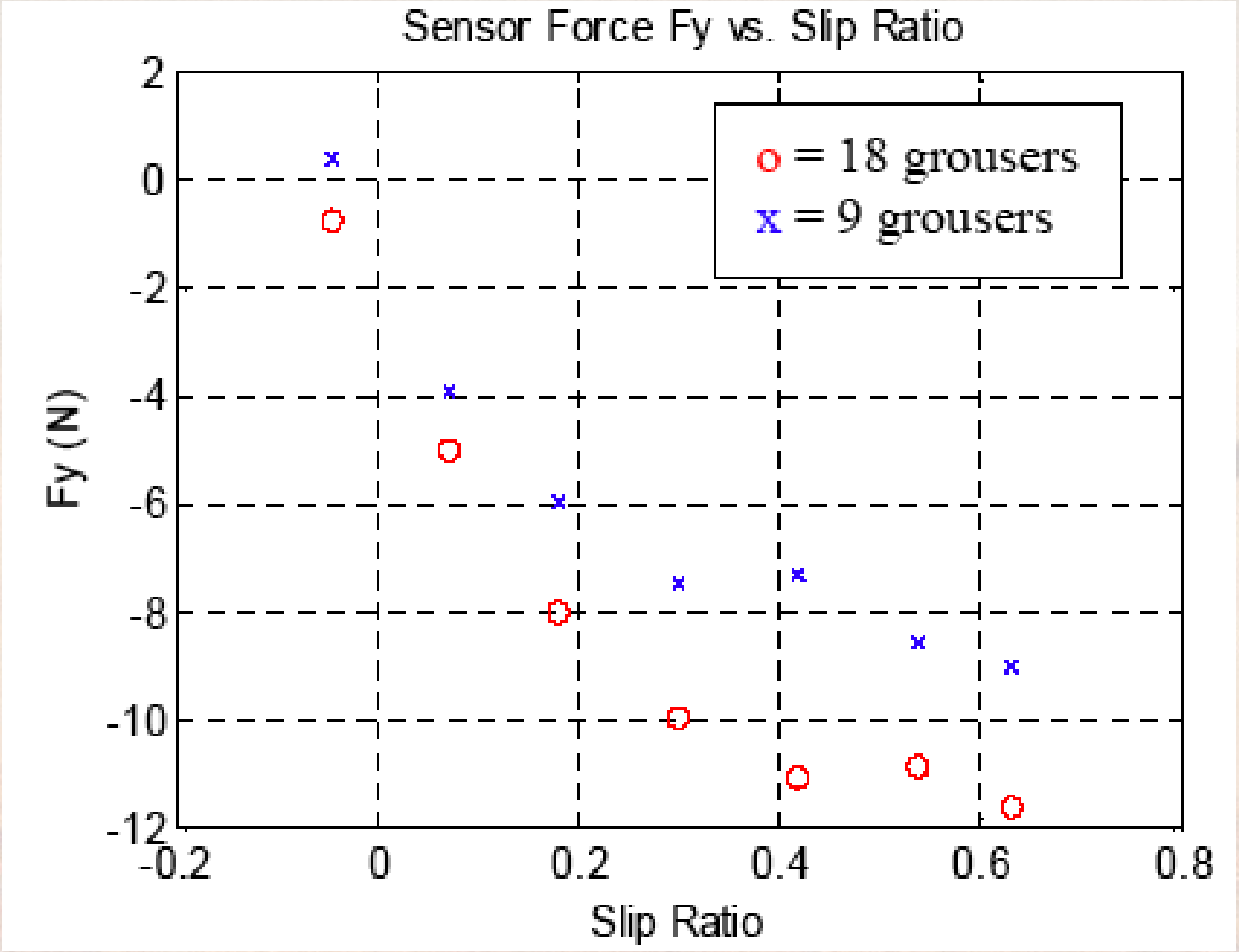
18 grousers

φ=0.24

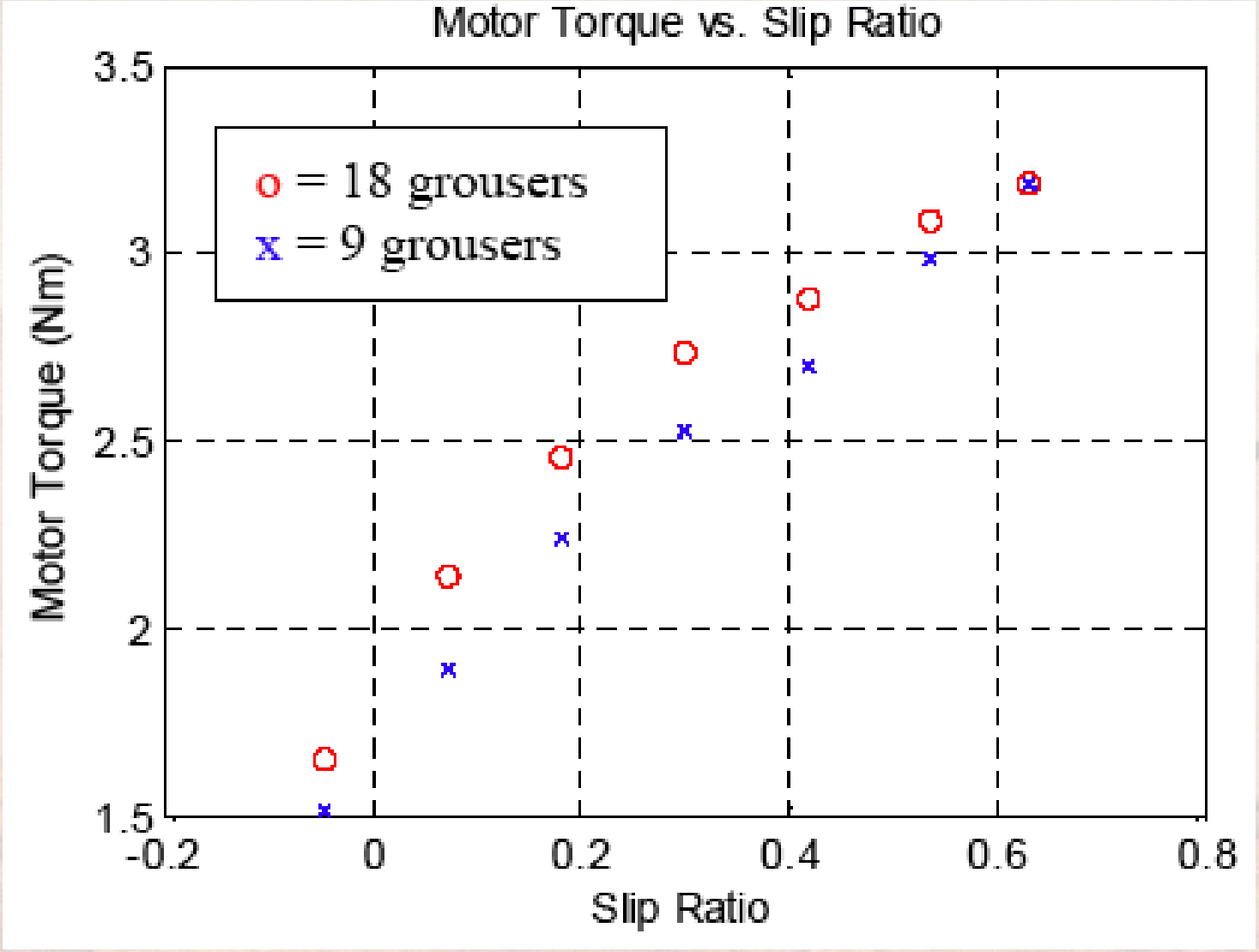
Sinkage vs. Slip Ratio



Drawbar Pull vs. Slip Ratio

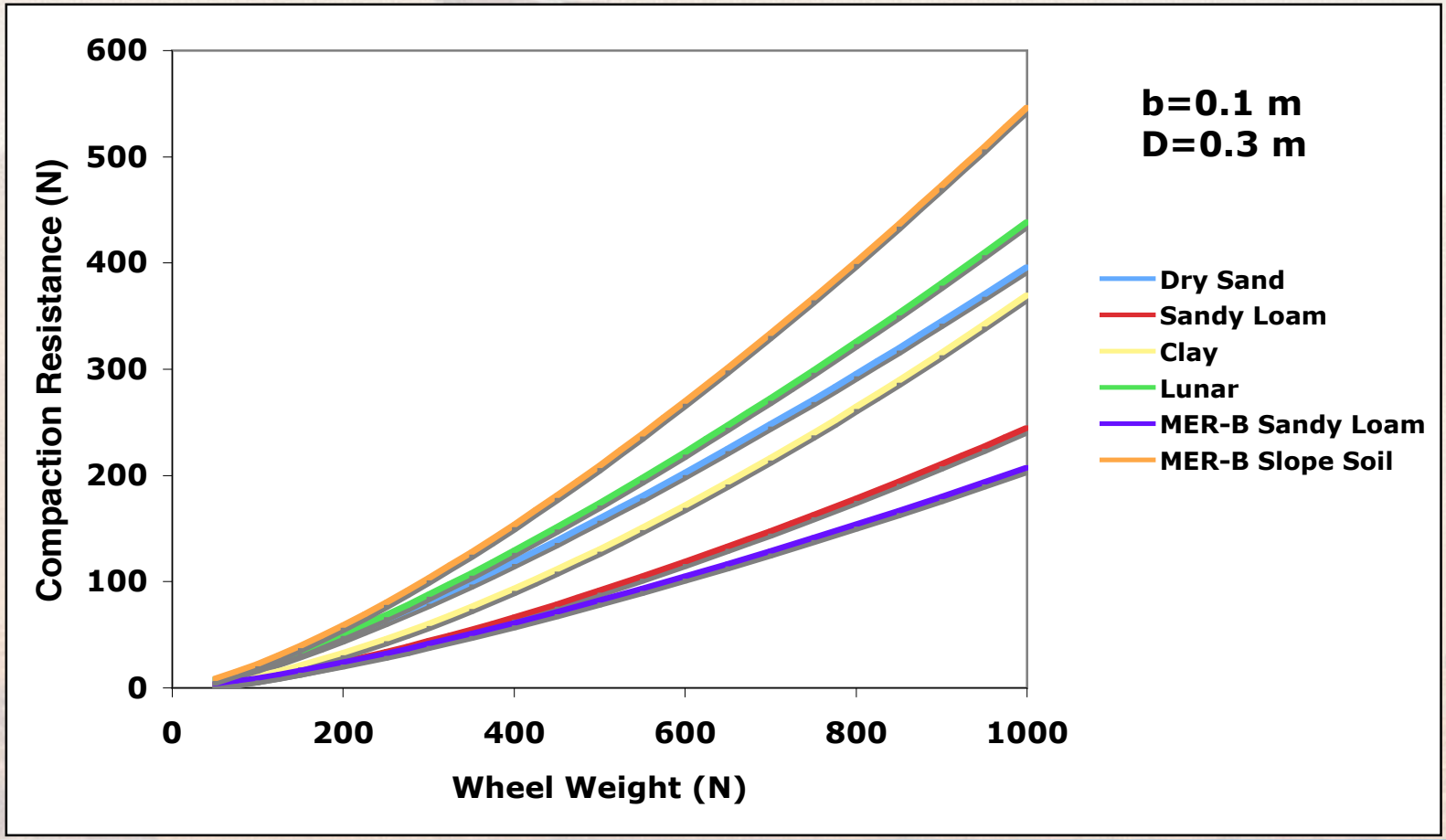


Motor Torque vs. Slip Ratio

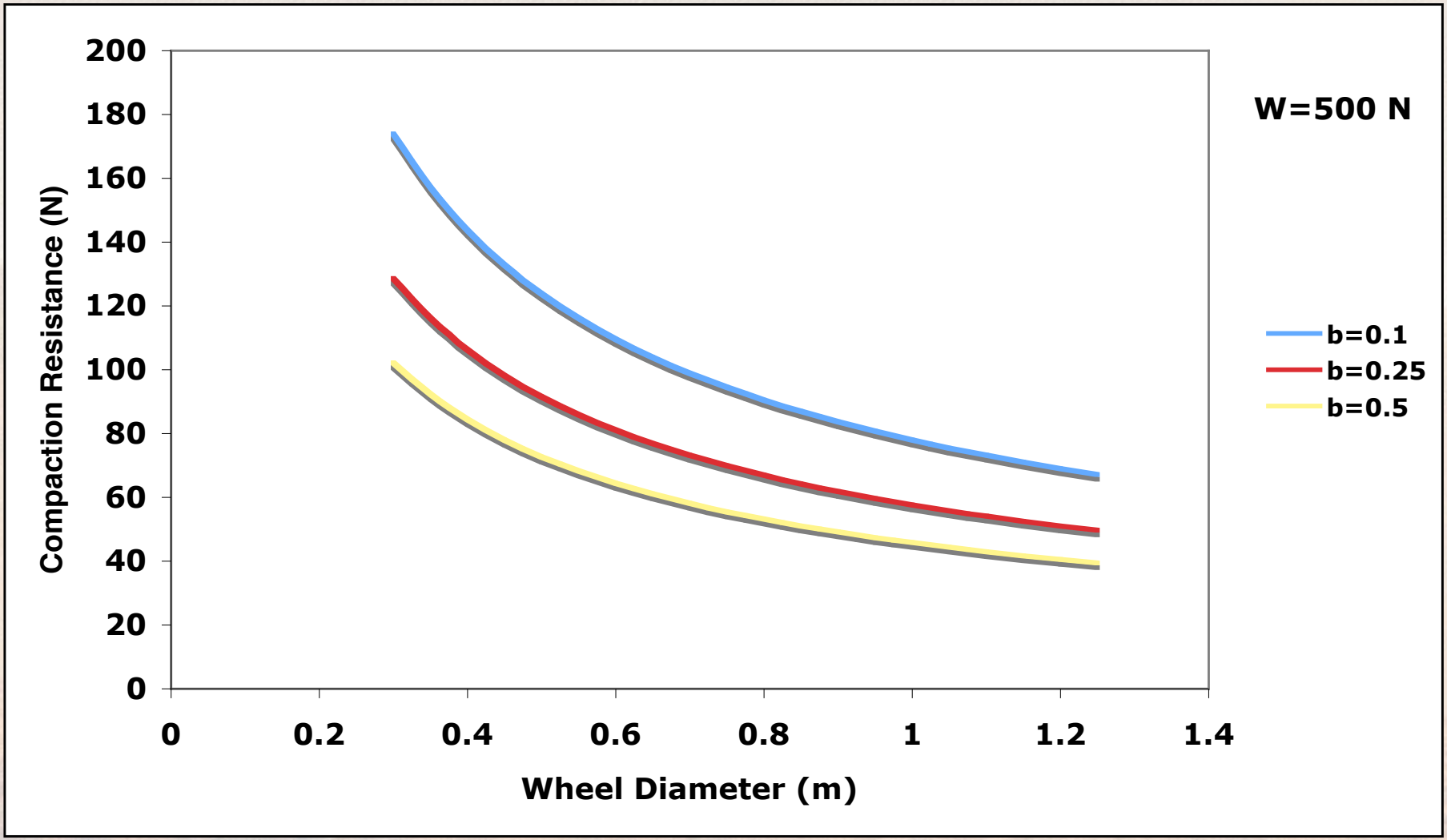


Example: Wheelbarrow (Single) Wheel

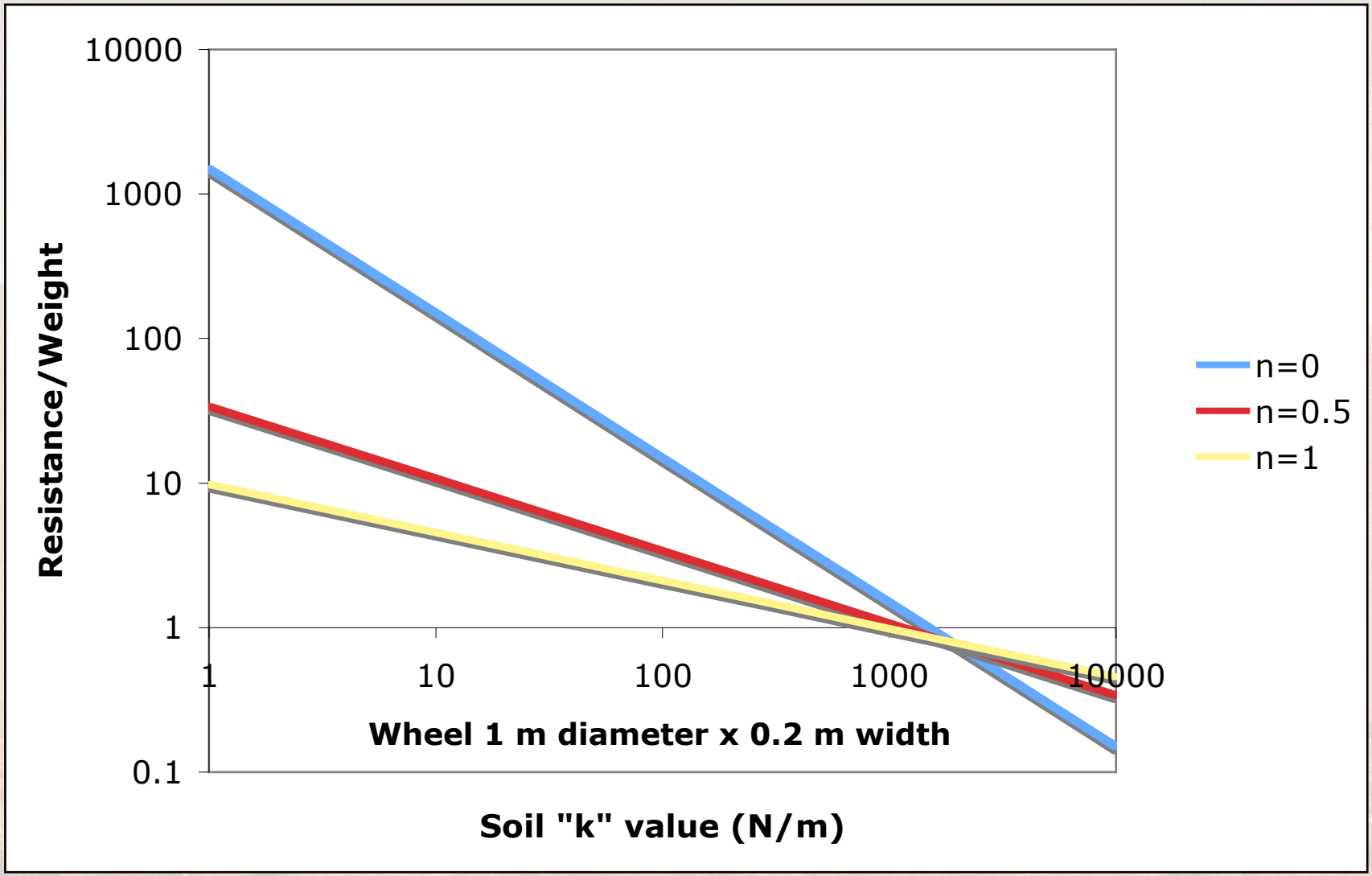
$$R = (k_c + k_\phi b)^{\frac{-1}{2n+1}} W^{\frac{2(n+1)}{2n+1}} \frac{1}{n+1} \left(\frac{3}{3-n} \right)^{\frac{2(n+1)}{2n+1}} D^{\frac{-(n+1)}{2n+1}}$$



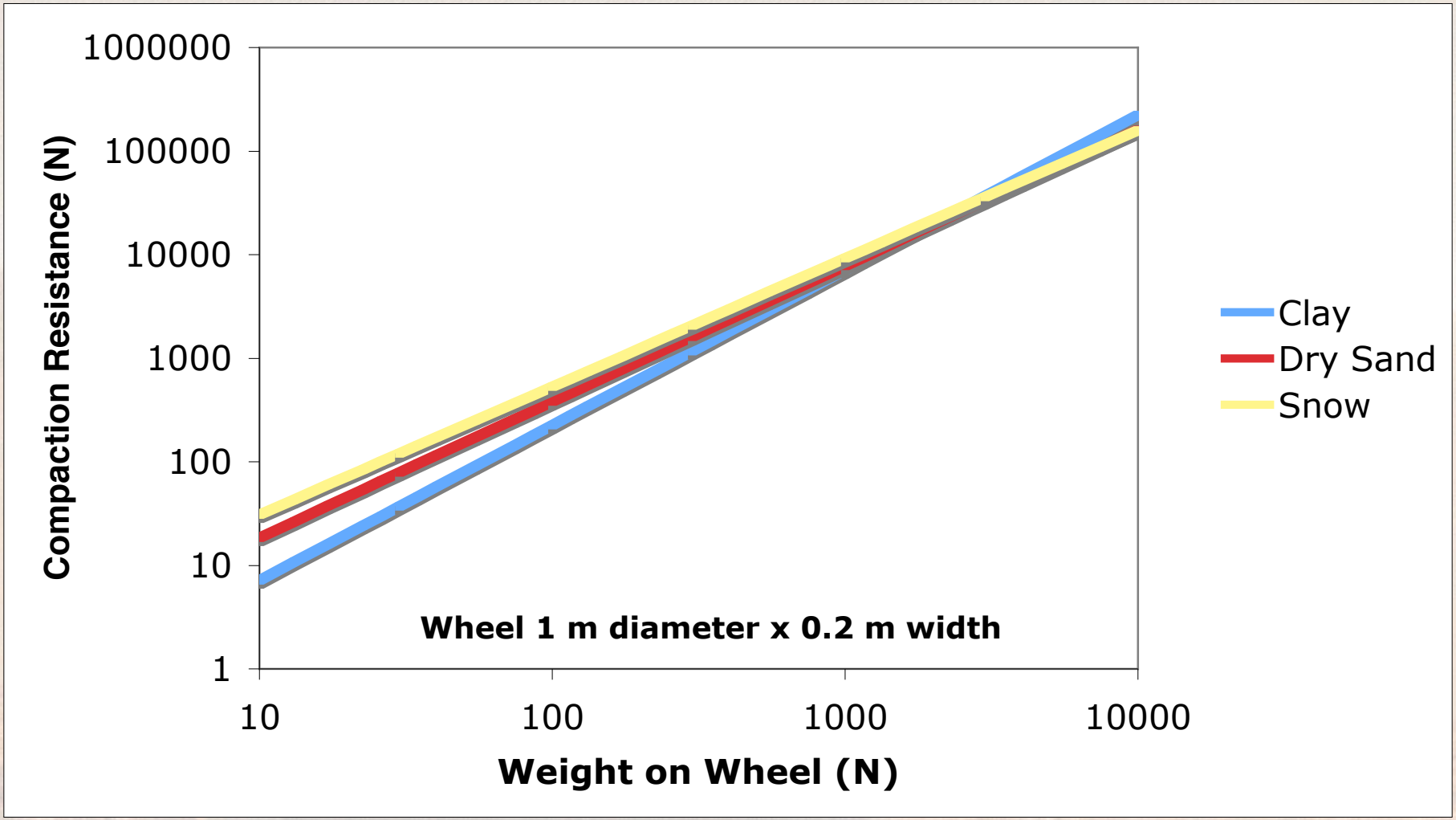
Effects of Wheel Parameters



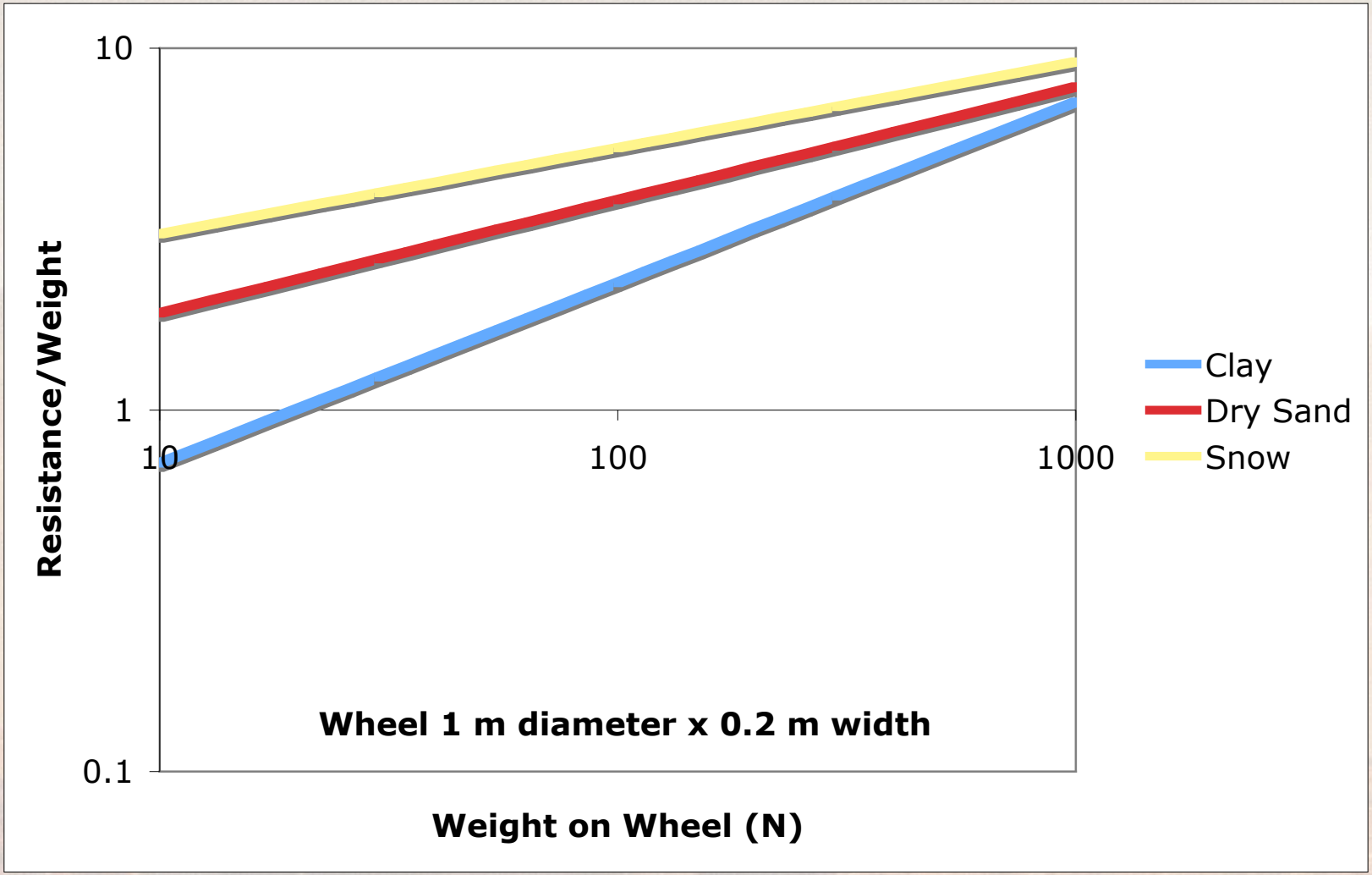
Effect of Soil "Spring Constant" on R/W



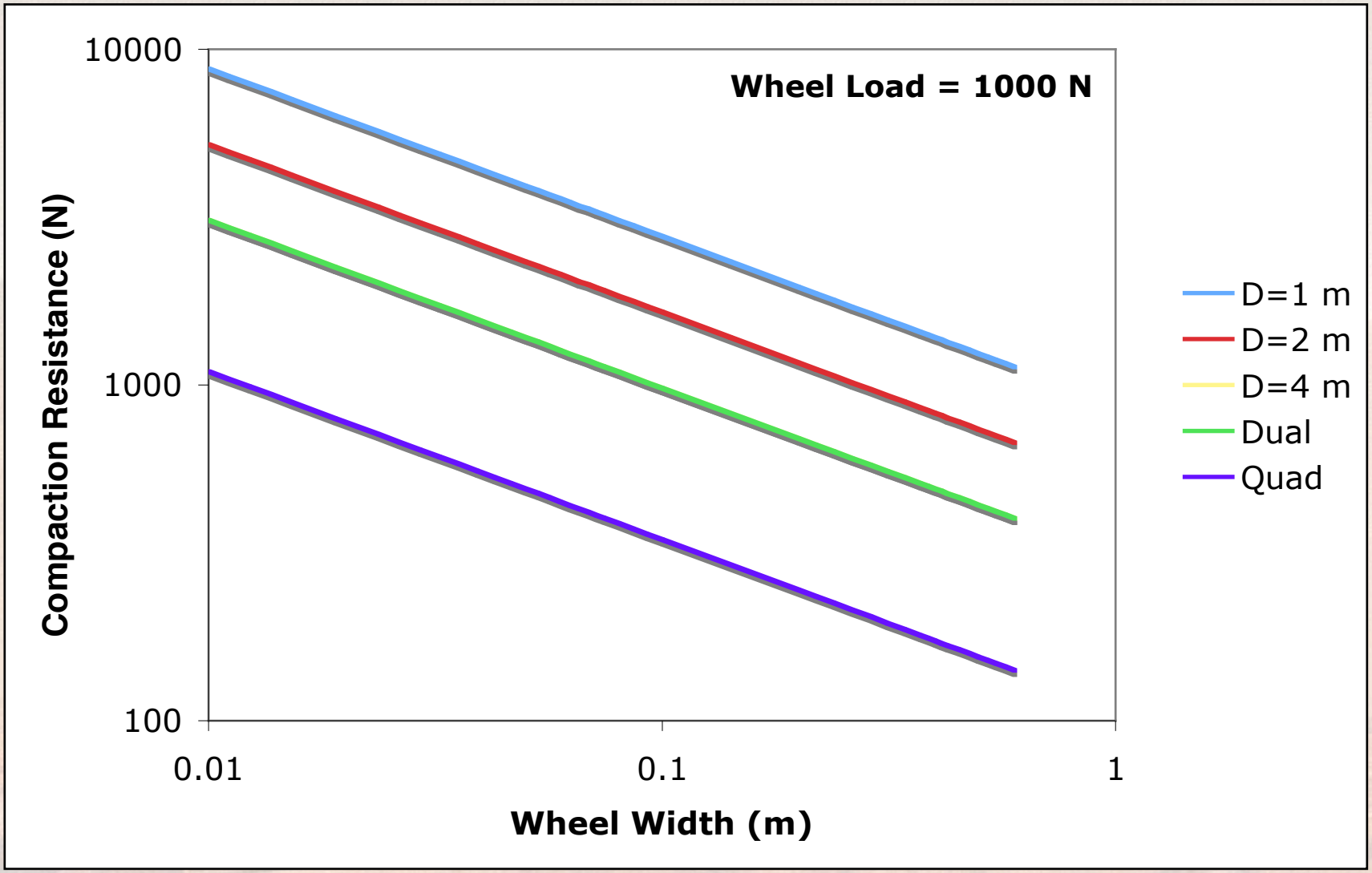
Soil Type and Wheel Load



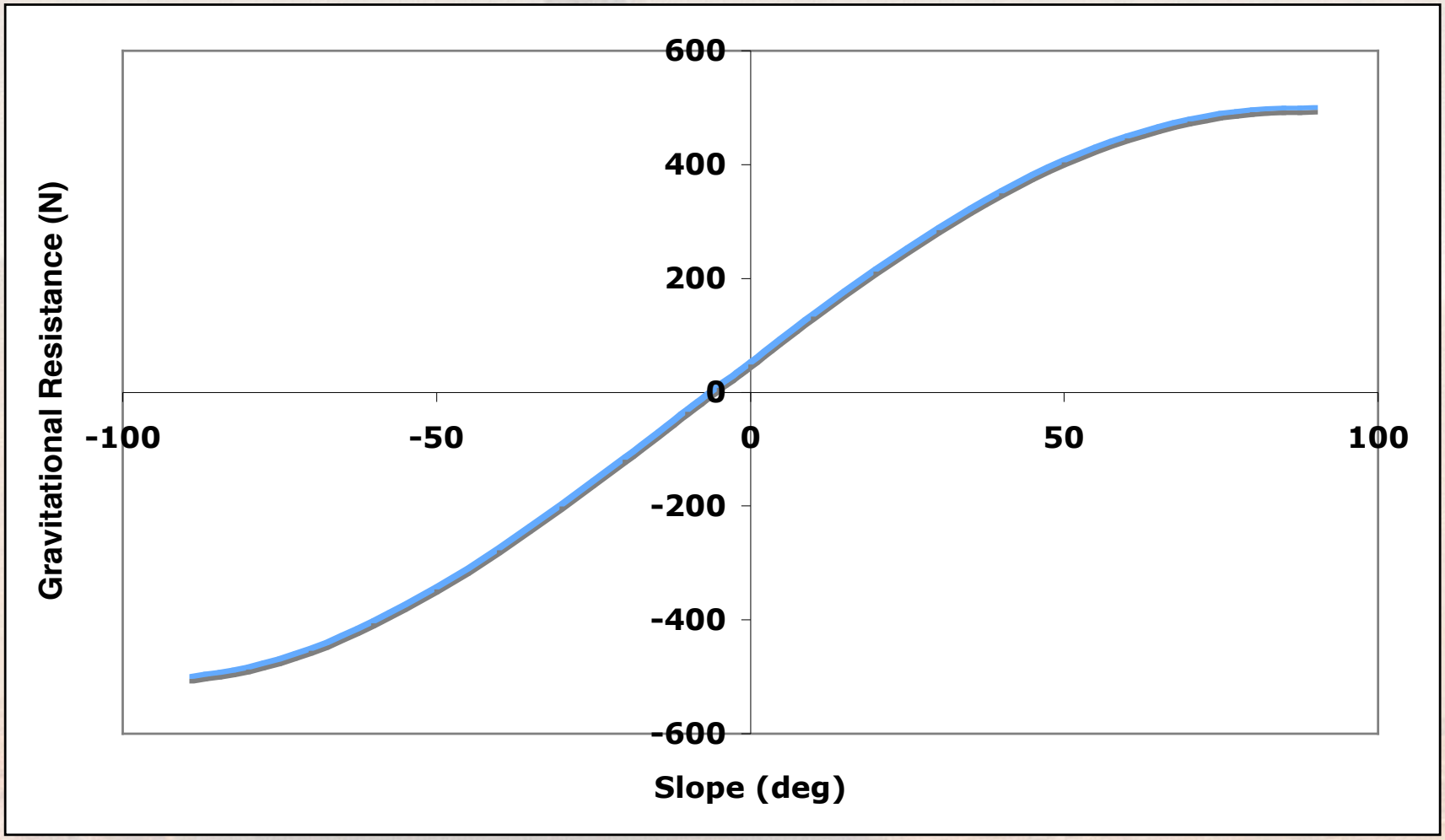
Soil Type and Specific Resistance



Effect of Wheel Diameter and Width



Effect of Slope



Bulldozing Resistance (Reminder)

General case:

All angles in radians!

$$R_b = \frac{b \sin(\alpha + \phi)}{2 \sin\alpha \cos\phi} \left(2zcK_c + \gamma z^2 K_\gamma\right) + \frac{\ell_o^3 \gamma}{3}\left(\frac{\pi}{2} - \phi\right) + c\ell_o^2\left[1 + \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\right]$$

$$\ell_o = z \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

$\ell_o \equiv$ soil disruption depth, and is *not* the same as contact length ℓ

For tracked vehicles, only the first term applies:

$$R_b = \frac{b \sin(\alpha + \phi)}{2 \sin\alpha \cos\phi} \left(2zcK_c + \gamma z^2 K_\gamma\right)$$

Bulldozing Resistance (Rb)

Symbol	Description	Value
φ	Angle of internal friction	30-40 degrees for Lunar Regolith
γ	Soil density	0.002595 N/cm^3 for Lunar Regolith
Co	Cohesive strength of soil	0.017 N/cm^2 for Lunar Regolith
Lo (degrees)	Distance of rupture	$lo = ztan^2\left(45° - \frac{\varphi}{2}\right)$
Kc (degrees)	Modulus of cohesion of soil deformation	$kc = (Nc - \tan(\varphi))(\cos^2(\varphi))$
K_γ (degrees)	Modulus of density of soil deformation	$K_\gamma = \left(\frac{2N_\gamma}{\tan\varphi} + 1\right)\cos^2(\varphi)$
Nc (radians)	Coefficient of passive earth pressure	$Nc = \cot(\varphi)\left(\frac{e^{2\left(\frac{3\pi}{4} - \frac{\varphi}{2}\right)\tan(\varphi)}}{2\cos^2\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)} - 1\right)$
α (degrees)	Angle of approach of the wheel	$\alpha = \cos^{-1}\left(1 - \frac{2z}{D}\right)$

Bulldozing Example (1)

$$\phi = 33^o = 0.576\ rad$$

$$\alpha = \cos^{-1}\left(1 - \frac{2z}{d}\right) = \cos^{-1}\left(1 - \frac{2(1.812)}{81.2}\right) = 17.18^o = 0.2999\ rad$$

$$\ell_o = z_1 \tan^2\left(\frac{\pi}{2} - \frac{\phi}{4}\right) = 0.5341$$

$$N_q = \frac{e^{(1.5\pi - \phi)\tan\phi}}{2\cos^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)} = 32.23$$

$$N_c = \frac{N_q - 1}{\tan\phi} = 48.09$$

Bulldozing Example (2)

$$N_\gamma = \frac{2(N_q + 1)\tan\phi}{1 + 0.4\sin 4\phi} = 33.27$$

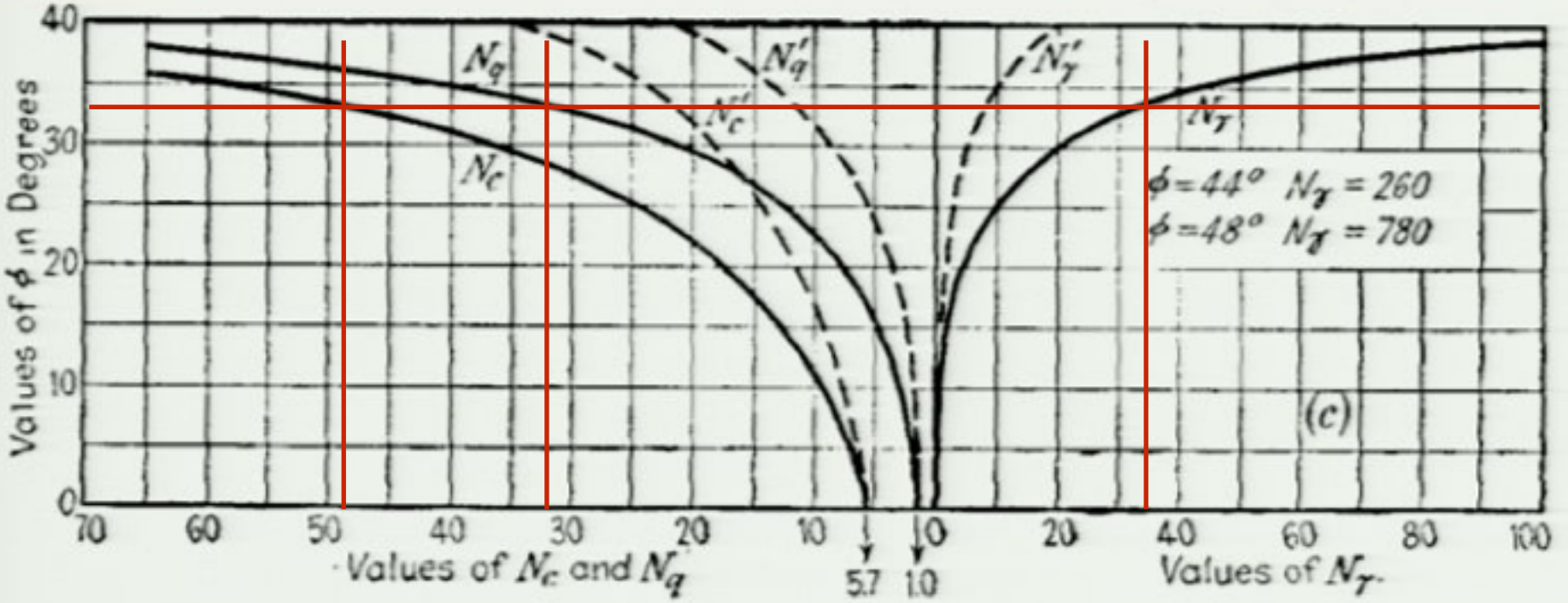
$$K_c = (N_c - \tan\phi)\cos^2\phi = 33.37$$

$$K_\gamma = \left(\frac{2N_\gamma}{\tan\phi} + 1\right)\cos^2\phi = 72.77$$

$$\ell = \frac{d}{2}\cos^{-1}\left(1 - \frac{2z}{d}\right) = 12.18\ cm$$

$$\gamma = 0.002595\ \frac{N}{cm^3}; \quad c_o = 0.017\ \frac{N}{cm^2}$$

Terzhagi Parameters



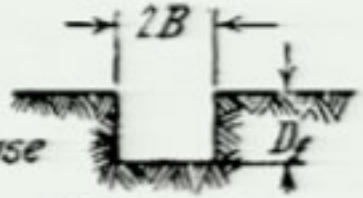
General shear failure: $Q_D = 2B(cN_c + \gamma D_f N_q + \gamma B N_\gamma)$

Local shear failure: $Q_D' = 2B(\frac{2}{3}cN_c' + \gamma D_f N_q' + \gamma B N_\gamma')$

} per unit length of footing

Circular footing, Diameter 2R,

Total critical load: $Q_{Dr} \cong R^2\pi(1.3cN_c + \gamma D_f N_q + 0.6\gamma R N_\gamma)$



Unit weight of earth = γ

Unit shear. resistance $s = c + \sigma \tan\phi$

Red lines represent values calculated for this example

Bulldozing Example (3)

$$R_b = \frac{b \sin(\alpha + \phi)}{2 \sin\alpha \cos\phi} \left(2zcK_c + \gamma z^2 K_\gamma\right) + \frac{\ell_o^3 \gamma}{3}\left(\frac{\pi}{2} - \phi\right) + c\ell_o^2\left[1 + \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\right]$$

$$\langle R_b \rangle = cm\left(cm\ \frac{N}{cm^2} + \frac{N}{cm^3}cm^2\right) + cm^3\frac{N}{cm^3} + \frac{N}{cm^2}cm^2$$

$$R_b = 94.98 + 0.000131 + 0.014 = 95.00\ N \text{ per leading wheel}$$

$$R_{b,total} = 190.0\ N$$

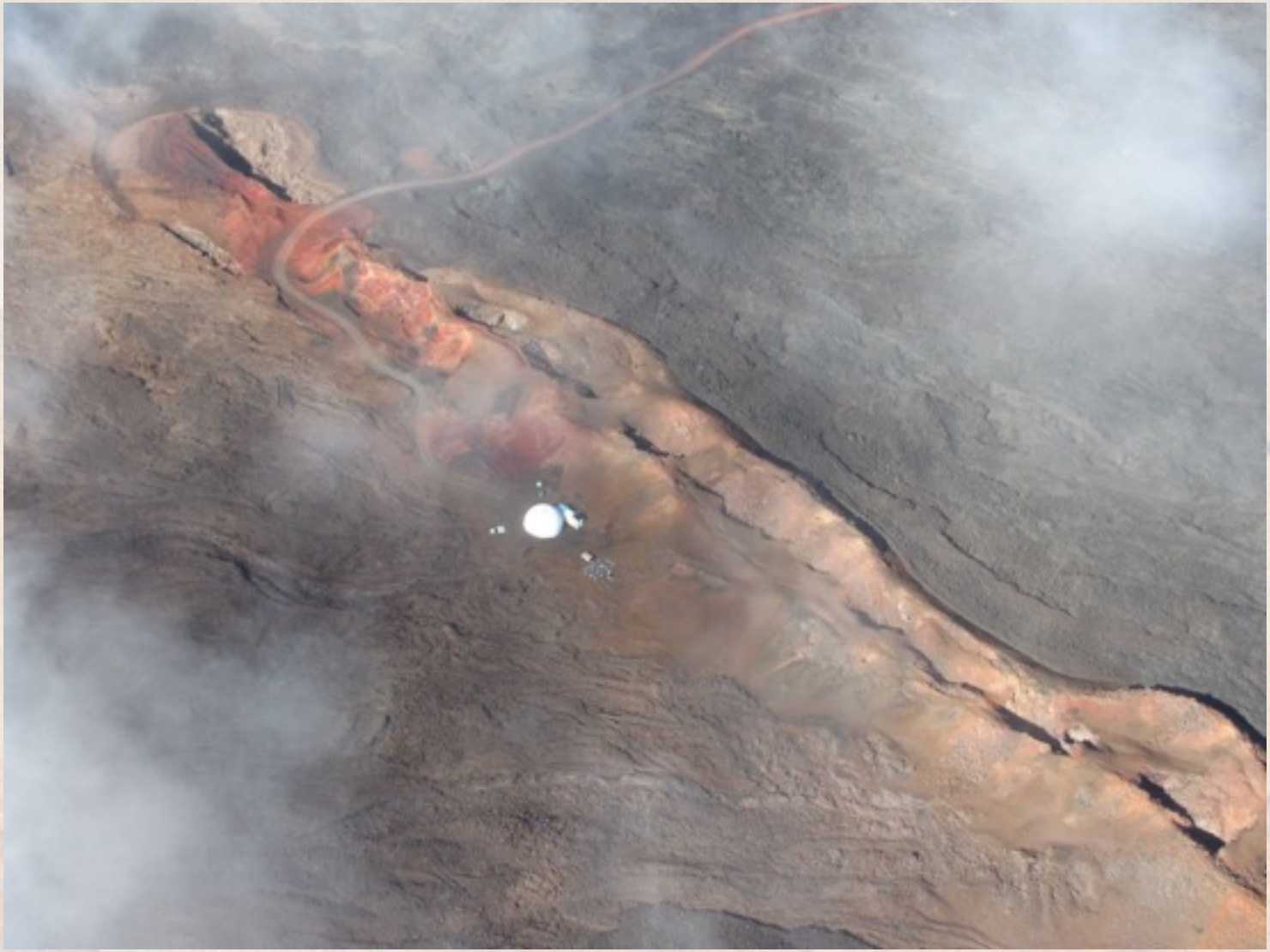
AAR Design Project Statement

- Perform a detailed design of a small astronaut assistance rover, emphasizing mobility systems
  - Chassis systems (e.g., wheels, steering, suspension…)
  - Support systems (e.g., energy storage)
  - Navigation and guidance system (e.g., sensors, algorithms...)
- Design for Moon, then assess feasibility of systems for Mars, and conversion to Earth analogue rover
- This is not a hardware project - focus is on detailed design (but may be built later!)

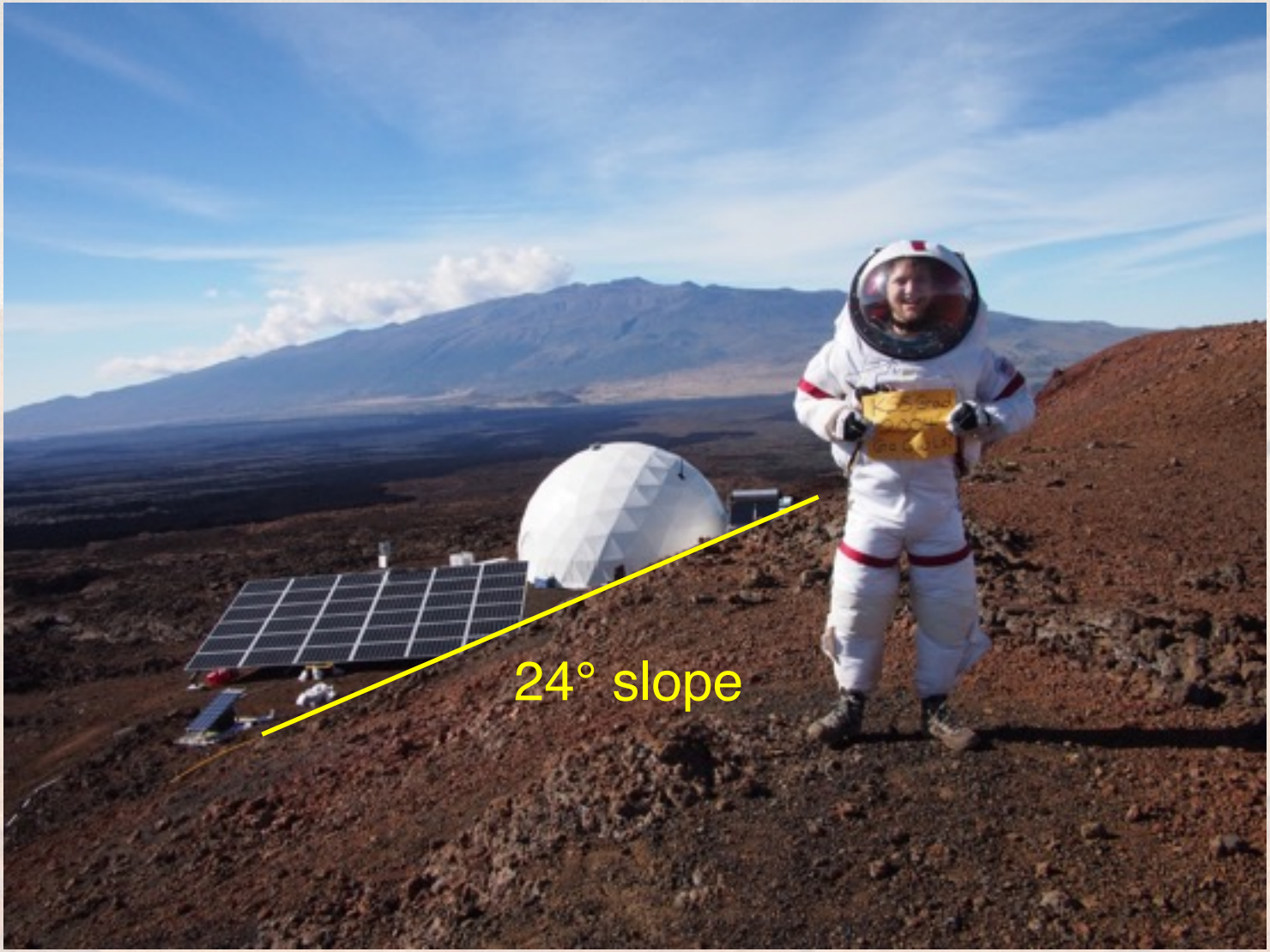
Objectives for Design Project (1)

- UMd SSL is proposing to do month-long simulations of lunar/Mars science exploration missions to examine impact of robotics
- Primary missions would be held at HI-SEAS in Hawaii
- Rover design should facilitate shipping to/from Hawaii
  - Minimal size and mass
  - Modular construction for packing (ideally each less than 50 lbs with packaging)

Aerial View of HI-SEAS Site



Sample HI-SEAS Terrain



Sample HI-SEAS Terrain

Sample HI-SEAS Terrain

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Sample HI-SEAS Terrain

First Steps in Rover Design

- Assume a vehicle gross weight
- Examine rolling resistance for design parameters
  - Number of wheels
  - Diameter of wheels
  - Width of wheels
- Calculate torque transfer to ground
  - Required torque for climbing maximum slope
  - Design of grousers for required torque
- Next step: chassis design