Multi-Wheel Systems

- Obstacle climbing with multiwheel systems
- Planar rocker analysis
- Planar rocker-bogey analysis
- Suspension dynamics

Slopes & Obstacles

wheel coefficient of friction
with ground $= \mu$

$N \equiv$ normal force to surface

$\tau = \mu r N = \mu r W \sin \alpha$

$T = \mu N = \frac{\tau}{r} = W \sin \alpha$

$\mu W \cos \alpha = W \sin \alpha$

$\tan \alpha = \mu$

Assume $\tau > \mu_{limit} N r$ (friction limited,
not torque limited)

Longitudinal Dynamic Solutions

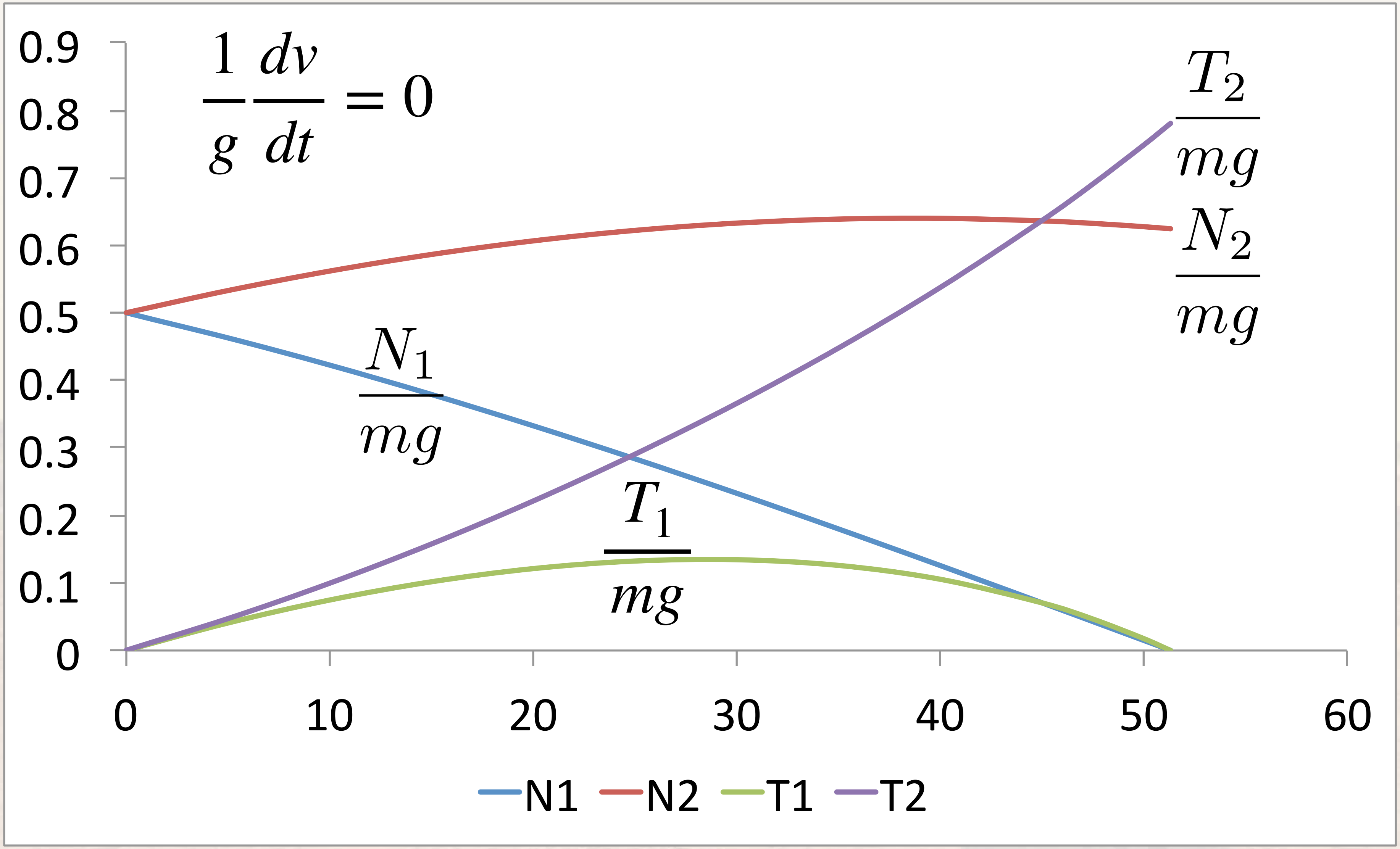
$$N_1 = mg\left[\left(1 - \frac{a}{\ell}\right)\cos\theta - \left(\frac{h}{\ell} + \frac{r}{\ell}\right)\sin\theta - \frac{1}{g}\frac{dv}{dt}\right]$$

$$N_2 = mg\left[\frac{a}{\ell}\cos\theta + \left(\frac{h}{\ell} + \frac{r}{\ell}\right)\sin\theta + \frac{1}{g}\frac{dv}{dt}\right]$$

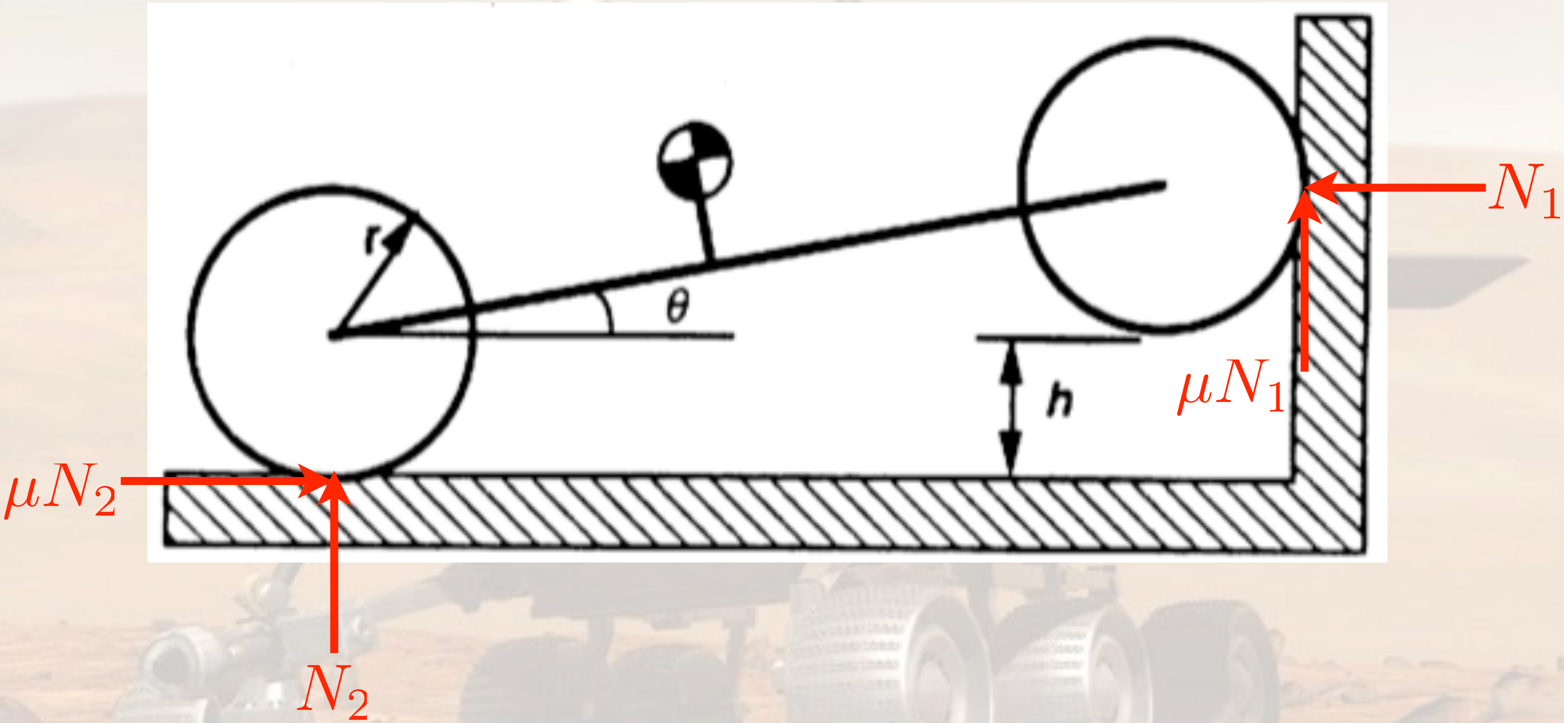
$$T_2 = \frac{N_2}{N_1 + N_2}\left(mg\sin\theta + m\frac{dv}{dt}\right)$$

$$T_1 = \frac{N_1}{N_1 + N_2}\left(mg\sin\theta + m\frac{dv}{dt}\right)$$

Normal and Shear Wheel Force w/Slope

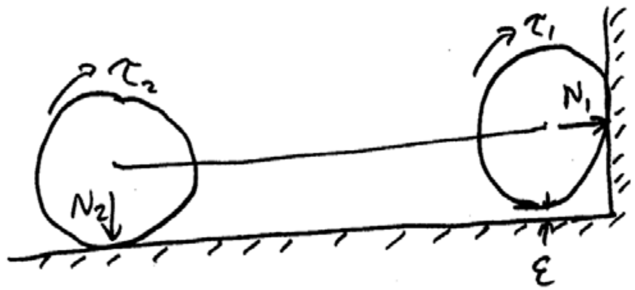


Four-Wheeled Vehicle Climbing a Wall



from Howard Eisen, "Scale and Computer Modeling of Wheeled Vehicles for Planetary Exploration" S.M. Thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, May, 1990

Wall Climbing



$$\sum F_{vertical} \Rightarrow N_2 + \mu N_1 = W$$

$$\sum F_{horizontal} \Rightarrow \mu N_2 = N_1$$

$$\sum M_{rear\ axle} \Rightarrow \mu N_2 r + \mu N_1 (l + r) = W(l - a)$$

$$N_2 + \mu^2 N_2 = W \Rightarrow N_2 = \frac{W}{1+\mu^2} \Rightarrow N_1 = \frac{\mu}{1+\mu^2} W$$

$$\frac{\mu}{1+\mu^2} Wr + \frac{\mu^2}{1+\mu^2} W(l+r) = W(l-a)$$

$$(r+a)\mu^2 + r\mu - (l-a) = 0$$

$$\mu = \frac{-r \pm \sqrt{r^2 + 4(r+a)(\ell - a)}}{2(r+a)}$$

Let $\alpha \equiv \frac{a}{r}$ $\quad \lambda \equiv \frac{\ell}{r}$

$$\mu = \frac{-1 \pm \sqrt{1 + 4(1+\alpha)(\lambda - a)}}{2(1+\alpha)}$$

Assume $\alpha = \frac{\lambda}{2}$

$$\mu = -\frac{1}{2+\lambda} \pm \frac{\sqrt{1 + 4\left(1 + \frac{\lambda}{2}\right)\left(\frac{\lambda}{2}\right)}}{2+\lambda}$$

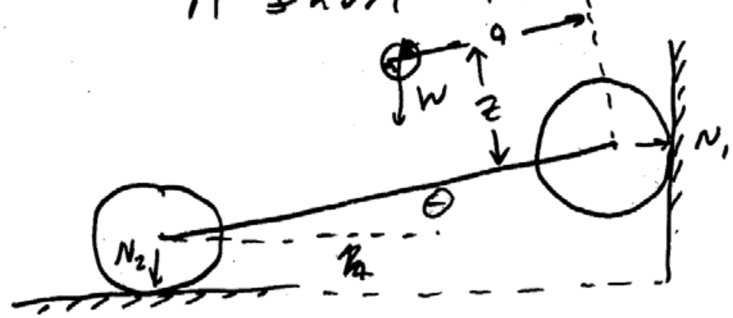
$$1 + 4\frac{\lambda}{2} + 4\frac{\lambda^2}{4}$$
$$= 1 + 2\lambda + \lambda^2$$
$$= (\lambda + 1)^2$$

$$= -\frac{1 \pm (\lambda+1)}{2+\lambda} = \frac{\lambda}{2+\lambda},\ -1$$

$\lambda \to 0 \quad \mu_{limit} \to 0 \qquad \lambda \to \infty \quad \mu_{limit} \to 1$

Shorter is better!

A Short Time Later...



$h = l \sin\theta$

$$\left.\begin{aligned} \Sigma F_{vert} &\Rightarrow \mu N_1 + N_2 = W \\ \Sigma F_{horiz} &\Rightarrow \mu N_2 = N_1 \end{aligned}\right\} \quad \begin{aligned} N_1 &= \frac{\mu}{1+\mu^2} W \\ N_2 &= \frac{W}{1+\mu^2} \end{aligned}$$

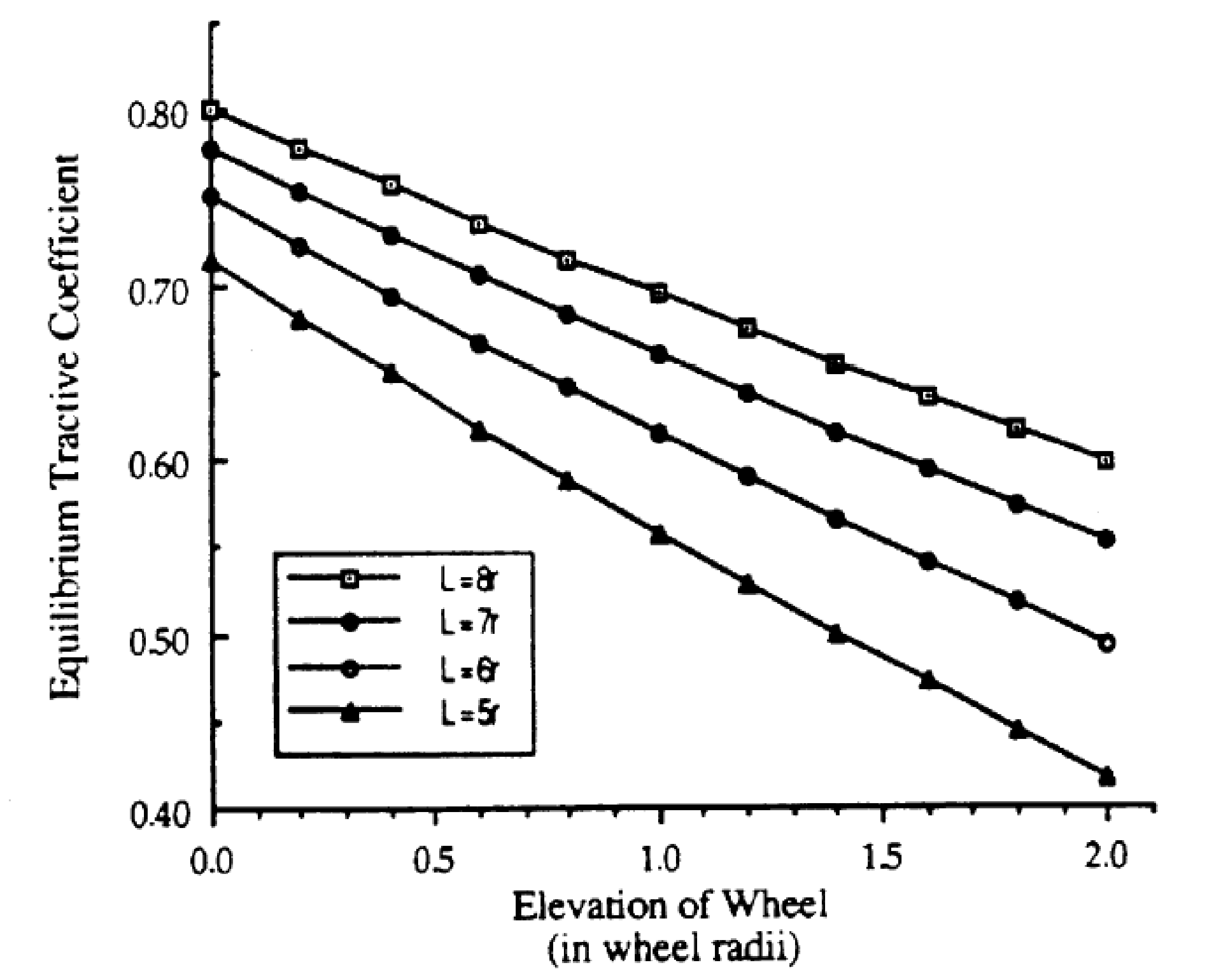
$$\Sigma M_{rear} \Rightarrow \mu N_2 r + N_1 l \sin\theta + \mu N_1 (r + l\cos\theta) = W\left[(l-a)\cos\theta - z\sin\theta\right]$$

As θ increases, effect of W decreases and

effect of N_1 increases $\Rightarrow$

hardest point of the climb is at the start!

Required Traction for Wall Climbing



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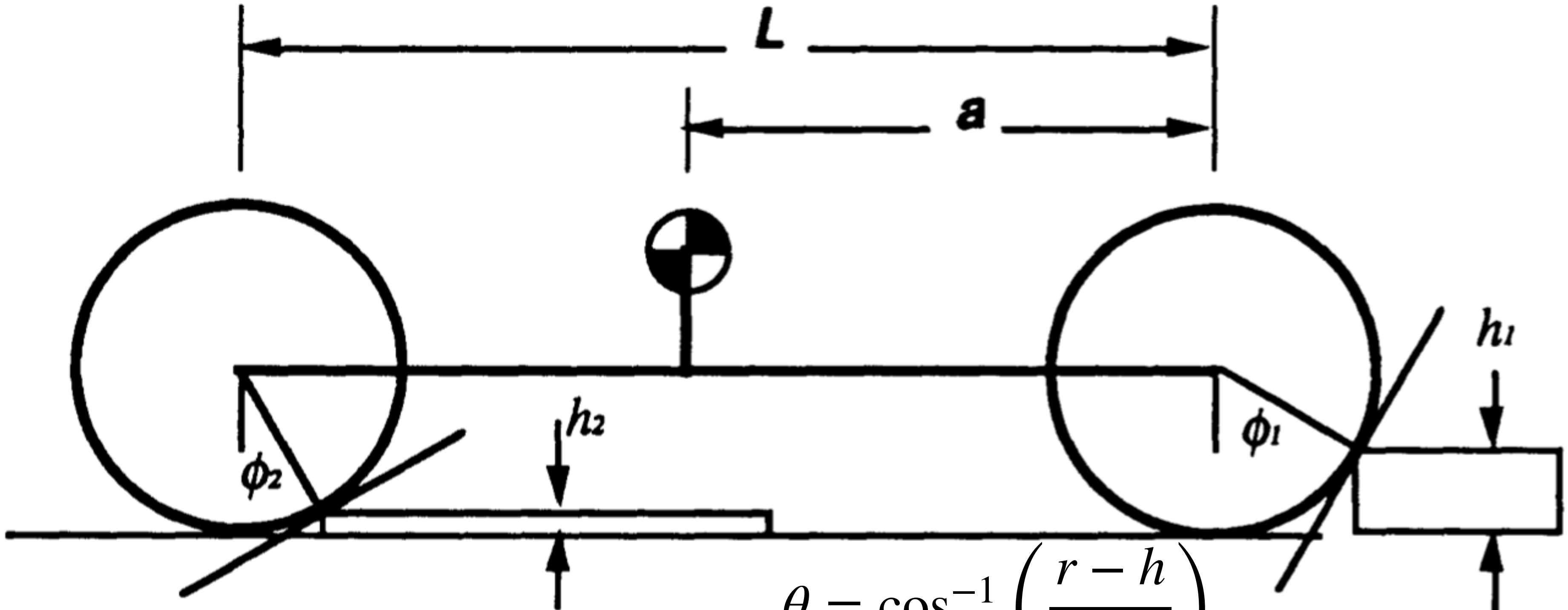
Wheel Interaction with Slope



$$\theta = \cos^{-1}\left(\frac{r-h}{r}\right)$$

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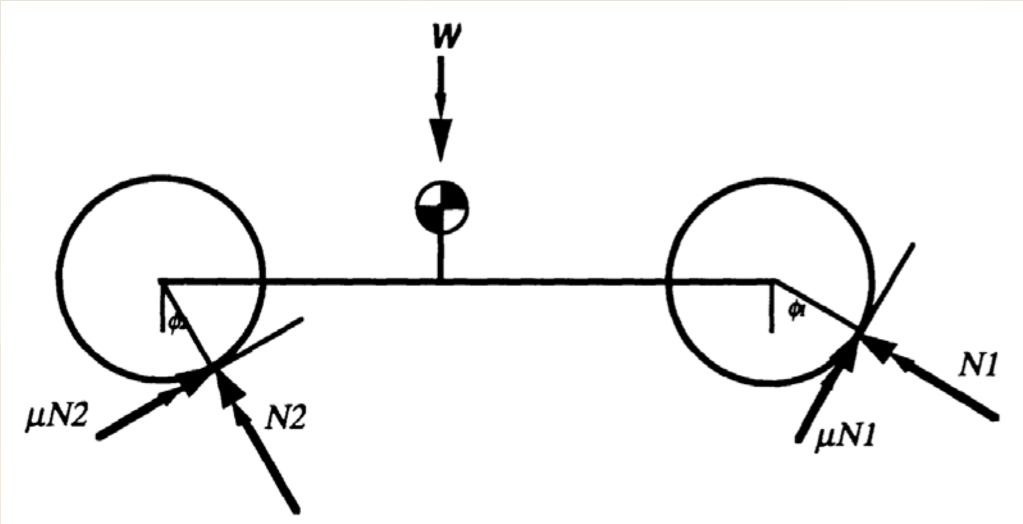
Equations for Slopes under Wheels



Sum of Horizontal forces:

$$\mu N_2 \sin\phi_2 + N_2 \cos\phi_2 + \mu N_1 \sin\phi_1 + N_1 \cos\phi_1 - W = 0$$

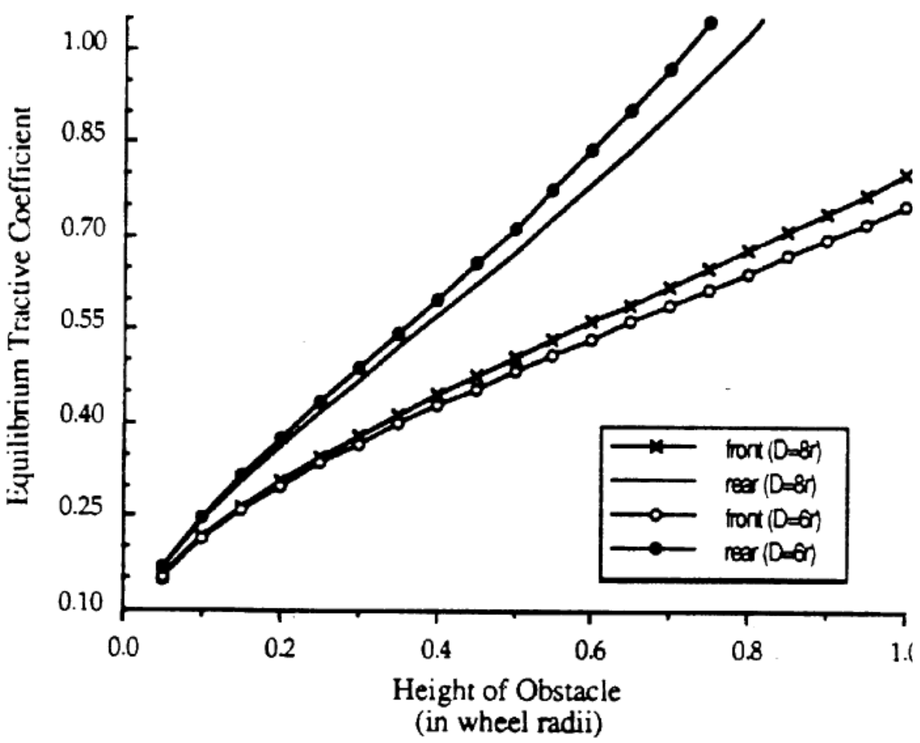
Sum of vertical forces:

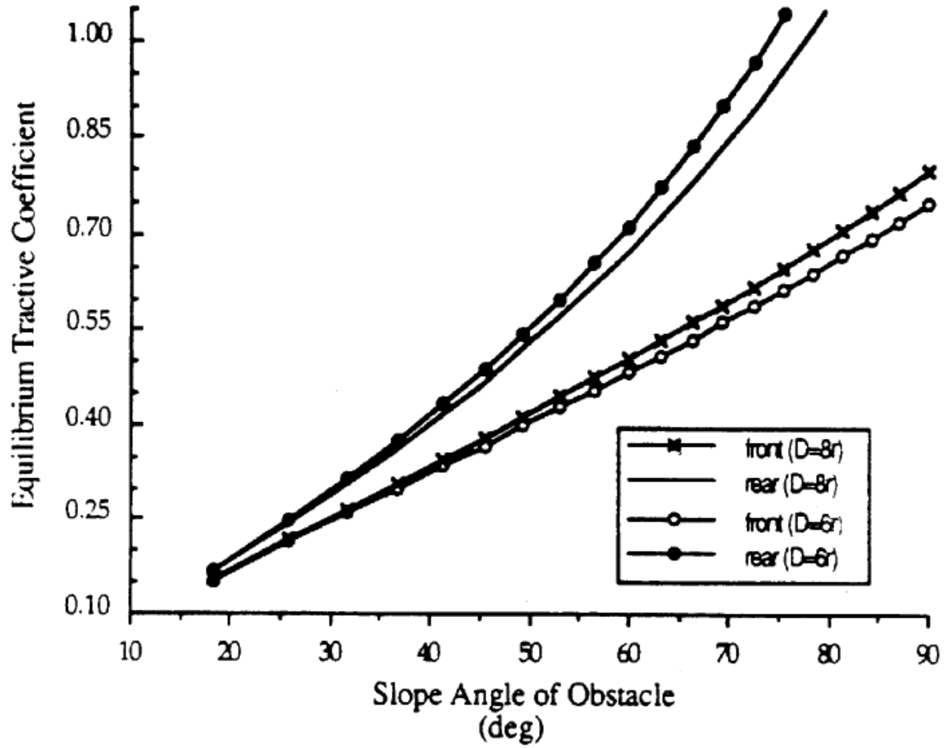
$$\mu N_2 \cos\phi_2 - N_2 \sin\phi_2 + \mu N_1 \cos\phi_1 - N_1 \sin\phi_1 = 0$$

Sum of forces around the rear axle:

$$\Big(\mu N_2 r - W(L-a) + N_1 L \cos\phi_1 + \mu N_1 \left(r + L\sin\phi_1\right) = 0$$

Bump/Slope Traction Requirements





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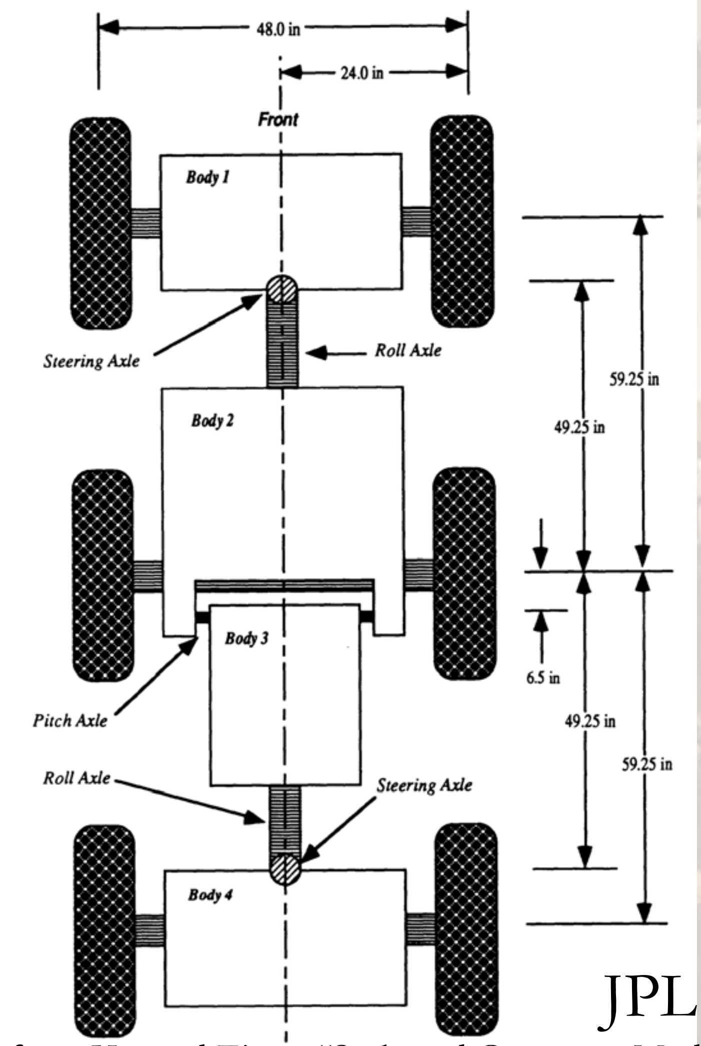
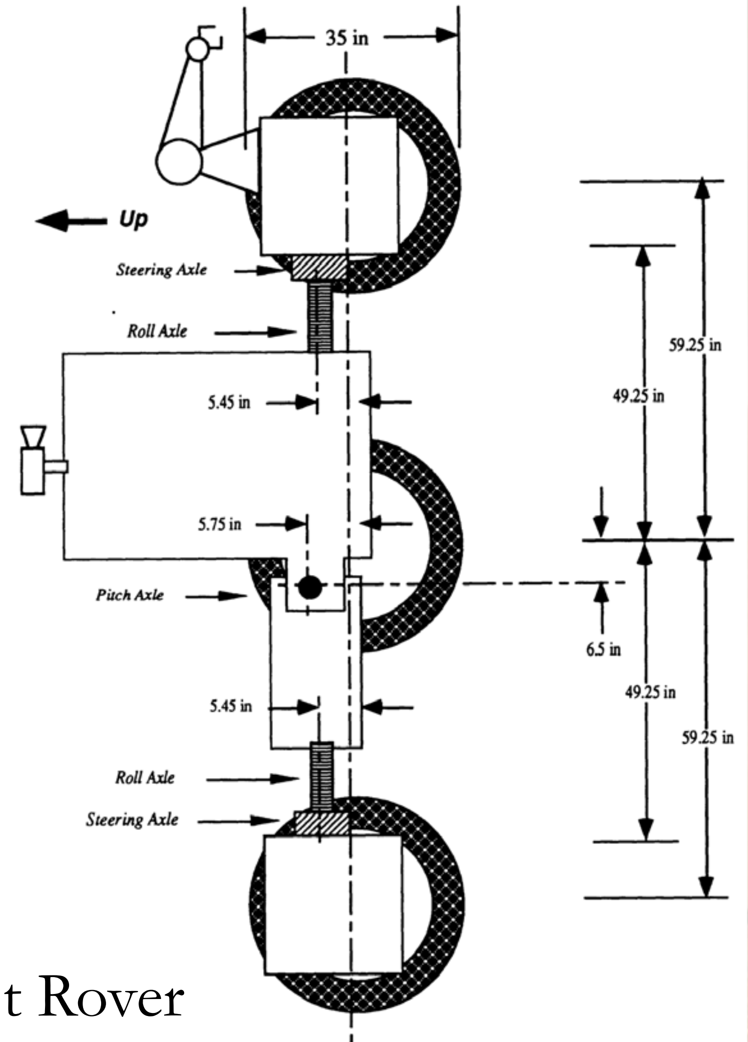
Six-Wheel Articulated Body Rover

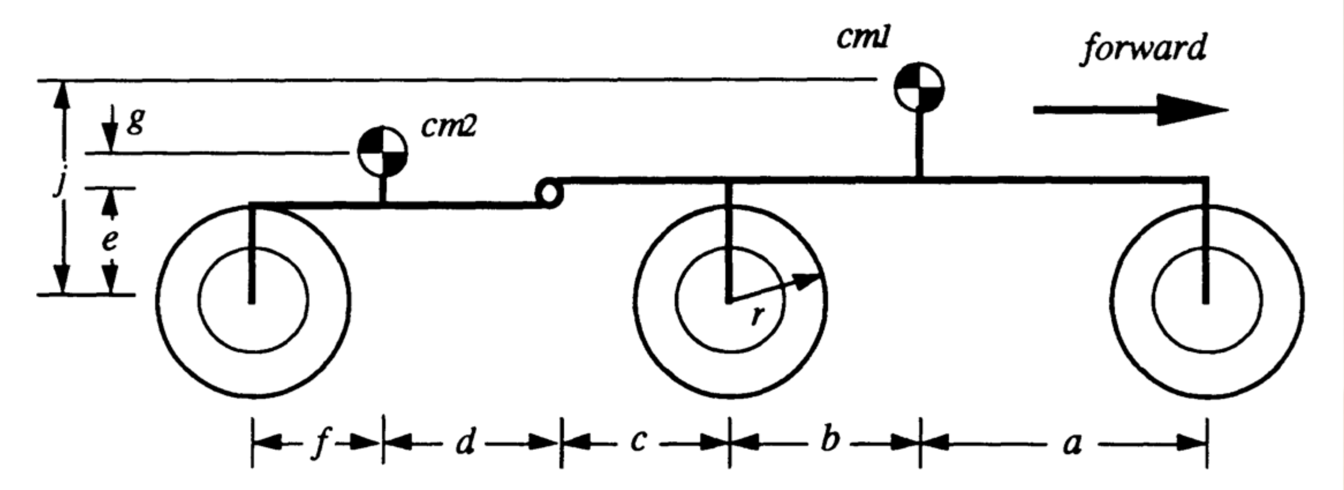




JPL Navtest Rover

from Howard Eisen, "Scale and Computer Modeling of Wheeled Vehicles for Planetary Exploration" S.M. Thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, May, 1990

Model of Six-Wheel Vehicle



Sum of vertical forces:

$$N_3 + \mu N_2 + N_1 - Wf - Wb = 0$$

Sum of horizontal forces:

$$\mu N_3 - N_2 + \mu N_1 = 0$$

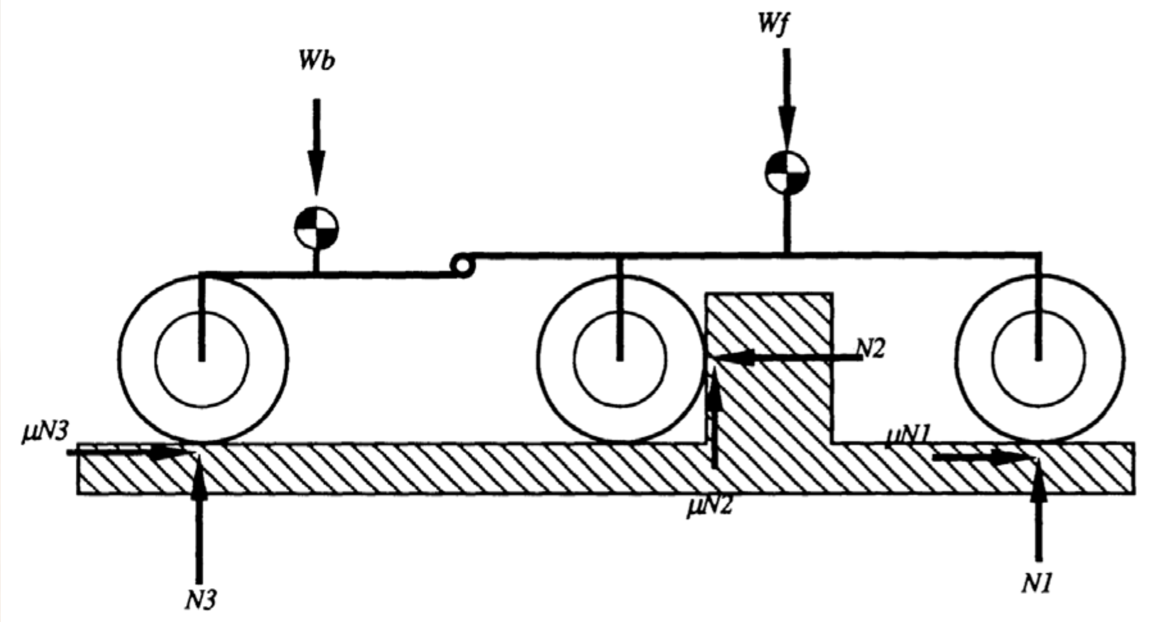
Sum of moments for front body around pitch axis

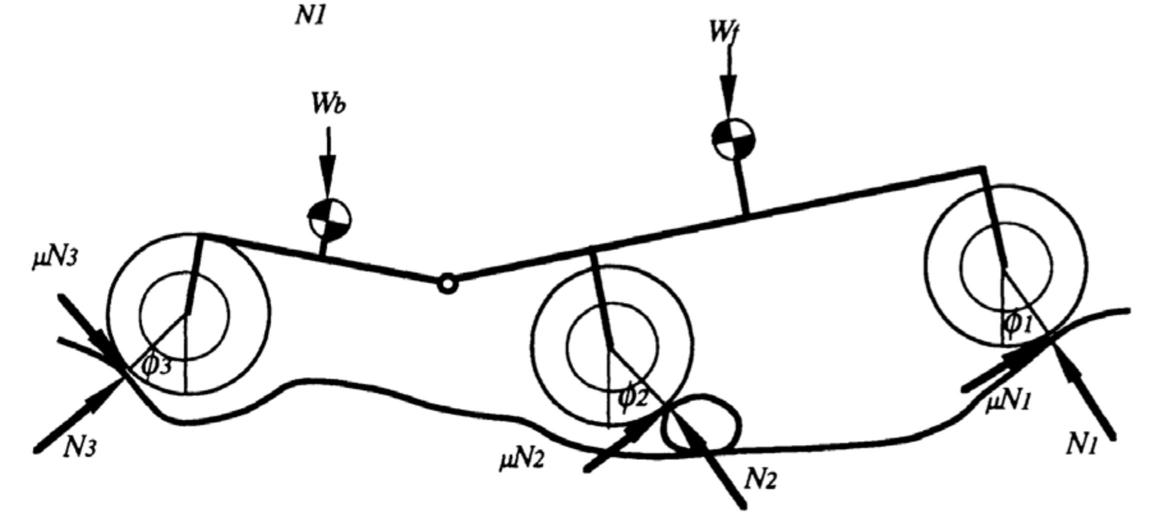
$$\mu N_1(r + e) + N_1(a + b + c) + \mu N_2(r + c) + -N_2 e - Wf(b + c) = 0$$

Sum of moments for rear body around pitch axis

$$Wbd + \mu N_3(r + e) - N_3(d + f) = 0$$

Navtest Rover with Walls and Slopes





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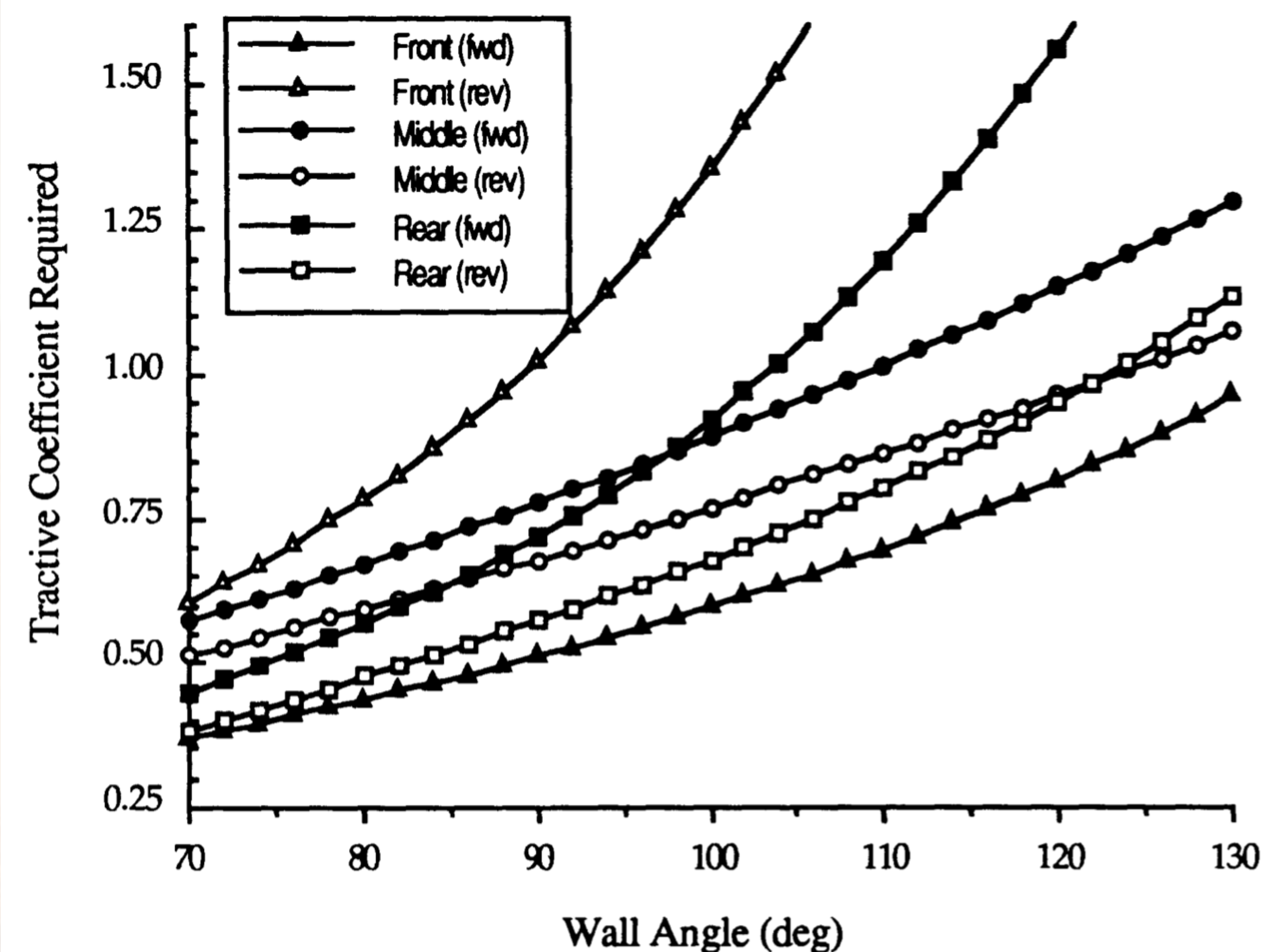
Six-Wheel Rover, Slope Climbing



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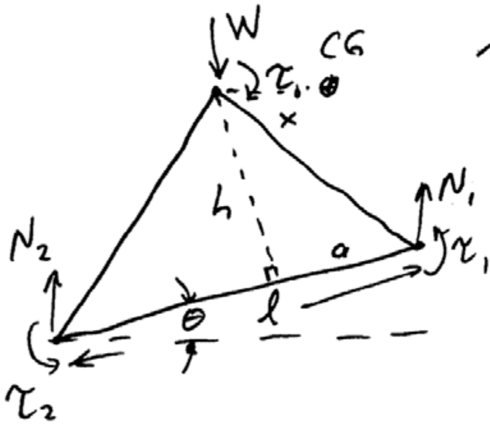
Four-Wheel Rocker Suspension



$\tau_0 = xW$

Planar Rocker Analysis

Σ Forces: $N_1 + N_2 = W$

Σ Moment (rear axle)

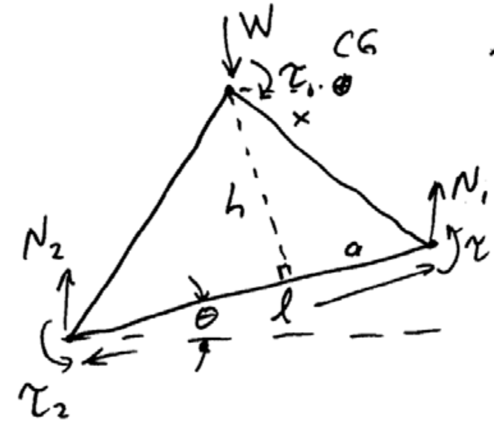
$$\tau_1 + \tau_2 + N_1 \ell \cos\theta = \tau_0 + W[(\ell - a)\cos\theta \mp h \sin\theta]$$

$$N_1 \ell \cos\theta = \tau_0 - \tau_1 - \tau_2 + W[(\ell - a)\cos\theta - h\sin\theta]$$

$$\boxed{N_1 = \frac{\tau_0 - \tau_1 - \tau_2}{\ell \cos\theta} + W\left(\frac{\ell - a}{\ell} - \frac{h}{\ell}\tan\theta\right)}$$

$$N_2 = W - N_1 = W\left(1 - \frac{\ell - a}{\ell} + \frac{h}{\ell}\tan\theta\right) - \frac{\tau_0 - \tau_1 - \tau_2}{\ell\cos\theta}$$

$$\boxed{N_2 = W\left(\frac{a}{\ell} + \frac{h}{\ell}\tan\theta\right) + \frac{\tau_1 + \tau_2 - \tau_0}{\ell\cos\theta}}$$



$\tau_0 = xW$

Nondimensionalize

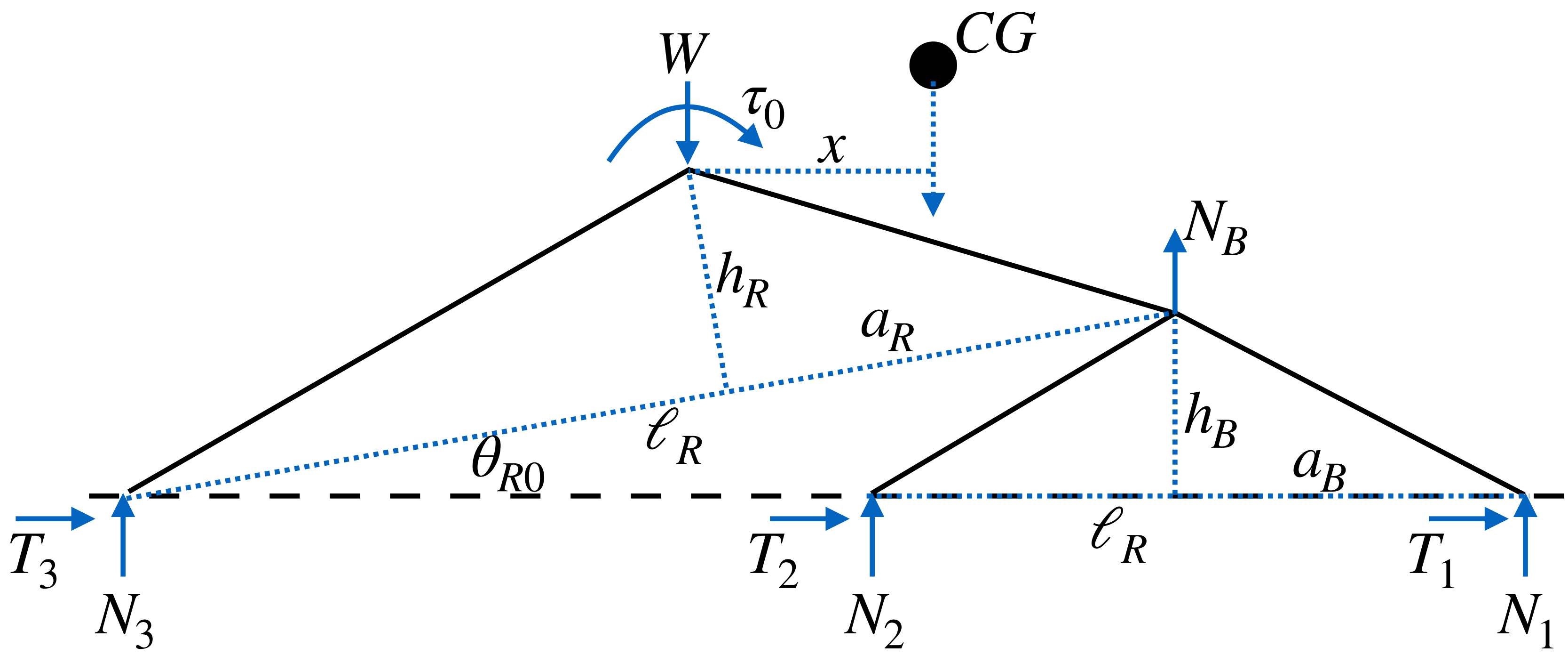
$$\tau_0 = Wx \Rightarrow \frac{\tau_0}{W\ell} = \frac{x}{\ell}$$

$$\tau_{wheel} = Tr \Rightarrow \frac{\tau_{wheel}}{W\ell} = \frac{T}{W}\frac{r}{\ell}$$

$$\frac{N_1}{W} = 1 - \frac{a}{\ell} - \frac{h}{\ell}\tan\theta + \left(\frac{x}{\ell} - \frac{T_1}{W}\frac{r_1}{\ell} - \frac{T_2}{W}\frac{r_2}{\ell}\right)\frac{1}{\cos\theta}$$

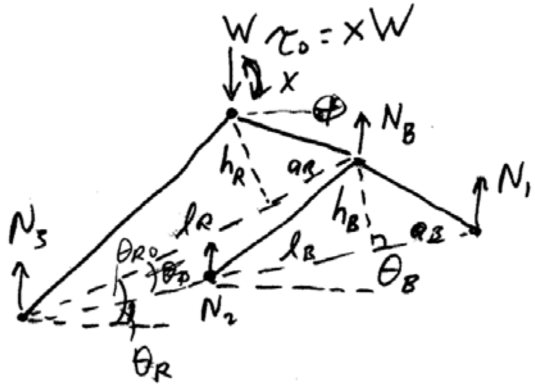
$$\frac{N_2}{W} = \frac{a}{\ell} + \frac{h}{\ell}\tan\theta + \left(\frac{T_1}{W}\frac{r_1}{\ell} + \frac{T_2}{W}\frac{r_2}{\ell} - \frac{x}{\ell}\right)\frac{1}{\cos\theta}$$

Six-Wheel Rocker-Bogey Suspension



Kinematics of Planar Rocker-Bogey

Planar Rocker-Bogey Analysis



For bogey,

$$N_1 = N_B\left(\frac{l_B - a_B}{l_B} - \frac{h_B}{l_B}\tan\theta_B\right) - \frac{\tau_1 + \tau_2}{l_B\cos\theta_B}$$

$$N_2 = N_B\left(\frac{a_B}{l_B} + \frac{h_B}{l_B}\tan\theta_B\right) + \frac{\tau_1 + \tau_2}{l_B\cos\theta_B}$$

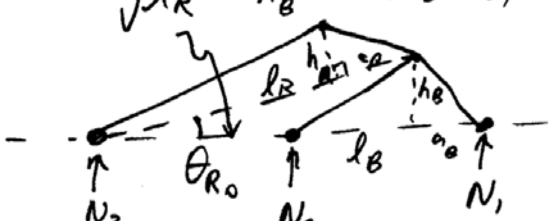
($\tau_B = 0$ always)

For rocker,

$$N_B = \frac{\tau_0 - \tau_3}{l_R\cos(\theta_R + \theta_{R_0})} + W\left(\frac{l_R - a_R}{l_R} - \frac{h_R}{l_R}\tan(\theta_R + \theta_{R_0})\right)$$

$$N_3 = W\left(\frac{a_R}{l_R} + \frac{h_R}{l_R}\tan(\theta_R + \theta_{R_0})\right) + \frac{\tau_3 - \tau_0}{l_R\cos(\theta_R + \theta_{R_0})}$$

$$\sqrt{l_R^2 - h_B^2} - (l_B - a_B) \equiv l_3 \qquad \theta_{R_0} = \sin^{-1}\frac{h_B}{l_R}$$



$$\Rightarrow \sqrt{(l_3 + l_B - a_B)^2 + h_B^2} = l_R \Rightarrow \frac{l_R}{l_B} = \sqrt{\left(\frac{l_3}{l_B} + 1 - \frac{a_B}{l_B}\right)^2 + \left(\frac{h_B}{a_B}\right)^2}$$

Normalize by W and l_B

$$\frac{N_3}{W} = \frac{a_R}{l_R} + \frac{h_R}{l_R}\tan(\theta_R + \theta_{R_0}) + \left(\frac{T_3}{W}\frac{r_3}{l_B}\frac{l_B}{l_R} - \frac{x}{l_B}\frac{l_B}{l_R}\right)\frac{1}{\cos(\theta_R + \theta_{R_0})}$$

$$\frac{N_B}{W} = 1 - \frac{a_R}{l_R} - \frac{h_R}{l_R}\tan(\theta_R + \theta_{R_0}) + \left(\frac{x}{l_B}\frac{l_B}{l_R} - \frac{T_3}{W}\frac{r_3}{l_B}\frac{l_B}{l_R}\right)\frac{1}{\cos(\theta_R + \theta_{R_0})}$$

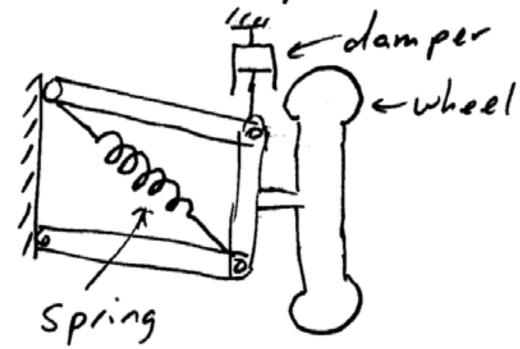
$$\frac{N_1}{W} = \frac{N_B}{W}\left(1 - \frac{a_B}{l_B} - \frac{h_B}{l_B}\tan\theta_B\right) - \left(\frac{T_1}{W}\frac{r_1}{l_B} + \frac{T_2}{W}\frac{r_2}{l_B}\right)\frac{1}{\cos\theta_B}$$

$$\frac{N_2}{W} = \frac{N_B}{W}\left(\frac{a_B}{l_B} + \frac{h_B}{l_B}\tan\theta_B\right) + \left(\frac{T_1}{W}\frac{r_1}{l_B} + \frac{T_2}{W}\frac{r_2}{l_B}\right)\frac{1}{\cos\theta_B}$$

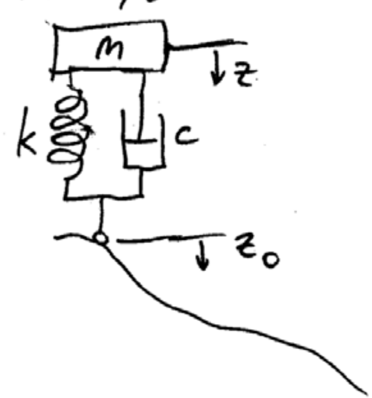
Suspension Systems

- Current planetary rovers (e.g., MER, MSL) have little or no shock absorption

- Notional car suspension



Analyze one wheel first



$$m\ddot{z} + c\dot{z} + kz = c\dot{z}_0 + kz_0$$

Undamped force-free solution

$$m\ddot{z} + kz = 0 \qquad z = Z_1 \cos \omega_n t$$

$$-m\omega_n^2 + k = 0 \qquad \ddot{z} = -Z_1 \omega_n^2 \cos \omega_n t$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

Rover Example

$M_{TOT} = 500\,kg \Rightarrow$ wheel $m = \frac{M_{TOT}}{4} = 125\,kg$

d = deflection of suspension (at rest) $\sim 0.1m$

$$k = \frac{F}{d} = \frac{mg}{d}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$f_n = \frac{\omega_n}{2\pi}$$

	Earth	Moon
k	12,250 N/m	2000 N/m
ω_n	9.9 rad/sec	4 rad/sec
f_n	1.6 Hz	0.64 Hz

ℓ_{crit} = critical distance between bumps

$$= \frac{V}{f_n}$$

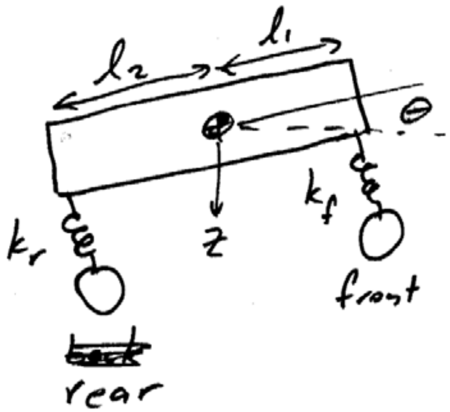
@10kph (2.8 m/sec)

	Earth	Moon
ℓ_{crit}	1.8m	4.3m

Multiwheel Analysis

Responses to two-wheels hitting a bump

We can excite both of these modes



Equation of Motion
(assuming no damping)

Bounce: $m\ddot{z} + k_f(z - l_1\theta) + k_r(z + l_2\theta) = 0$

(small angles)

Pitch: $I_y\ddot{\theta} + k_f l_1(z - l_1\theta) + k_r l_2(z + l_2\theta) = 0$

let $I_y = m r_y^2$ $\quad r_y$ = radius of gyration

Solve this set of coupled differential eqns

let $D_1 \equiv \dfrac{k_f + k_r}{m}$ $\quad D_2 = \dfrac{k_r l_2 - k_f l_1}{m}$ $\quad D_3 \equiv \dfrac{k_f l_1^2 + k_r l_2^2}{I_y}$

Rewrite in terms of D_1, D_2, D_3

$$\ddot{z} + D_1 z + D_2 \theta = 0$$

$$\ddot{\theta} + D_3 \theta + \frac{D_2}{r_y^2} z = 0$$

D_2 = coupling coefficient

Equations are independent if $D_2 = 0 \Rightarrow k_f l_1 = k_r l_2$

If $D_2 = 0$, force @ CG only produces bounce $\omega_{n_z} = \sqrt{D_1}$

force elsewhere produces pitch $\omega_{n_\theta} = \sqrt{D_3}$

Assume $D_2 \neq 0$

$z = Z_1 \cos \omega_n t \qquad \theta = \Theta \cos \omega_n t$

$$\left.\begin{aligned} (D_1 - \omega_n^2) Z_1 + D_2 \Theta &= 0 \\ \frac{D_2}{r_y^2} Z_1 + (D_3 - \omega_n^2)\Theta &= 0 \end{aligned}\right\} \quad \begin{vmatrix} D_1 - \omega_n^2 & D_2 \\ \frac{D_2}{r_y^2} & D_3 - \omega_n^2 \end{vmatrix} = 0$$

$$\omega_n^4 - (D_1 + D_3)\,\omega_n^2 + \left(D_1 D_3 - \frac{D_2^2}{r_y^2}\right)$$

$$\omega_n^4 - (D_1 + D_3)\,\omega_n^2 + \left(D_1 D_3 - \frac{D_2^2}{r_y^2}\right) = 0$$

$$\omega_{n_1}^2 = \frac{D_1 + D_3}{2} \pm \frac{1}{2}\sqrt{(D_1 + D_3)^2 - 4\left(D_1 D_3 - \frac{D_2^2}{r_y^2}\right)}$$

$$\omega_n^2 = \frac{D_1 + D_3}{2} \pm \frac{1}{2}\sqrt{(D_1 + D_3)^2 - 4\left(D_1 D_3 - \frac{D_2^2}{r_\gamma^2}\right)}$$

$\vdots$ (Insert algebra here)

$$\omega_{n_1}^2 = \frac{D_1 + D_3}{2} + \sqrt{\frac{1}{4}(D_1 - D_3)^2 + \frac{D_2^2}{r_\gamma^2}}$$

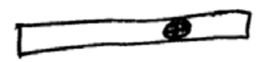
$$\omega_{n_1}^2 = \frac{D_1 + D_3}{2} - \sqrt{\frac{1}{4}(D_1 - D_3)^2 + \frac{D_2^2}{r_y^2}}$$

$$\omega_{n_2}^2 = \frac{D_1 + D_3}{2} - \sqrt{\frac{1}{4}(D_1 - D_3)^2 + \frac{D_2^2}{r_\gamma^2}}$$

$$\omega_{n_2}^2 = \frac{D_1 + D_3}{2} + \sqrt{\frac{1}{4}(D_1 - D_3)^2 + \frac{D_2^2}{r_y^2}}$$

Example: $k_f = k_r = 2000\ \text{N/m}$ (moon)

$l_1 = 1\,m \quad l_2 = 2\,m$



$I = \frac{ml^2}{12} \Rightarrow I_y = 375\ kg\,m^2$

$r_y = 0.75\,m$

$$D_1 = \frac{4000}{500} = 8\ \frac{N/m}{kg} = \left\langle \frac{1}{sec^2} \right\rangle$$

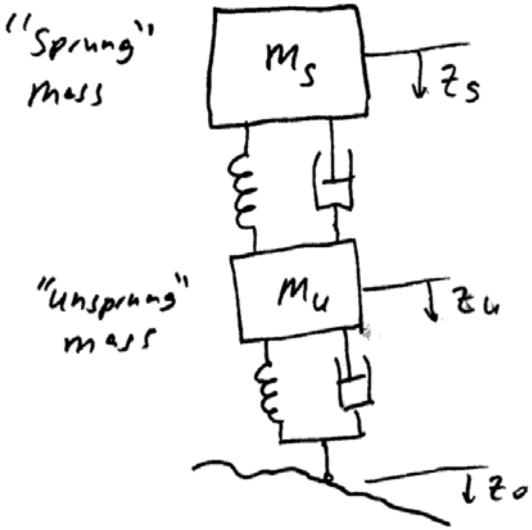
$$D_2 = \frac{2000\,\frac{N}{m}(2m) - 2000\,\frac{N}{m}(1m)}{500\,kg} = 4\ \frac{N}{kg} = 4\ m/sec^2$$

$$D_3 = \frac{2000\,\frac{N}{m}(1m)^2 + 2000\,\frac{N}{m}(2m)^2}{375\ kg\,m^2} = 26.7\ \frac{\frac{N}{m}\,m^2}{kg\,m^2} = \left\langle \frac{1}{sec^2} \right\rangle$$

$$\omega_n^2 = 17.33 \pm 10.43$$

$$\boxed{\begin{aligned} \omega_{n_1} &= 2.63\ \text{rad/sec} \Rightarrow 0.42\ \text{Hz} \\ \omega_{n_2} &= 5.67\ \text{rad/sec} \Rightarrow 0.84\ \text{Hz} \end{aligned}}$$

Add in Tire Mass & Stiffness



Sprung Mass

$$m_s \ddot{z}_s + C_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = 0$$

Unsprung Mass

$$m_u \ddot{z}_u + C_s(\dot{z}_u - \dot{z}_s) + k_s(z_u - z_s)$$
$$+ C_u \dot{z}_u + k_u z_u = F(t) = C_u \dot{z}_o + k_u z_o$$

Undamped Force-Free Solutions

$$m_s \ddot{z}_s + k_s(z_s - z_u) = 0$$

$$m_u \ddot{z}_u + k_s(z_u - z_s) + k_u z_u = 0$$

$$z_s = Z_s \cos \omega_n t \qquad z_u = Z_u \cos \omega_n t$$

$$\begin{vmatrix} k_s - m_s \omega_n^2 & -k_u \\ -k_s & k_s + k_u - m_u \omega_n^2 \end{vmatrix} = 0$$

$$\omega_n^4 (m_u m_s) + \omega_n^2 (-m_s k_s - m_s k_u - m_u k_s) + k_s k_u = 0$$

$$\omega_{n_1} = \frac{+B - \sqrt{B^2 - 4AC}}{2A} \qquad \omega_{n_2} = \frac{B + \sqrt{B^2 - 4AC}}{2A}$$

$$A = m_u m_s \qquad B = m_s (k_s + k_u) + m_u k_s \qquad C = k_s k_u$$

Example: $m_s = 100\ kg$ $\quad m_u = 25\ kg$

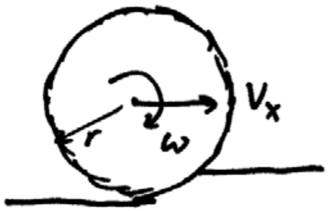
$k_s = 2000\ N/m$ $\quad k_u = 10{,}000\ N/m$

$A = 2500\ kg^2$ $\quad B = 1.25 \times 10^6\ kg^2/sec^2$ $\quad C = 2 \times 10^7\ N^2/m^2$

$\omega_{n_1} = 4.76\ \frac{rad}{sec} \Rightarrow 0.8\ Hz \Leftarrow$ suspension frequency

$\omega_{n_2} = 21.8\ \frac{rad}{sec} \Rightarrow 3.5\ Hz \Leftarrow$ wheel ~~response~~ stiffness frequency

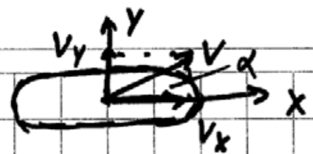
Wheel- Soil Interaction in Turn



Slip ratios

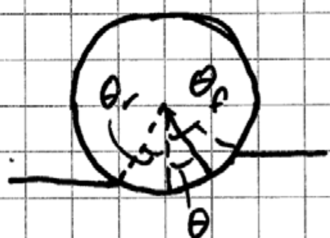
$$s = \frac{r\omega - V_x}{r\omega} \quad \text{for driving: } |r\omega| > |V_x|$$

$$s = \frac{r\omega - V_x}{V_x} \quad \text{for braking: } |r\omega| < |V_x|$$



$$-1 \leq s \leq 1 \qquad \text{Slip angle } \alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$

Wheel sinkage $\quad P(z) = \left(\frac{k_c}{b} + k_\phi\right) z^n$



$$z(\theta) = r(\cos\theta - \cos\theta_r)$$

for static sinkage, $\theta_f = \theta_r = \theta_s$

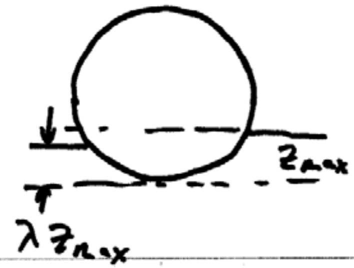
$$z(\theta) = r(\cos\theta - \cos\theta_s)$$

$$P(\theta) = \left(\frac{k_c}{b} + k_\phi\right) r^n (\cos\theta - \cos\theta_s)^n$$

Given weight W on wheel,

$$W = \int_{-\theta_s}^{\theta_s} P(\theta)\, b\, r \cos\theta\, d\theta = r^{n+1}(k_c + k_\phi b)\int_{-\theta_s}^{\theta_s} (\cos\theta - \cos\theta_s)^n \cos\theta\, d\theta$$

Static sinkage $\quad z_s = r(1 - \cos\theta_s)$



Soil entry angle $\theta_f = \cos^{-1}\left(1 - \frac{z_{max}}{r}\right)$

exit angle $\theta_r = \cos^{-1}\left(1 - \lambda \frac{z_{max}}{r}\right)$

λ = wheel sinkage ratio

= 0 when no soil restitution occurs

= 1 if soil completely rebounds (static sinkage)

can be > 1 if soil is transported to back of wheel

Wheel Stress Distribution

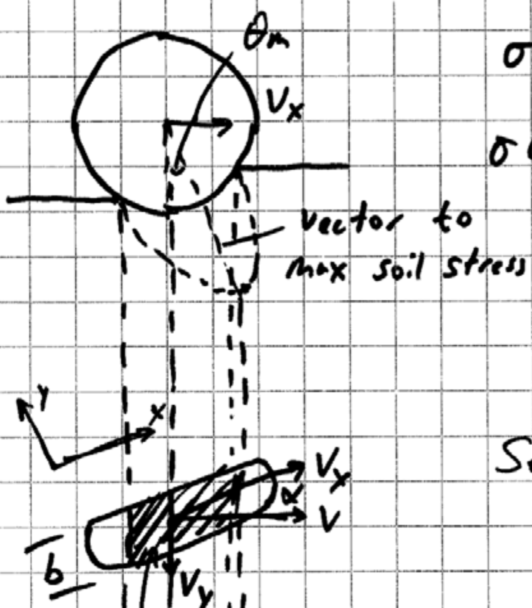
$$\sigma(\theta) = r^n\left(\frac{k_c}{b} + k_\phi\right)\left[\cos\theta - \cos\theta_f\right]^n \qquad \theta_m \le \theta < \theta_f$$

$$\sigma(\theta) = r^n\left(\frac{k_c}{b} + k_\phi\right)\left[\cos\left\{\theta_f - \frac{\theta - \theta_r}{\theta_m - \theta_r}(\theta_f - \theta_m)\right\} - \cos\theta_f\right]^n$$

$$\theta_r < \theta \le \theta_m$$

$$\theta_m = (a_0 + a_1 s)\,\theta_f$$

$a_0 \sim 0.4 \quad 0 \le a_1 \le 0.3$



Wheel-soil contact area

Soil Shear Stresses

$$\tau_x(\theta) = (c + \sigma(\theta)\tan\phi)\left[1 - e^{-j_x(\theta)/k_x}\right]$$

$$\tau_y(\theta) = (c + \sigma(\theta)\tan\phi)\left[1 - e^{-j_y(\theta)/k_y}\right]$$

Soil Deformations

$$j_x(\theta) = r\left[\theta_f - \theta - (1-s)(\sin\theta_f - \sin\theta)\right]$$

$$j_y(\theta) = r(1-s)(\theta_f - \theta)\tan\alpha$$

Resolve into orthogonal forces

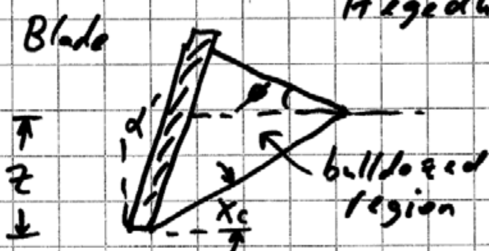
Drawbar pull $\quad F_x = rb\int_{\theta_r}^{\theta_f}\{\tau_x(\theta)\cos\theta - \sigma(\theta)\sin\theta\}\,d\theta$

Side force $\quad F_y = \int_{\theta_r}^{\theta_f}\{\underbrace{r_b\,\tau_y(\theta)}_{F_u} + \underbrace{R_b(r - z(\theta)\cos\theta)}_{F_s}\}\,d\theta$

F_u are side loads generated under the wheel

F_s are side loads generated by bulldozing by side of wheel

Hegedus bulldozing theory



$$R_b(z) = D_1\left[c\,z(\theta) + D_2\frac{\rho\,z^2(\theta)}{2}\right]$$

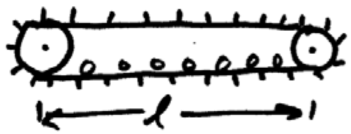
$$D_1(X_c,\phi) = \cot X_c + \tan(X_c + \phi)$$

$$D_2(X_c,\phi) = \cot X_c + \cot^2 X_c / \cot\phi$$

Bekker destructive angle $\quad X_c = \frac{\pi}{4} - \frac{\phi}{2}$

Vertical Force $\quad F_z = rb\int_{\theta_r}^{\theta_f}\{\tau_x(\theta)\sin\theta + \sigma(\theta)\cos\theta\}\,d\theta$

Tracked Vehicles



Bekker soil sinkage equation

$$p = k z_0^n = \left(\frac{k_c}{b} + k_\phi\right) z_0^n \Rightarrow z_0 = \left(\frac{p}{k_c/b + k_\phi}\right)^{1/n}$$

Uniform Pressure $\Rightarrow$ $p = \frac{W}{A} = \frac{W}{b\ell}$ $\quad (A = b\ell)$

$$z_0 = \left[\frac{W/b\ell}{\left(k_c/b + k_\phi\right)}\right]^{1/n}$$

$$\text{Work} = b\ell \int_0^{z_0} p\,dz = b\ell \int_0^{z_0} \left(\frac{k_c}{b} + k_\phi\right) z^n\,dz$$

$$= b\ell \left(\frac{k_c}{b} + k_\phi\right) \frac{z_0^{n+1}}{n+1}$$

Substituting, $\quad \text{Work} = \dfrac{b\ell}{(n+1)\left(\frac{k_c}{b} + k_\phi\right)^{1/n}} \left(\dfrac{W}{b\ell}\right)^{\frac{n+1}{n}} = R_c\,\ell$

$$R_c = \frac{b}{(n+1)\left(\frac{k_c}{b} + k_\phi\right)^{1/n}} \left(\frac{W}{b\ell}\right)^{\frac{n+1}{n}}$$

$$= \frac{1}{(n+1)\, b^{1/n} \left(\frac{k_c}{b} + k_\phi\right)^{1/n}} \left(\frac{W}{\ell}\right)^{\frac{n+1}{n}}$$

$$= \frac{1}{(n+1)\left(k_c + k_\phi b\right)^{1/n}} \left(\frac{W}{\ell}\right)^{\frac{n+1}{n}} \Leftarrow$$

motion resistance due to soil compaction by uniformly loaded tread