Robotic Mobility – Airless Bodies

- Free Space
- Relative Orbital Motion
- Mobility on Airless Major Bodies (moons)
- Landing dynamics (back-of-envelope)

Propulsive Motion in Free Space

- Basic motion governed by Newton's Law

$$F = ma \;\; (\text{Actually, } \overrightarrow{F} = m\ddot{x})$$

- Over a distance d and time t, assuming the motion is predominately coasting,

$$\text{Coast velocity } v = \frac{d}{t} \implies \Delta v = 2\frac{d}{t}$$

(required to accelerate and decelerate)

- The rocket equation (relates propellant to ΔV)

$$\frac{m_{final}}{m_o} = e^{-\frac{\Delta v}{v_{exhaust}}}$$

Cost of Propulsive Maneuvering

- Assuming $$\Delta V \ll V_{exhaust}$$
- Use the Taylor's Series expansion of e

$$\frac{m_{final}}{m_o} \approx 1 - \frac{\Delta v}{v_{exhaust}}$$

- Since $m_o = m_{initial} = m_{prop} + m_{final}$,

$$\frac{m_{\text{prop}}}{m_o} \approx \frac{2}{V_{\text{exhaust}}} \frac{d}{t} \quad \text{or} \quad \frac{m_{\text{prop}}}{m_o} \approx 2\frac{v_{\text{coast}}}{v_{\text{exhaust}}}$$

Thermal Rocket Exhaust Velocity

- Exhaust velocity is

$$V_e = \sqrt{\frac{2\gamma}{\gamma - 1}\frac{\Re T_0}{\bar{M}}\left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\bar{M} \equiv \text{average molecular weight of exhaust gases}$$

where

$$\Re \equiv \text{universal gas constant} = 8.3143\frac{Joules}{mole^{o}K}$$

$$\gamma \equiv \text{ratio of specific heats} \approx 1.2$$

$$p_e = \text{exit plane pressure}$$

$$p_0 \equiv \text{chamber pressure}$$

Typical Exhaust Velocities (in vacuum)

- Pressurized N_2 — 700 m/sec
- Hydrazine monopropellant. — 2300 m/sec
- N_2O_4/N_2H_4 biprop — 2900 m/sec
- LOX/RP-1 — 3200 m/sec
- LOX/LCH_4 — 3800 m/sec
- LOX/LH_2 — 4500 m/sec

Cold-gas Propellant Performance

Propellant	Molecular Mass	Density[a] (lb/ft^3)	Theoretical Specific Impulse (sec)
Hydrogen	2.0	1.21	296
Helium	4.0	2.37	179
Methane	16.0	12.10	114
Nitrogen	28.0	17.37	80
Air	28.9	19.3	74
Argon	39.9	27.60	57
Krypton	83.8	67.20	39
Freon 14	88.0	60.01	55
Carbon dioxide	44.0	Liquid	67

[a] At 3500 psia and 0°C.

From G. P. Sutton, Rocket Propulsion Elements (5th ed.) John Wiley and Sons, 1986

Total Impulse

- Total impulse I_t is the total thrust-time product for the propulsion system, with units <N-sec>

$$I_t = Tt = \dot{m} v_e t$$

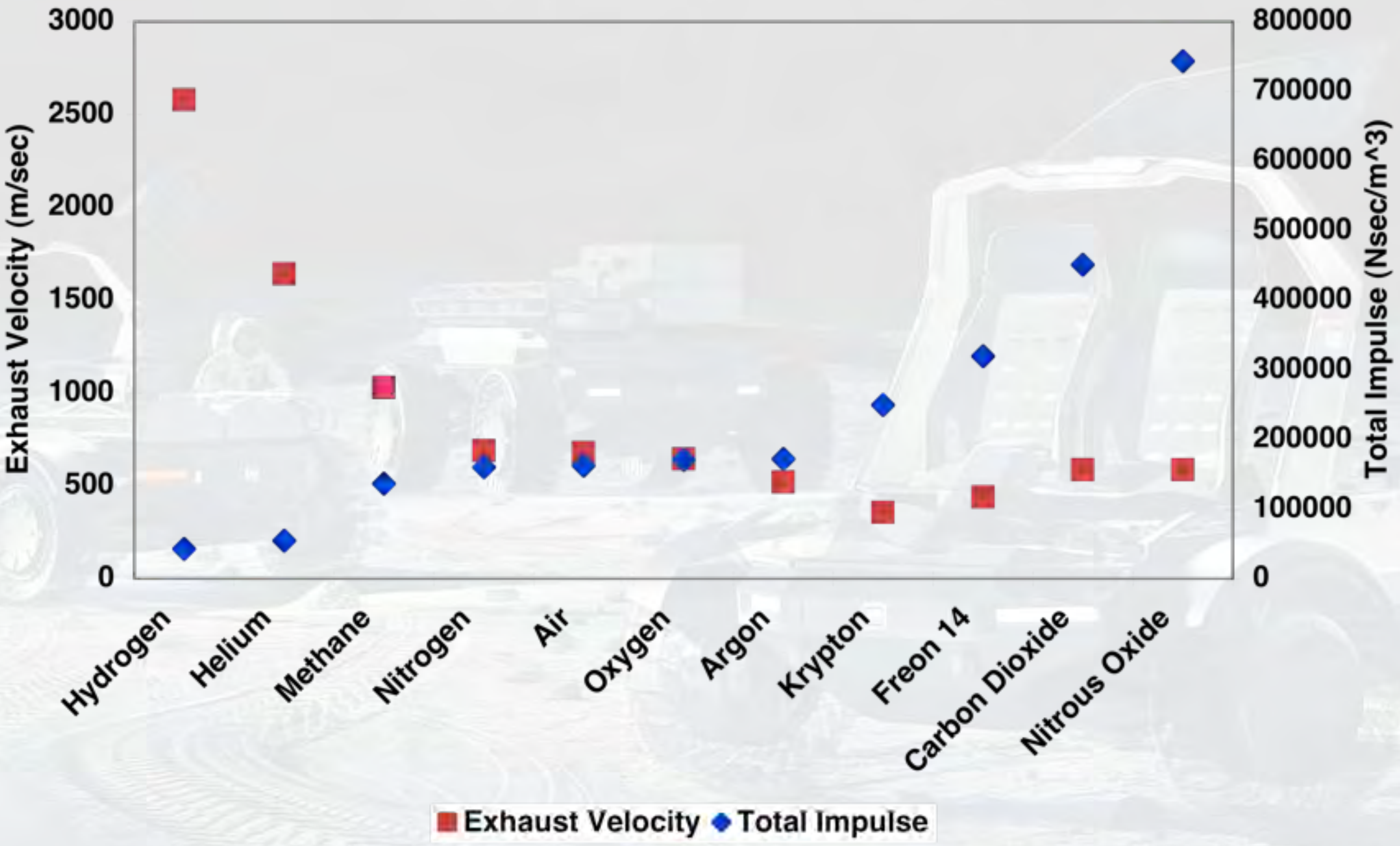
$$t = \frac{\rho V}{\dot{m}}$$

$$I_t = \rho V v_e$$

- To assess cold-gas systems, we can examine total impulse per unit volume of propellant storage

$$\frac{I_t}{V} = \rho v_e$$

Performance of Cold-Gas Systems



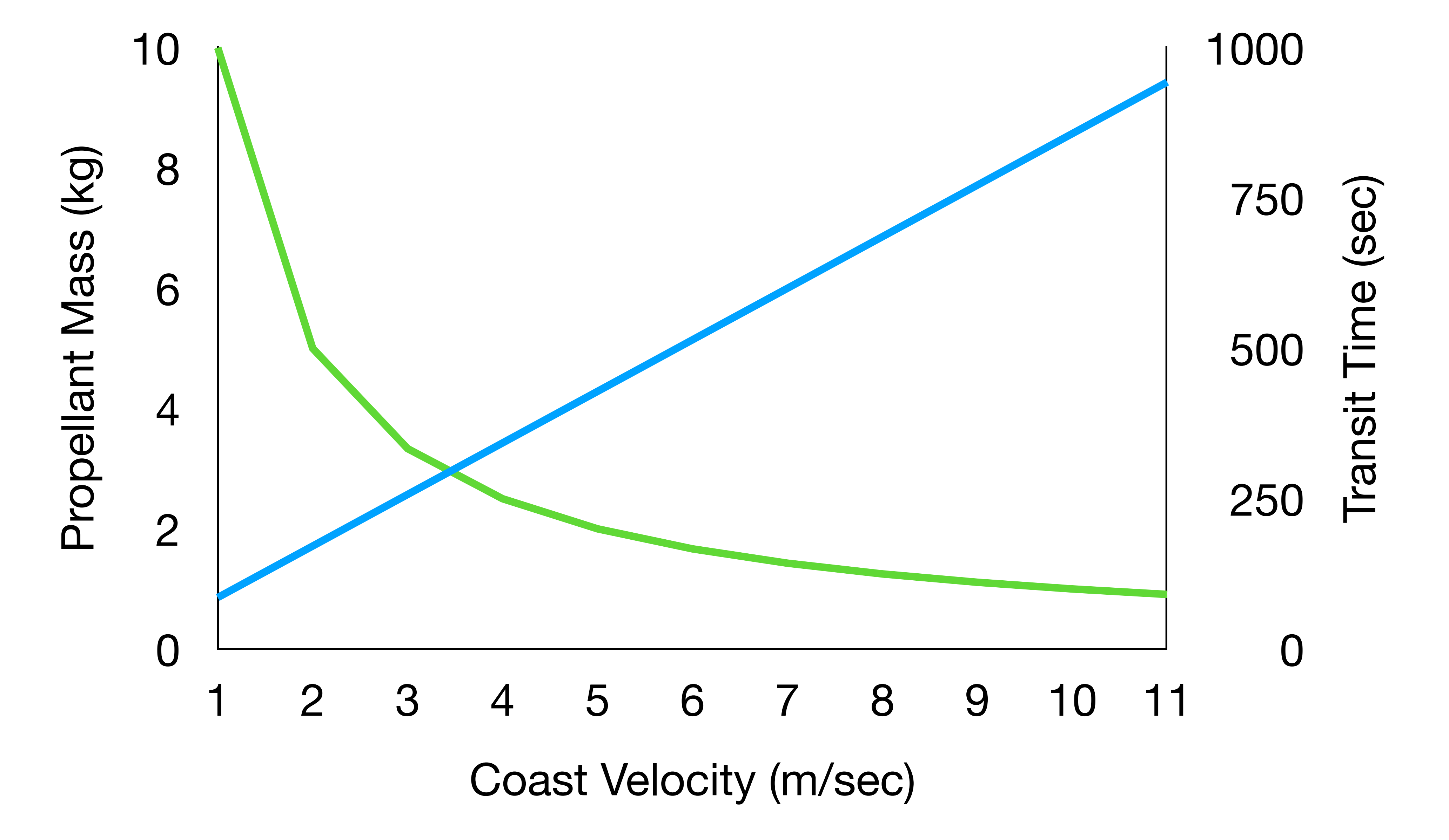
Example – Free Space Approach

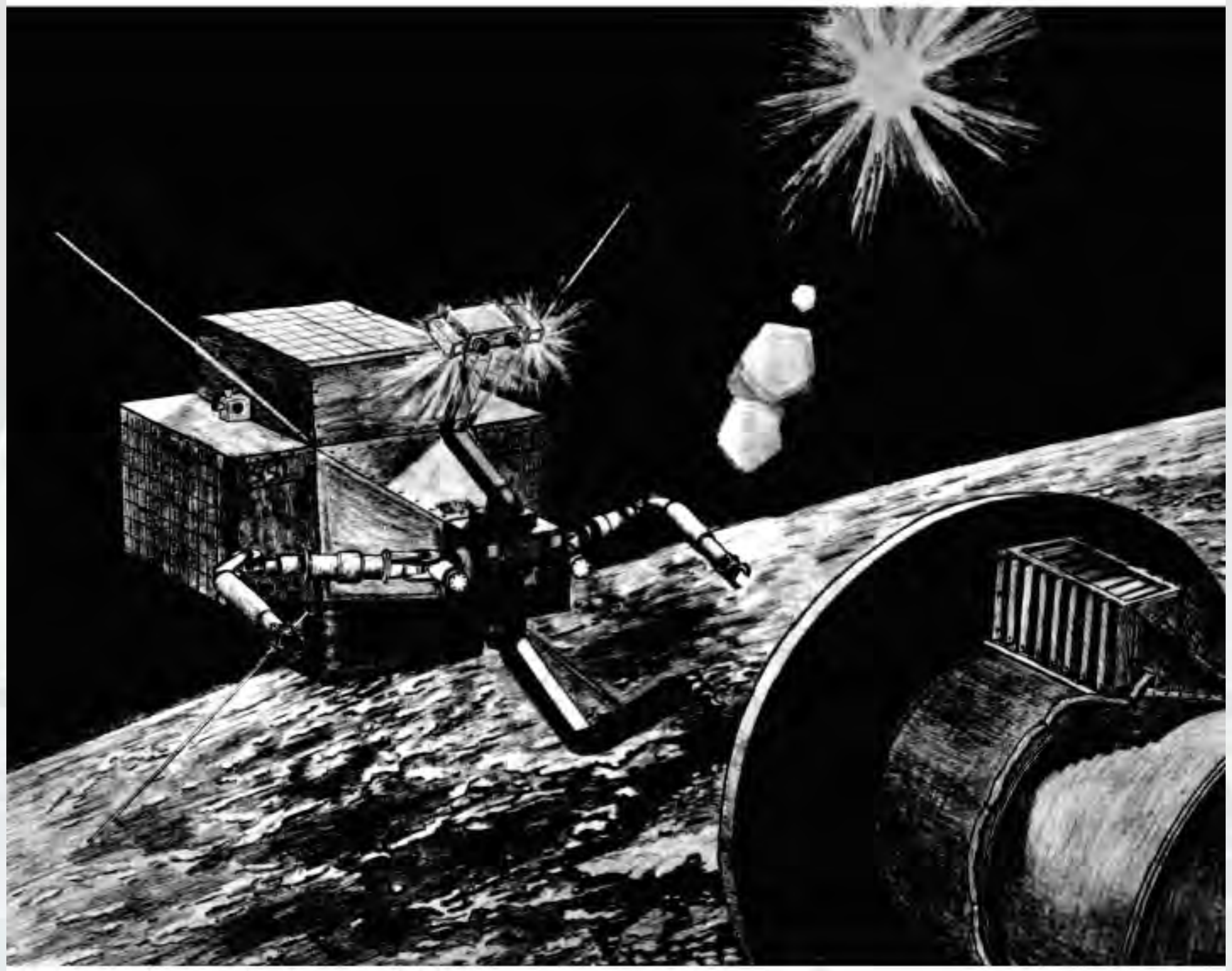
$$\text{Mass} = 300\ kg$$

N_2O cold gas thrusters

$$v_e = 700\ m/sec$$

$$\text{Separation } 1000\ m$$





Ranger Telerobotic Flight Experiment (TFX) Concept

Reaction Control Systems

- Thruster control of vehicle attitude and translation
- "Bang-bang" control algorithms
- Design goals:
  - Minimize coupling (pure forces for translation; pure moments for rotation)except for pure entry vehicles
  - Minimize duty cycle (use propellant as sparingly as possible)
  - Meet requirements for maximum rotational and linear accelerations

Single-Axis Equations of Motion

$$\tau = I\ddot{\theta}$$

$$\frac{\tau}{I}t = \dot{\theta} + C_1$$

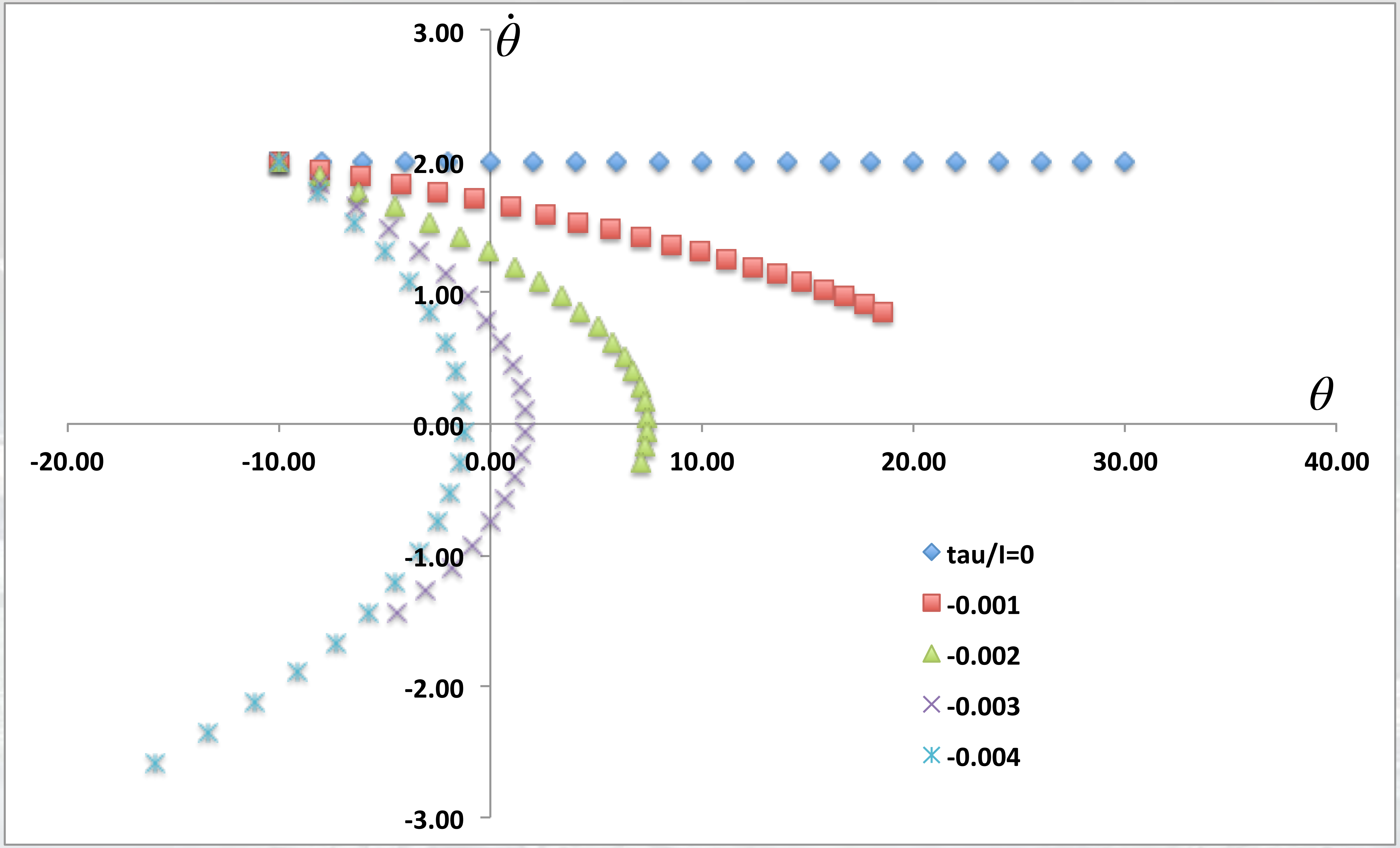
$$\text{at } t = 0,\ \dot{\theta} = \dot{\theta}_o \implies \frac{\tau}{I}t = \dot{\theta} - \dot{\theta}_o$$

$$\frac{1}{2}\frac{\tau}{I}t^2 + \dot{\theta}_o t = \theta + C_2$$

$$\text{at } t = 0,\ \theta = \theta_o \implies \frac{1}{2}\frac{\tau}{I}t^2 + \dot{\theta}_o t = \theta - \theta_o$$

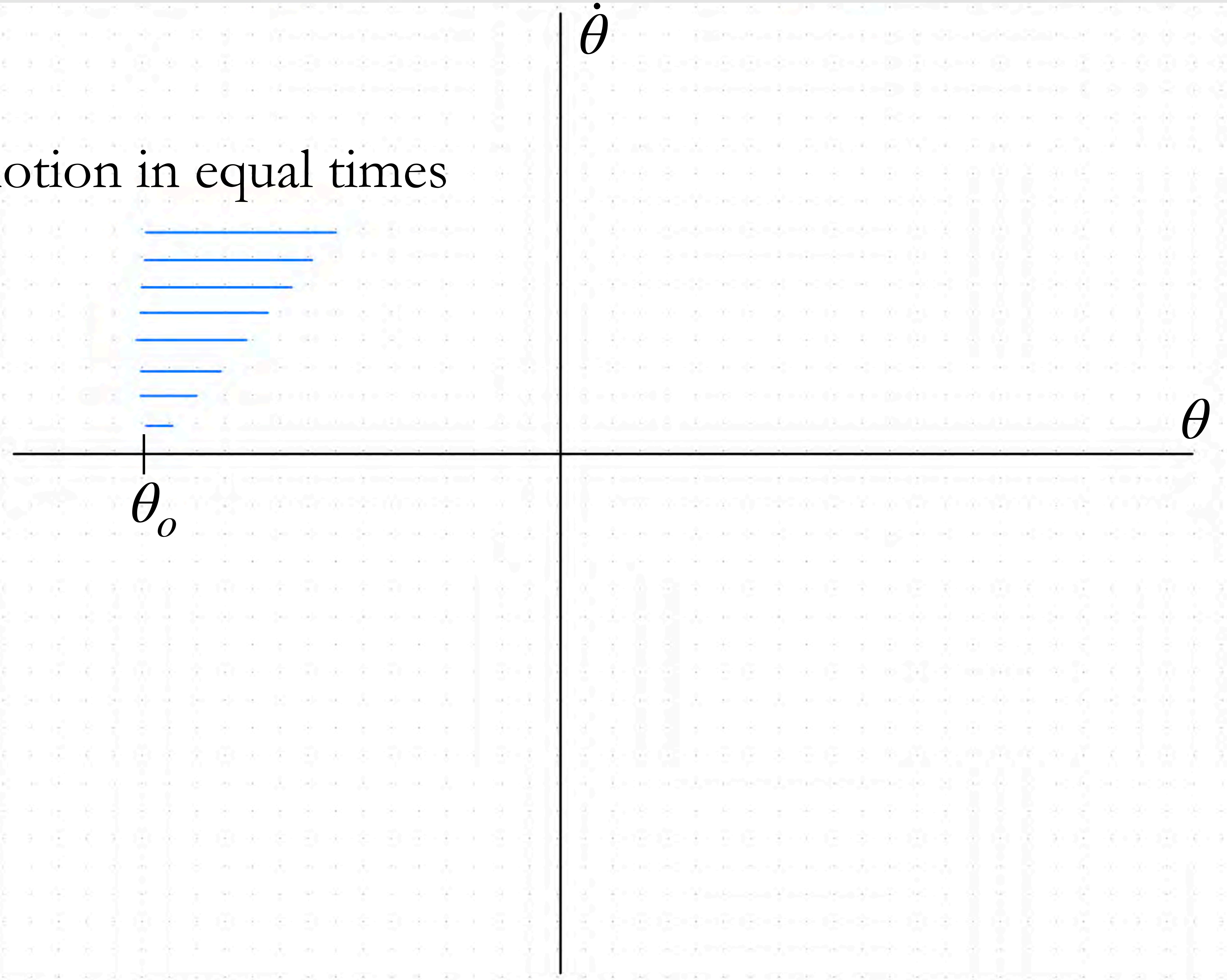
$$\boxed{\frac{1}{2}\left(\dot{\theta}^2 - \dot{\theta}_o{}^2\right) = \frac{\tau}{I}\left(\theta - \theta_o\right)}$$

Attitude Trajectories in the Phase Plane

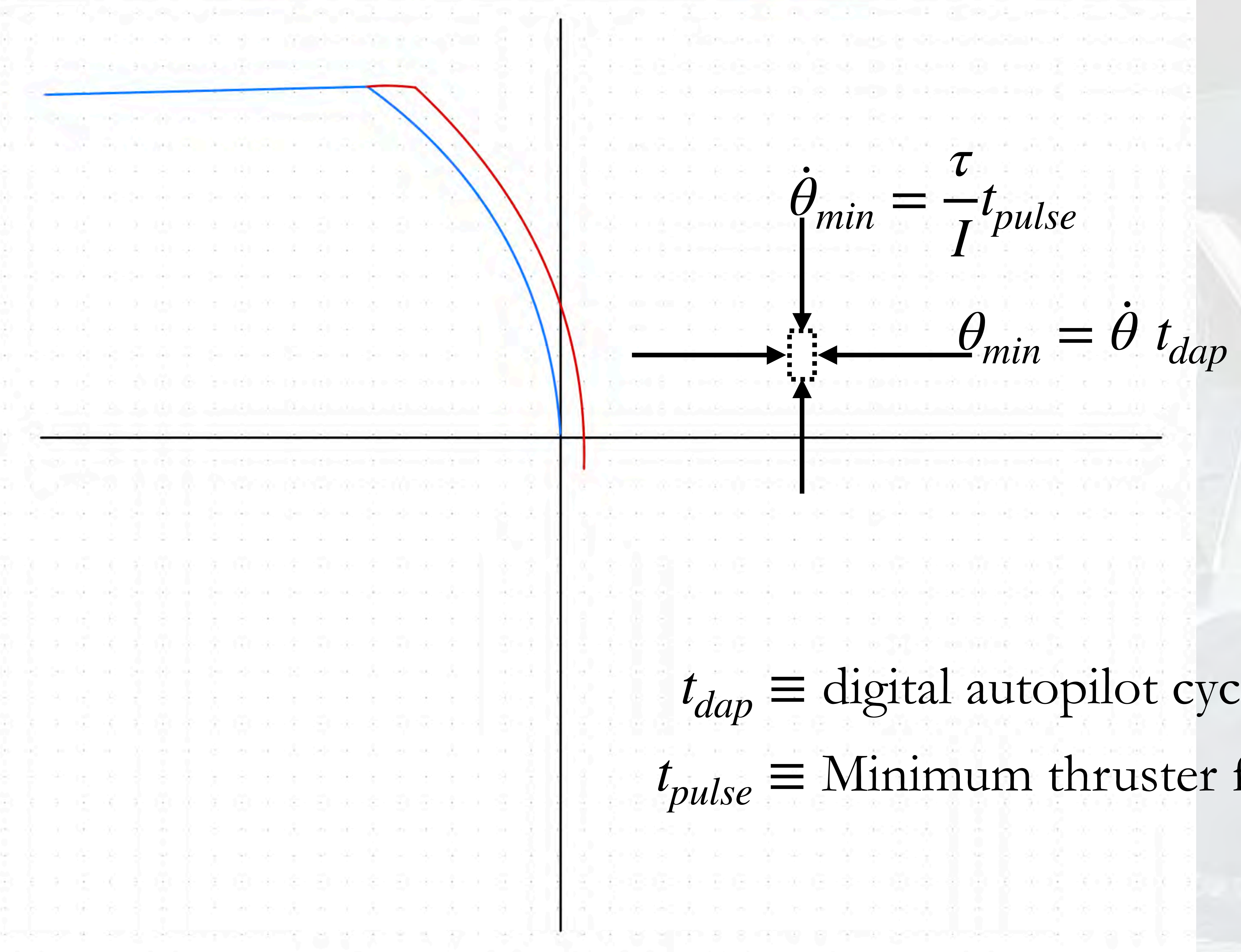


Comparative Motion on Phase Plane

Spacecraft motion in equal times



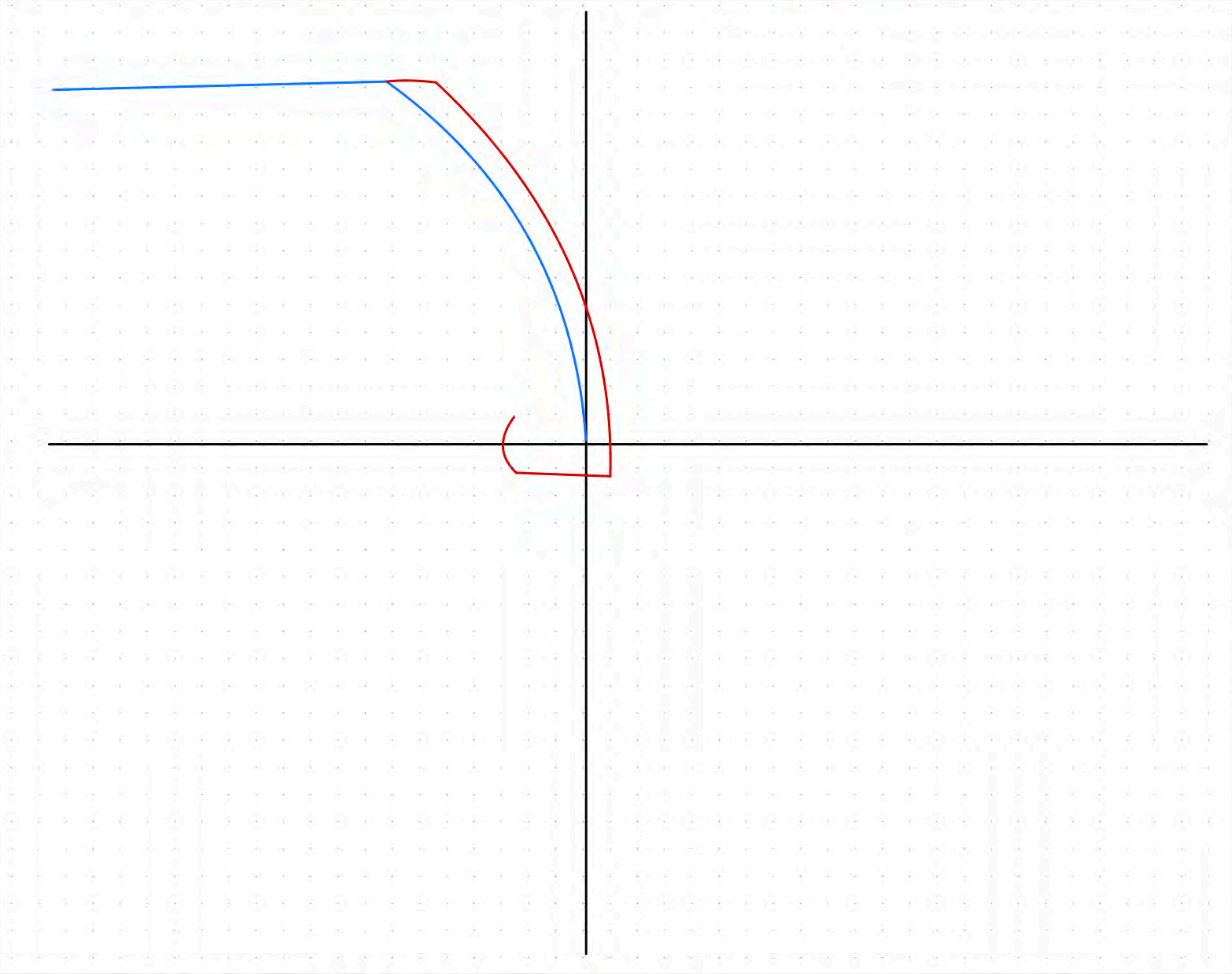
Reality Impacts on Phase Plane



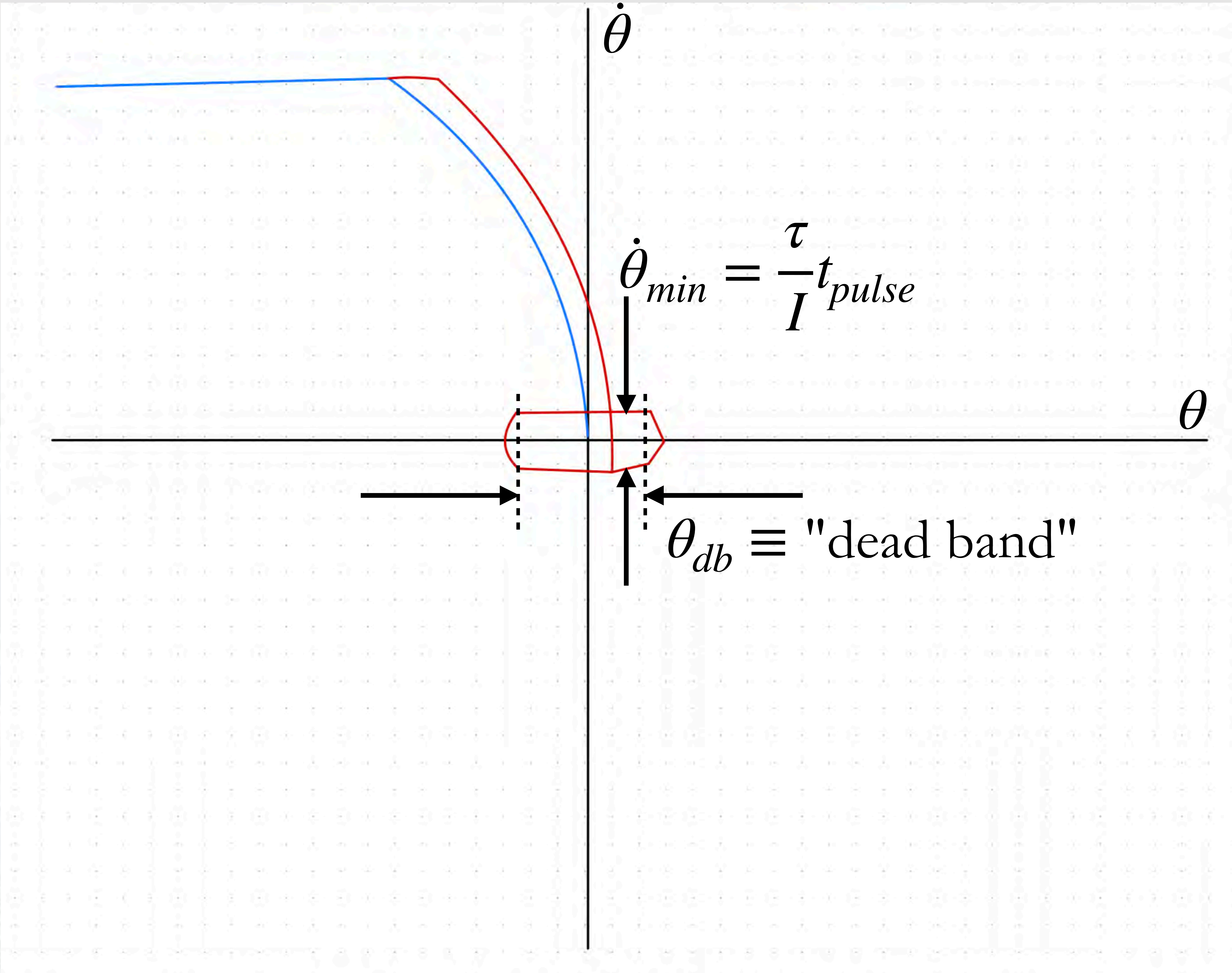
$t_{dap} \equiv$ digital autopilot cycle period

$t_{pulse} \equiv$ Minimum thruster firing time

First Correction to Phase Plane Errors



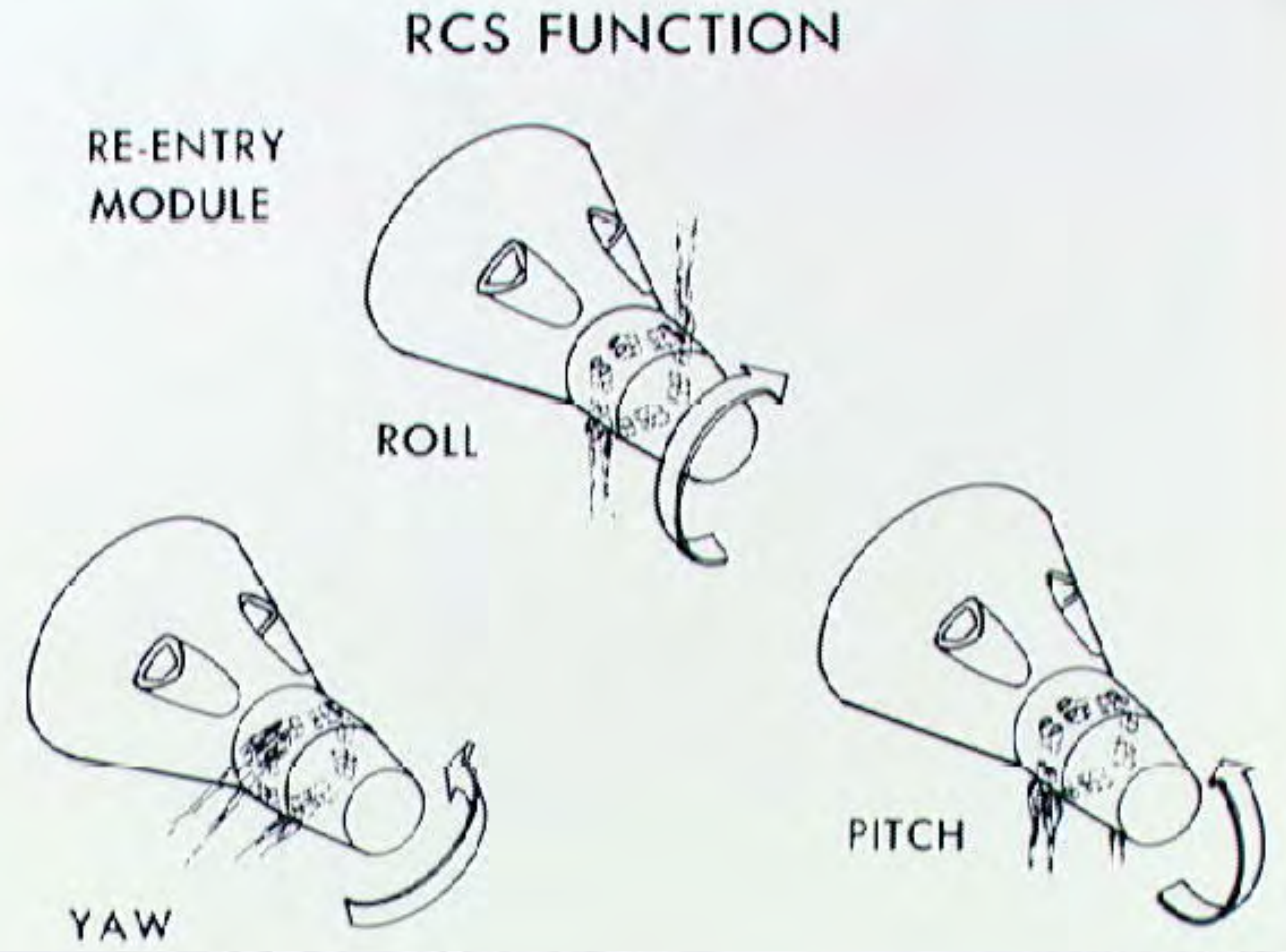
Limit Cycling in Attitude Hold



$$t_{drift} = 2\frac{\theta_{db}}{\dot{\theta}_{min}}$$

$$\text{Duty cycle} = \frac{t_{pulse}}{t_{drift}}$$

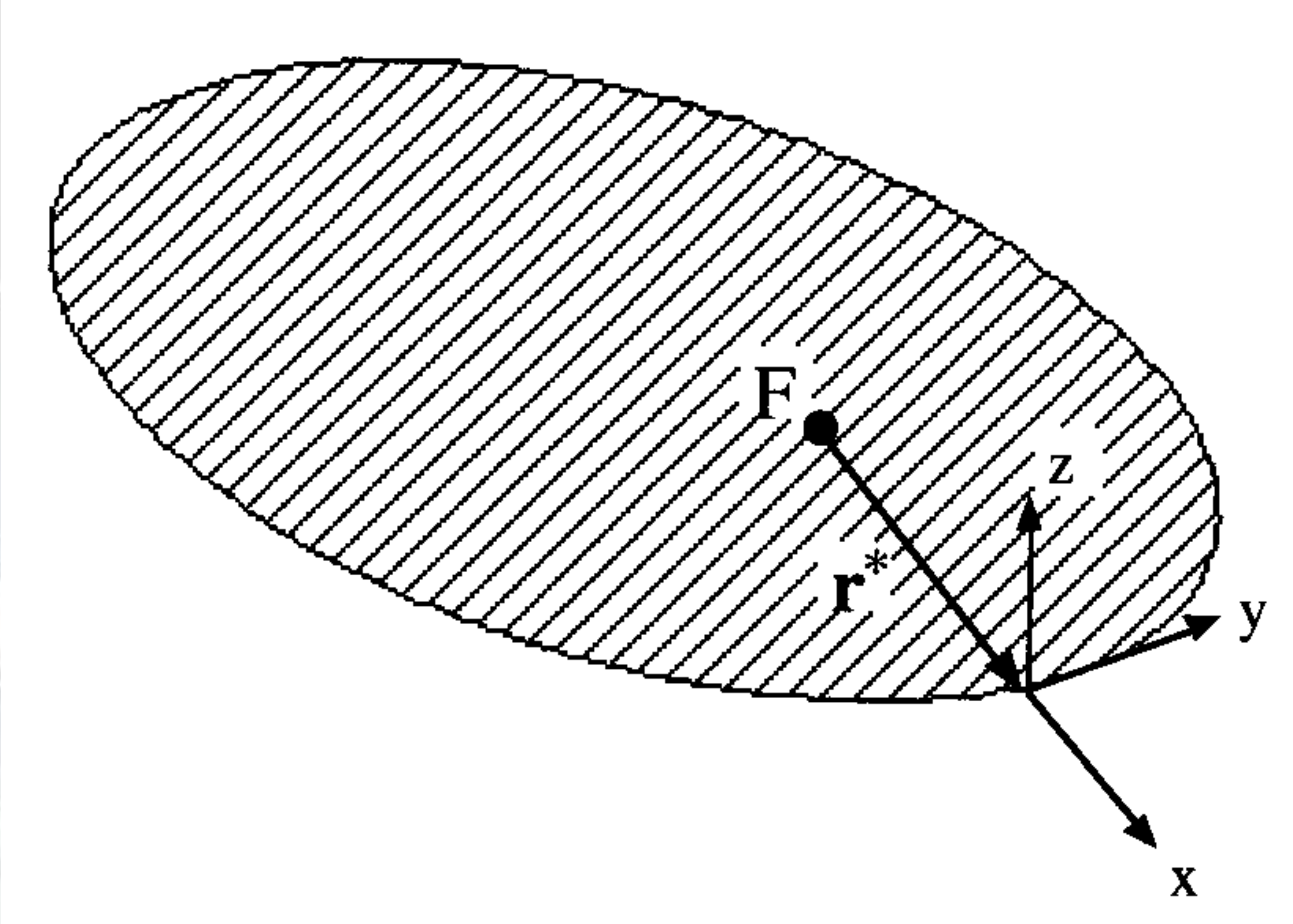
Gemini Entry Reaction Control System



Apollo Reaction Control System Thrusters

Hill's Equations (Proximity Operations)

Linearized equations of motion relative to a target in circular orbit in a rotating Cartesian reference frame



Ref: J. E. Prussing and B. A. Conway, *Orbital Mechanics*
Oxford University Press, 1993

$$\ddot{x} = 3n^2 x + 2n\dot{y} + a_{dx}$$

$$\ddot{y} = -2n\dot{x} + a_{dy}$$

$$\ddot{z} = -n^2 z + a_{dz}$$

$$n = \sqrt{\frac{\mu}{a^3}}$$

a_{dx}, a_{dy}, a_{dz} are disturbing accelerations (e.g., thrust, solar pressure)

Clohessy-Wiltshire ("CW") Equations

Force-free solutions to Hill's Equations

$$x(t) = [4 - 3\cos(nt)]x_o + \frac{\sin(nt)}{n}\dot{x}_o + \frac{2}{n}[1 - \cos(nt)]\dot{y}_o$$

$$y(t) = 6[\sin(nt) - nt]x_o + y_o - \frac{2}{n}[1 - \cos(nt)]\dot{x}_o + \frac{4\sin(nt) - 3nt}{n}\dot{y}_o$$

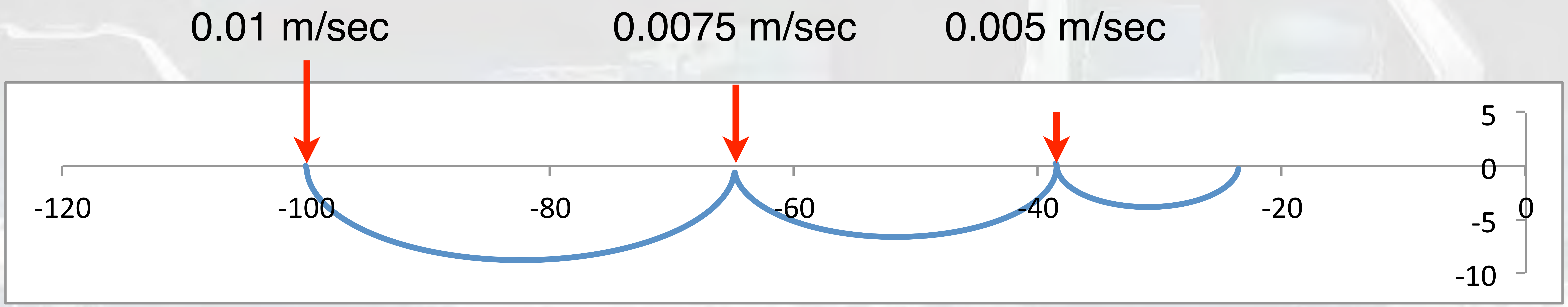
$$\dot{x}(t) = 3n\sin(nt)x_o + cos(nt)\dot{x}_o + 2\sin(nt)\dot{y}_o$$

$$\dot{y}(t) = -6n\left[1 - \cos(nt)\right]x_o - 2sin(nt)\dot{x}_o + \left[4\cos(nt) - 3\right]\dot{y}_o$$

$$z(t) = z_o\cos(nt) + \frac{\dot{z}_o}{n}\sin(nt)$$

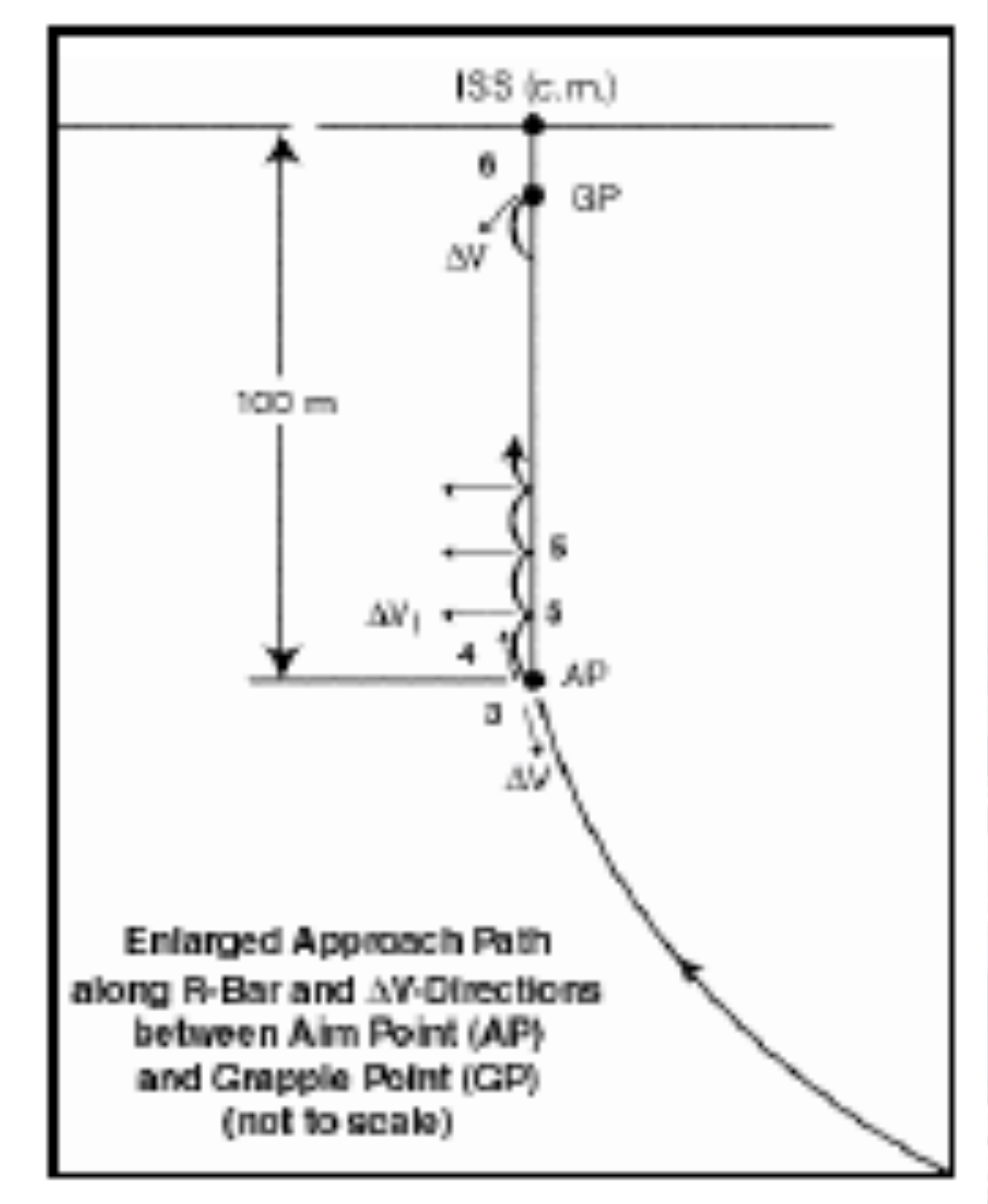
$$\dot{z}(t) = -z_o n\sin(nt) + \dot{z}_o\cos(nt)$$

“V-Bar” Approach



"R-Bar" Approach

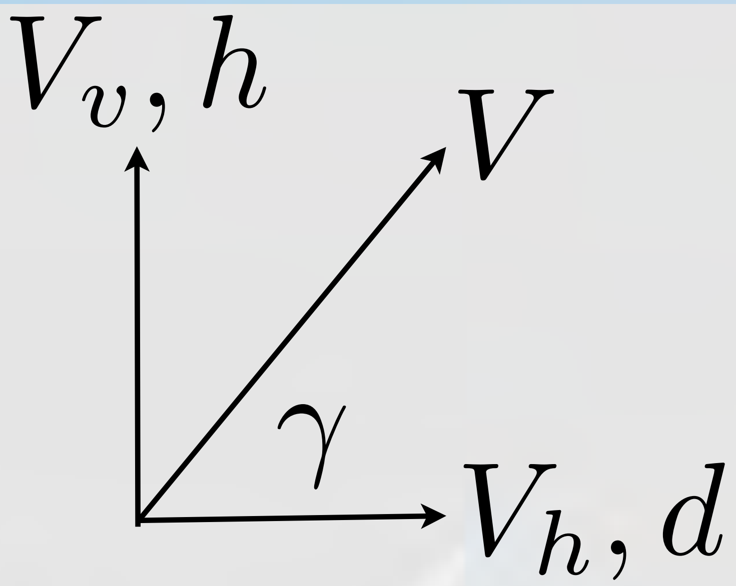
- Approach from along the radius vector ("R-bar")
- Gravity gradients decelerate spacecraft approach velocity - low contamination approach
- Used for Mir, ISS docking approaches



Ref: Collins, Meissinger, and Bell, *Small Orbit Transfer Vehicle (OTV) for On-Orbit Satellite Servicing and Resupply*, 15th USU Small Satellite Conference, 2001

Lunar Flying Vehicle Concept

Ballistic Hopping (Airless Flat Planet)



Use *F*=*ma* for vertical motion

$$\dot{V}_v = -g \qquad h = V_v t - \frac{1}{2} g t^2$$

$$t_{flt} = 2V_v/g$$

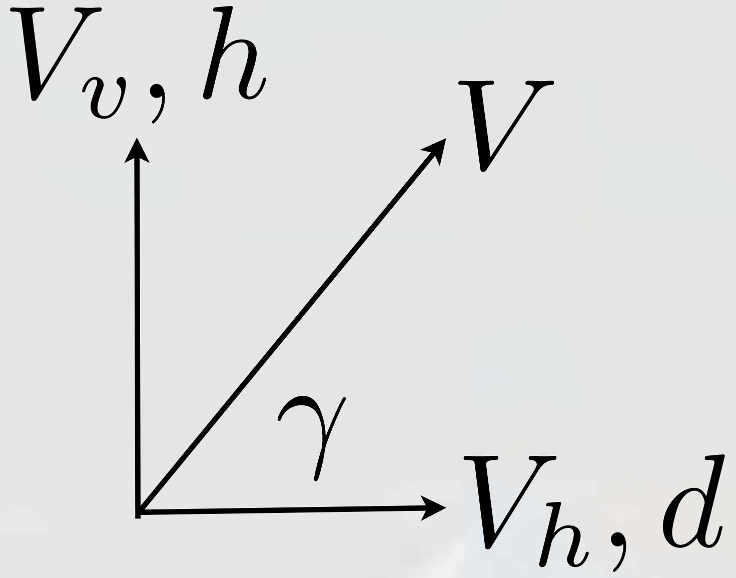
Constant velocity in horizontal direction produces

$$d = V_h t_{flt} = 2\frac{V_h V_v}{g}$$

$$V_h = V\cos\gamma; V_v = V\sin\gamma$$

$$d = 2\frac{V^2 \sin\gamma\cos\gamma}{g} = \frac{V^2}{g}\sin(2\gamma)$$

Ballistic Hopping (Airless Flat Planet)



Horizontal distance is maximized when $\sin(2\gamma) = 1$

$$\gamma_{opt} = \frac{\pi}{2} = 45^o \qquad d_{max} = \frac{V^2}{g}$$

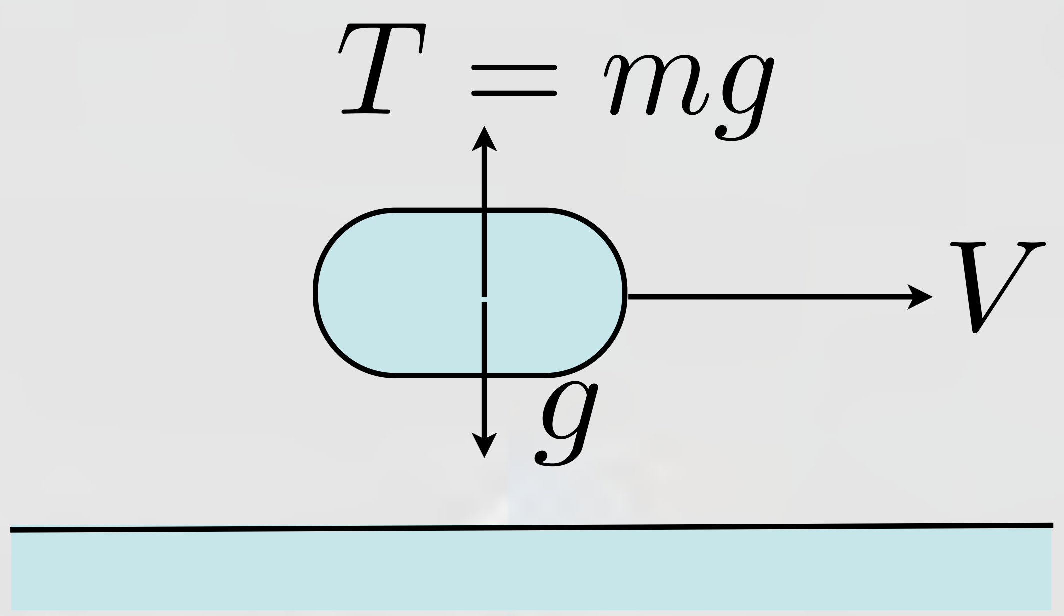
$$V = \sqrt{gd} \qquad \Delta V_{total} = 2V = 2\sqrt{gd}$$

$$h_{max} = V_v \frac{V_v}{g} - \frac{1}{2} g \left(\frac{V_v}{g} \right)^2 \qquad V_v = \frac{V}{\sqrt{2}}$$

$$h_{max} = \frac{V^2}{4g} = \frac{\sqrt{gd}^2}{4g} = \frac{d}{4}$$

An Example of Propulsive Gliding

Propulsive Gliding (Airless Flat Planet)



Assume horizontal velocity is V

$$\Delta V_h = 2V$$

(includes acceleration and deceleration)

$$t_{flt} = d/V$$

$$\Delta V_v = g t_{flt} = \frac{gd}{V}$$

Total ΔV becomes

$$\Delta V_{total} = \Delta V_v + \Delta V_h = 2V + \frac{gd}{V}$$

Propulsive Gliding (Airless Flat Planet)

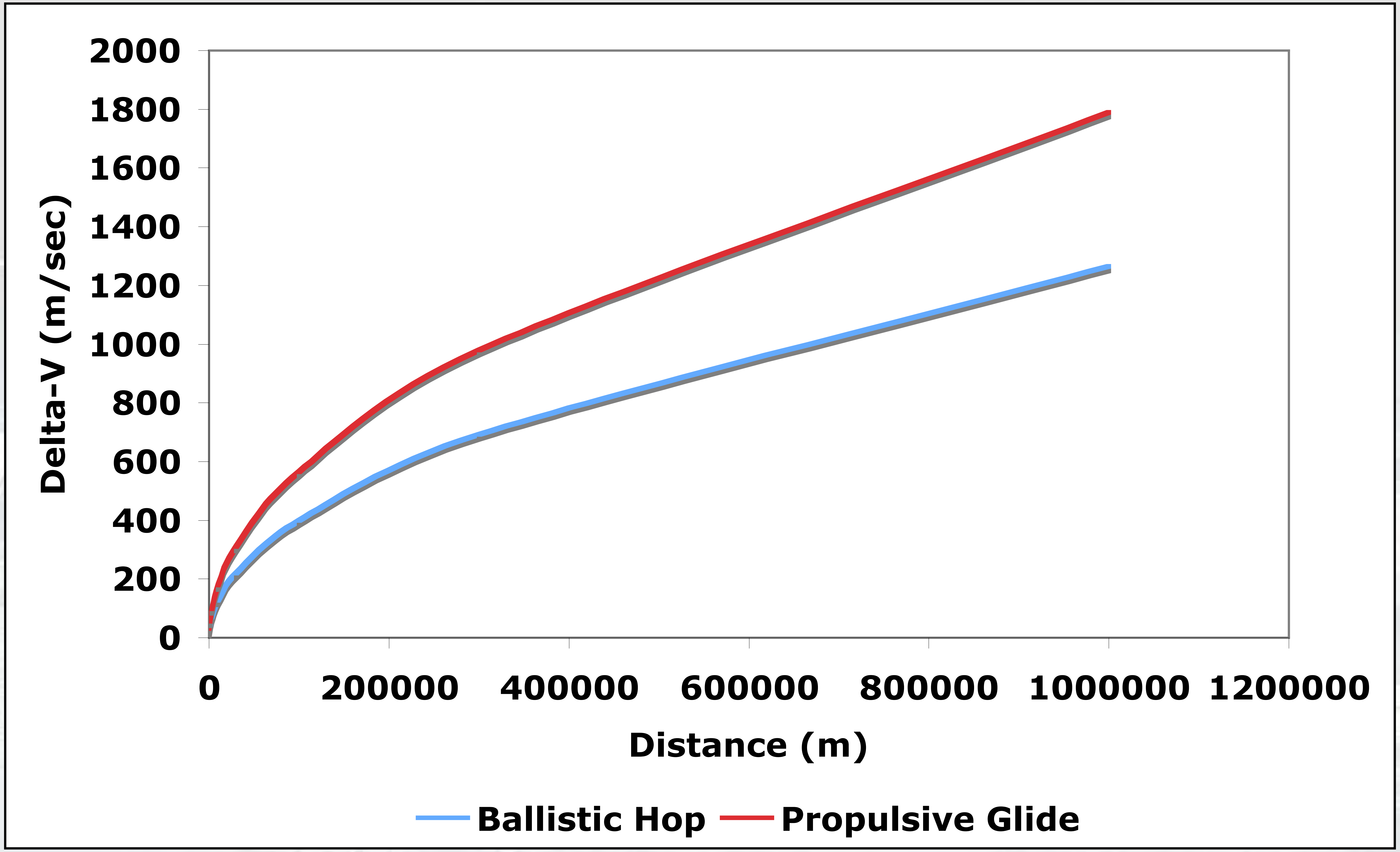
Want to choose V to minimize

$$\frac{\partial}{\partial V}\left(2V + \frac{gd}{V}\right) = 0 \qquad 2 - \frac{gd}{V^2} = 0$$

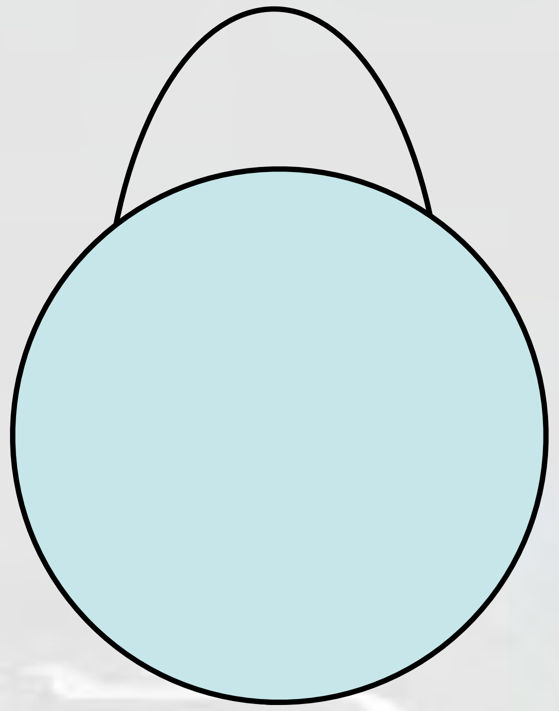
$$V_{opt} = \sqrt{\frac{gd}{2}}$$

$$\Delta V_{total} = 2\sqrt{\frac{gd}{2}} + gd\sqrt{\frac{2}{gd}} = \boxed{2\sqrt{2}\sqrt{gd}}$$

Delta-V for Hopping and Gliding



Ballistic Hopping (Spherical Planet)



$$\Delta v = 2v_o$$

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$r = \frac{p}{1 + e\cos\nu} = \frac{a(1 - e^2)}{1 - e\cos\theta}$$

$$a = r\left(\frac{1 - e\cos\theta}{1 - e^2}\right) \qquad v = \sqrt{\mu\left(\frac{2}{r} - \frac{1 - e^2}{r(1 - e\cos\theta)}\right)}$$

$$\frac{\partial v}{\partial e} = 0 \Rightarrow \frac{-r(1 - e\cos\theta)(-2e) + (1 - e^2)r(-\cos\theta)}{r^2(1 - e\cos\theta)^2} = 0$$

Ballistic Hopping (Spherical Planet)

$$2er - 2e^2 r\cos\theta - r\cos\theta + re^2\cos\theta = 0$$

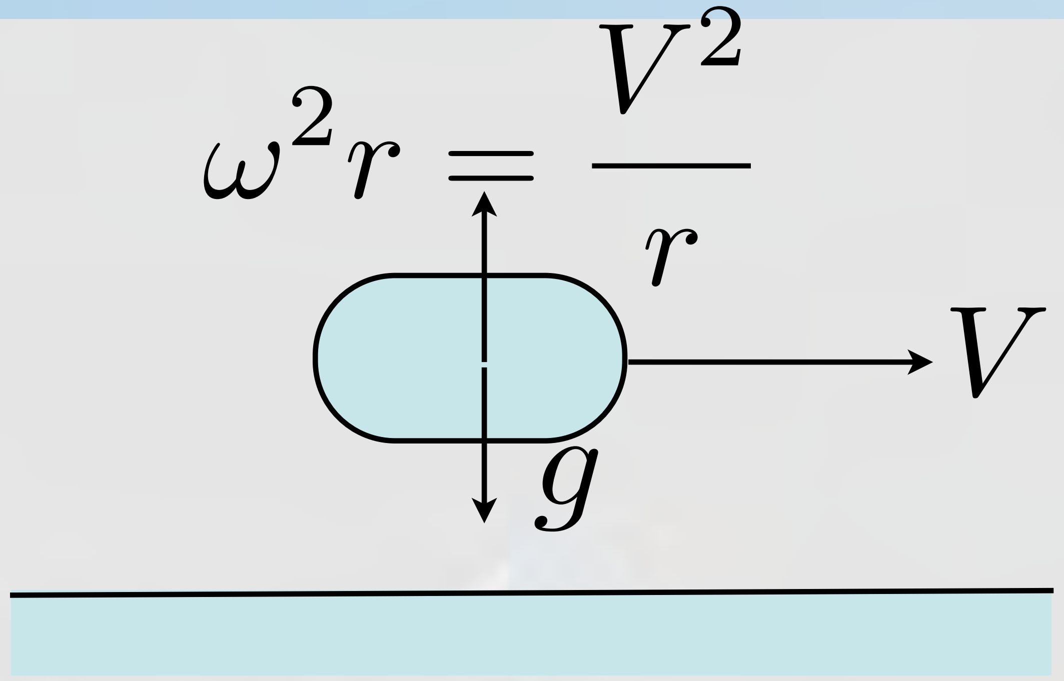
$$\cos\theta e^2 - 2e + \cos\theta = 0$$

$$e_{opt} = \frac{2 \pm \sqrt{2^2 - 4\cos^2\theta}}{2\cos\theta} = \frac{1 \pm \sin\theta}{\cos\theta}$$

$+$ produces $e > 1$ (hyperbolic orbit); $-$ gives elliptical orbit

$$e_{opt} = \frac{1 - \sin\theta}{\cos\theta} \qquad a_{opt} = r\left(\frac{1 - e_{opt}\cos\theta}{1 - e_{opt}^2}\right)$$

Propulsive Gliding (Airless Spherical Planet)



Assume horizontal velocity is V

$$\Delta V_h = 2V$$

(includes acceleration and deceleration)

$$t_{flt} = d/V \qquad \Delta V_v = \left(g - \frac{V^2}{r}\right) t_{flt} = \frac{gd}{V} - \frac{dV}{r}$$

Total ΔV becomes

$$\Delta V_{total} = \Delta V_v + \Delta V_h = 2V + \frac{gd}{V} - \frac{dV}{r}$$

Propulsive Gliding (Airless Spherical Planet)

Want to choose V to minimize

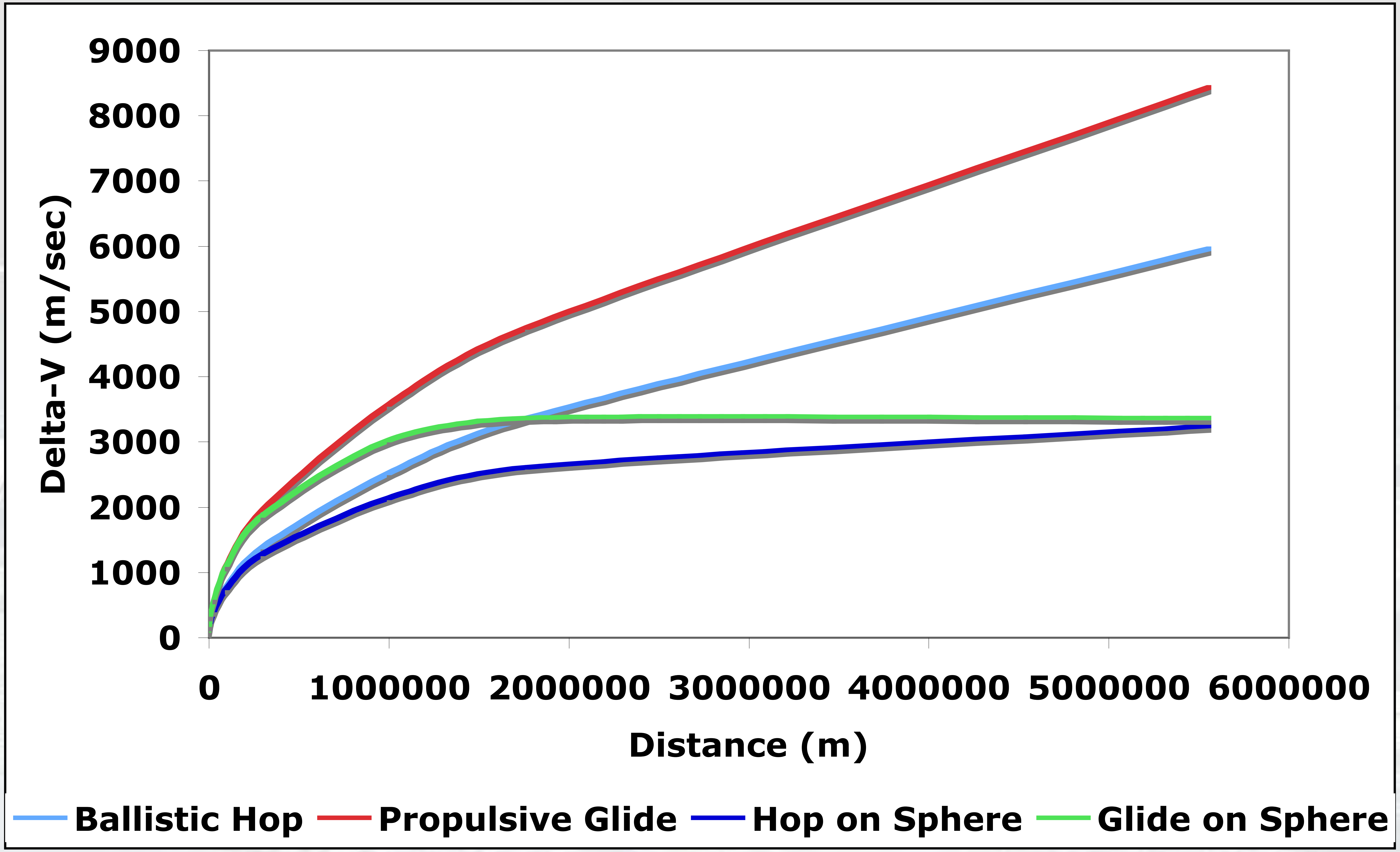
$$\frac{\partial}{\partial V}\left(2V + \frac{gd}{V} - \frac{dV}{r}\right) = 0 \qquad 2 - \frac{gd}{V^2} - \frac{d}{r} = 0$$

$$V_{opt} = \sqrt{\frac{gd}{2 - \frac{d}{r}}}$$

$$\Delta V_{total} = 2\sqrt{\frac{gd}{2 - \frac{d}{r}}} + gd\sqrt{\frac{2 - \frac{d}{r}}{gd}} - \frac{d}{r}\sqrt{\frac{gd}{2 - \frac{d}{r}}}$$

$$\Delta V_{total} = 2\sqrt{2 - \frac{d}{r}}\sqrt{gd}$$

Hopping on Flat and Round Bodies



Nondimensional Forms

$$\text{Define } \nu \equiv \frac{V}{\sqrt{dg}} \qquad \rho \equiv \frac{d}{r} \qquad \eta \equiv \frac{h_{max}}{d}$$

$$\nu_{flat\ glide} = 2\sqrt{2}$$

$$\nu_{flat\ hop} = 2 \qquad \eta = \frac{1}{4}$$

$$\nu_{spherical\ glide} = 2\sqrt{2-\rho} \qquad (0 \leq \rho \leq 1)$$

Multiple Hops

- Assume n hops between origin and destination
- At each intermediate "touchdown", v_v has to be reversed

$$\Delta V_{total} = 2V + 2(n-1)V_v$$

$$t_{peak} = \frac{V_v}{g} \qquad t_{total} = 2nt_{peak} = 2n\frac{V_v}{g}$$

$$d = V_h t_{total} = \frac{2n}{g}V_h V_v \qquad V_v = \sqrt{2gh_{max}} \qquad \nu_v = \sqrt{\frac{2\eta}{n}}$$

$$\nu \equiv \frac{V}{\sqrt{dg}} \qquad \eta \equiv \frac{h_{max}}{d/n} \qquad V_h = \frac{dg}{2nV_v} \qquad \nu_h = \frac{1}{2}\sqrt{\frac{1}{2n\eta}}$$

Multiple Hop Analysis

$$\Delta\nu = 2\nu + 2(n-1)\nu_v$$

$$\Delta\nu = 2\sqrt{\nu_v^2 + \nu_h^2} + 2(n-1)\nu_v$$

$$\Delta\nu = 2\sqrt{\frac{2\eta}{n} + \frac{1}{8n\eta}} + 2(n-1)\sqrt{\frac{2\eta}{n}}$$

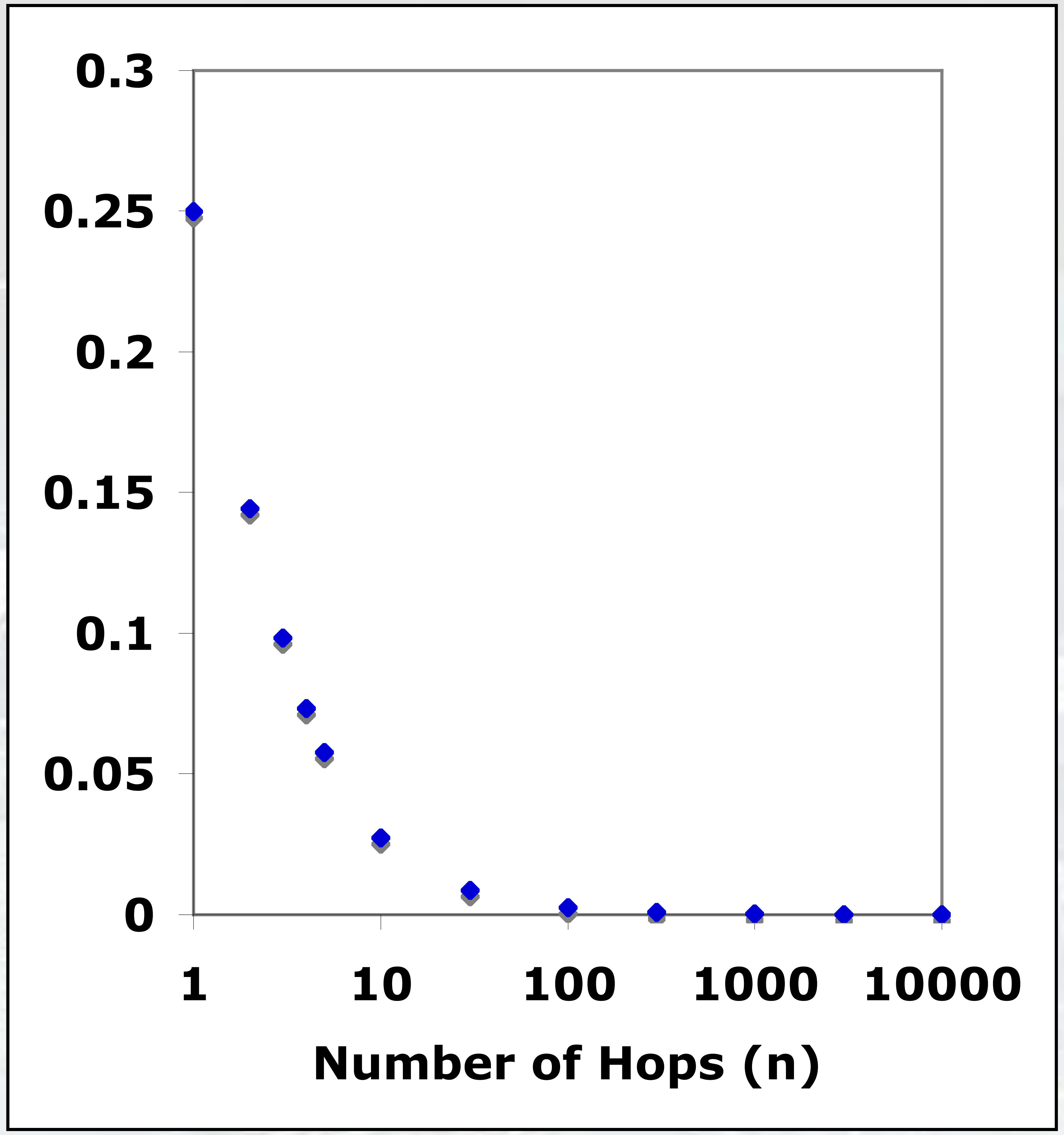
$$\frac{\partial\Delta\nu}{\partial\eta} = \left[\frac{1}{\sqrt{\frac{2\eta}{n} + \frac{1}{8n\eta}}}\left(\frac{2}{n} - \frac{1}{8n\eta^2}\right)\right] + (n-1)\sqrt{\frac{2}{n\eta}} = 0$$

Analytically messy, but note that for $n = 1 \Rightarrow \eta_{opt} = \frac{1}{4}$

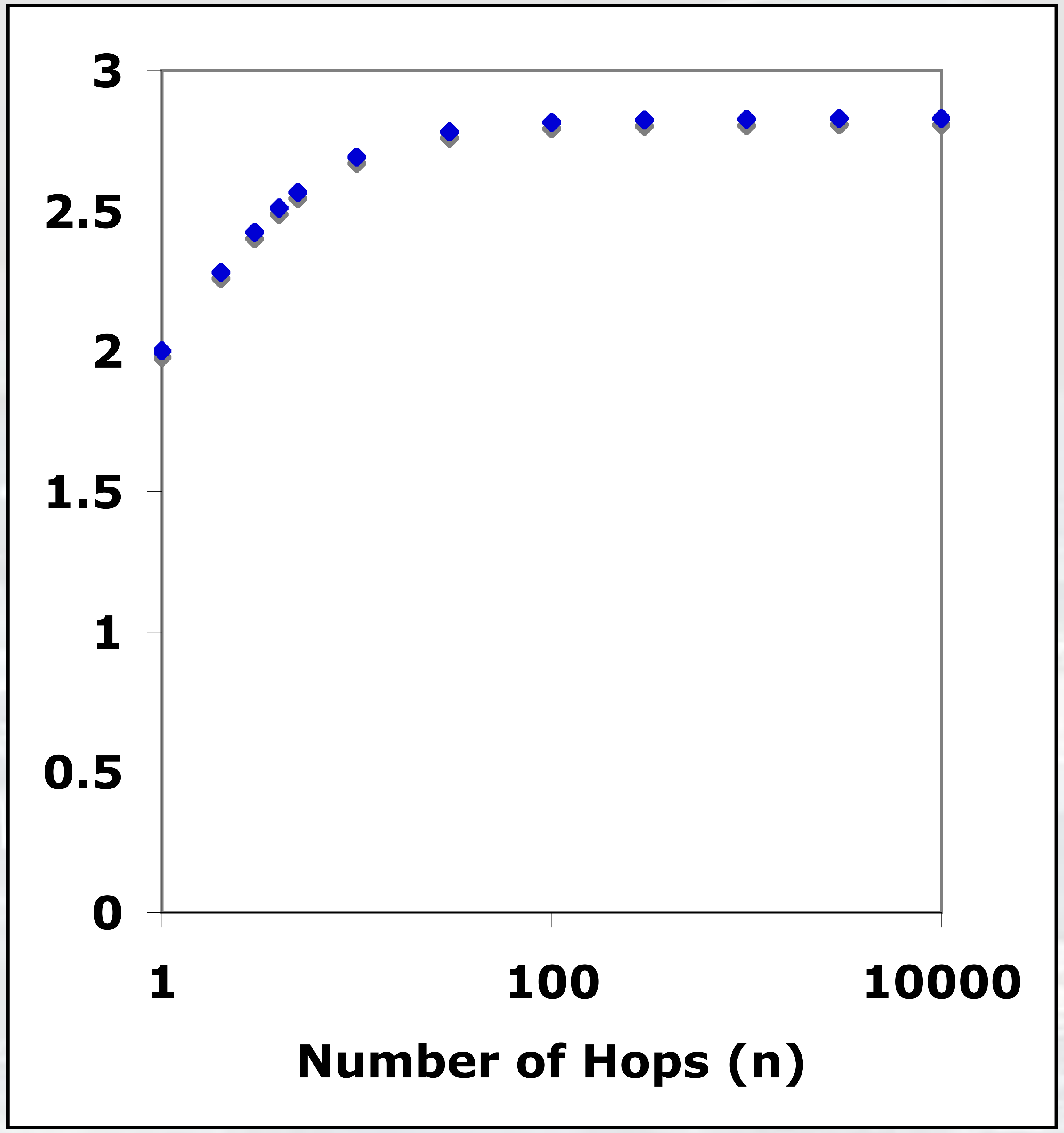
(In general, solve numerically)

Optimal Solutions for Multiple Hops

η_{opt}



$\Delta\nu$



A Few Notes on Hopping/Gliding

Optimum Ballistic Hop

$$v_h = \boxed{\sqrt{\frac{gd}{2}}}$$

$$V_v = V_h \Longrightarrow t_{flt} = \frac{2V_h}{g}$$

$$t_{flt} = \frac{2}{g}\sqrt{\frac{gd}{2}} = \boxed{\sqrt{\frac{2d}{g}}}$$

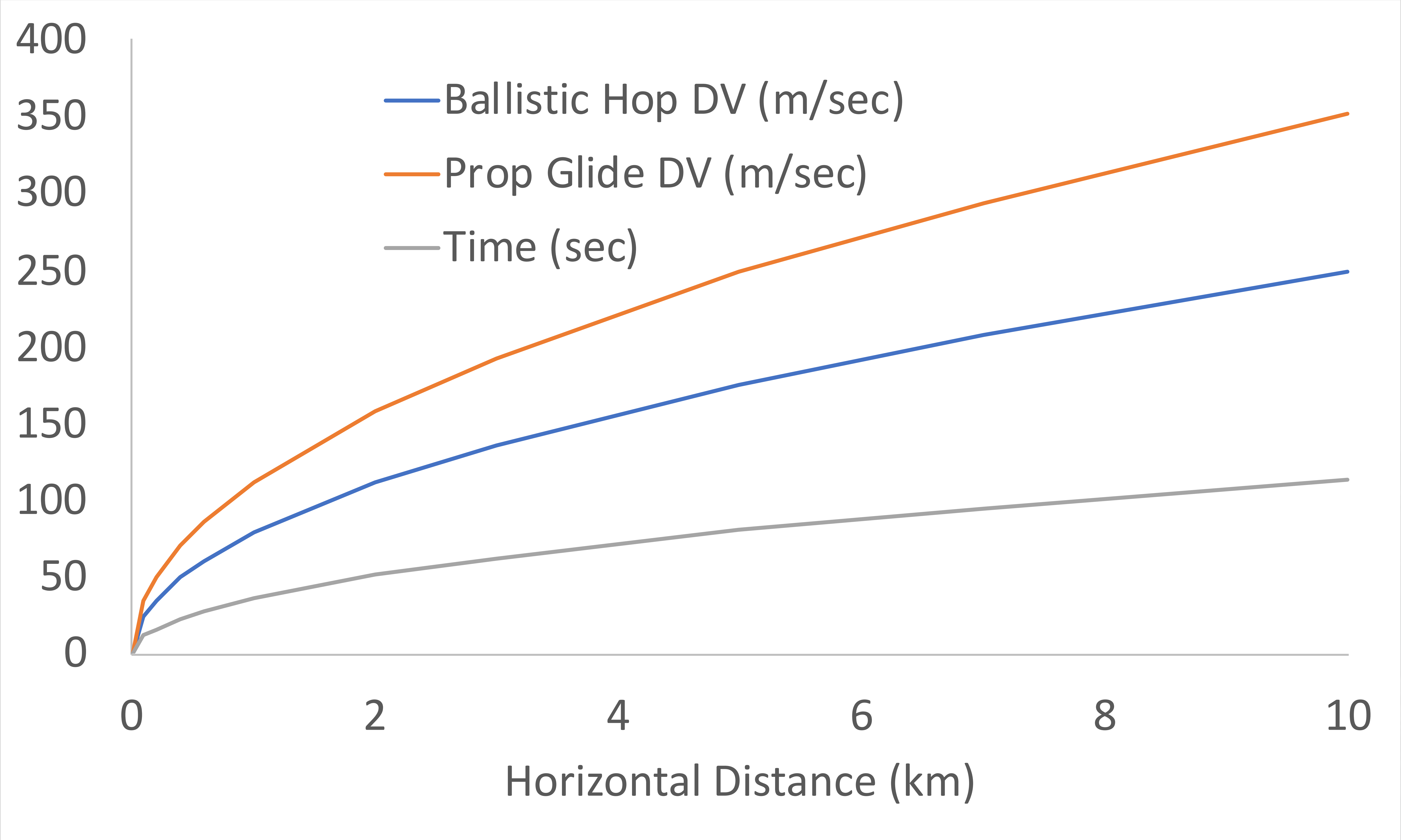
Optimum Propulsive Glide

$$v_{opt} = v_h = \boxed{\sqrt{\frac{gd}{2}}}$$

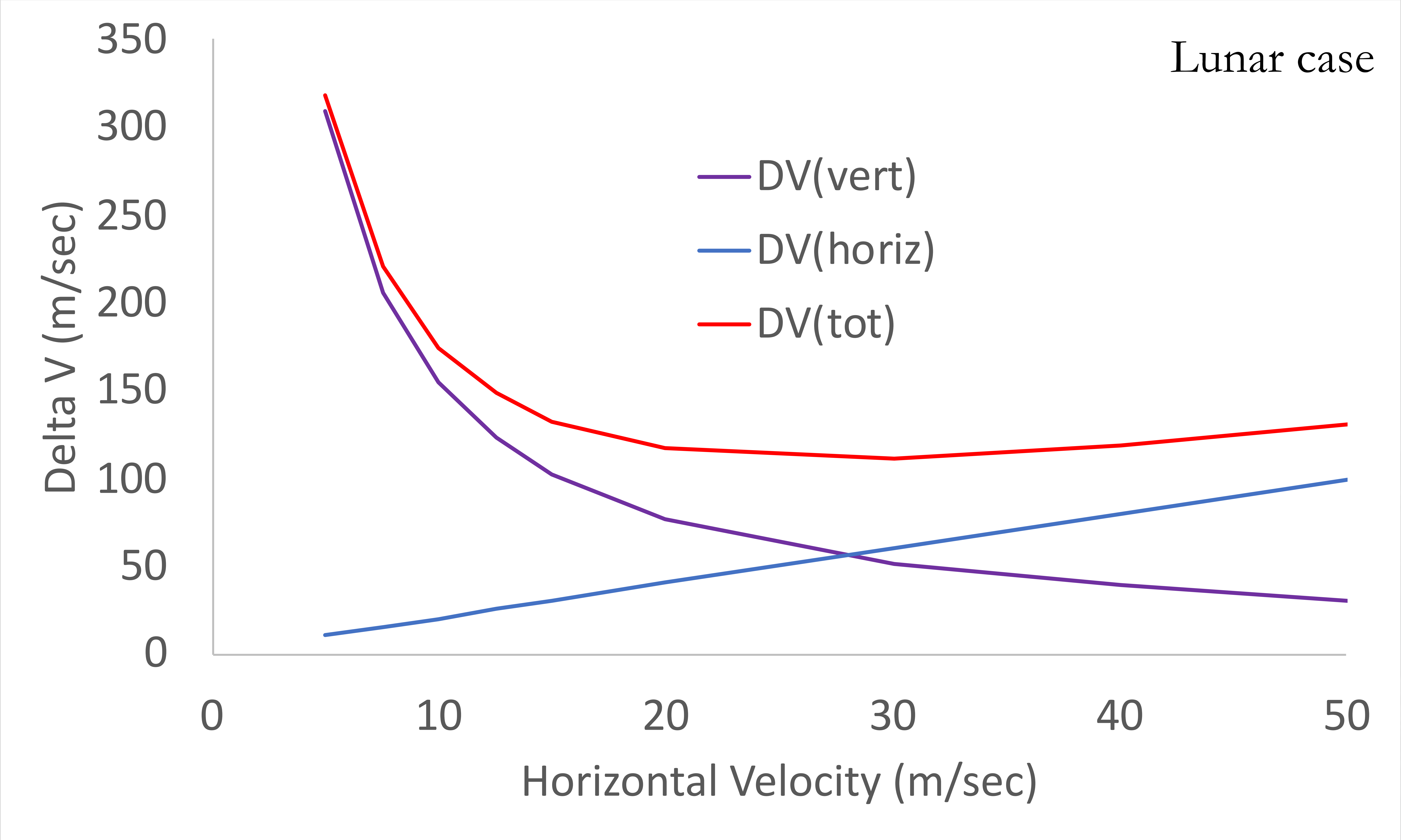
$$t_{flt} = \frac{d}{V_h} = \frac{d}{\sqrt{\frac{dg}{2}}}$$

$$t_{flt} = \boxed{\sqrt{\frac{2d}{g}}}$$

Short-Range Lunar Hops/Glides



Effects of Non-Optimum Propulsive Glide



Hopping Between Different Altitudes

Relative to starting point, landing elevation $\equiv h_2$

$$v_1 = (v_h, v_{v1}) \qquad v_2 = (v_h, v_{v2}) \qquad v_{v1} \neq v_{v2}$$

$$h = v_{v1}t - \frac{1}{2}gt^2 \qquad t_{peak} = \frac{v_{v1}}{g} \qquad h_{peak} = \frac{1}{2}\frac{v_{v1}^2}{g}$$

$$v_{v1} = \sqrt{2gh_{peak}}$$

From peak, $v_v = -gt_{fall}$; $h = h_{peak} - \frac{1}{2}gt_{fall}^2$

$$h_2 = h_{peak} - \frac{1}{2}\frac{v_{v2}^2}{g} \qquad t_{fall} = \sqrt{\frac{2}{g}(h_{peak} - h_2)}$$

$$v_{v2} = \sqrt{2g(h_{peak} - h_2)}$$

Optimal Hop with Altitude Change

$$d = v_h(t_{peak} + t_{fall}) = v_h\left(\frac{v_{v1}}{g} + \sqrt{\frac{2}{g}(h_{peak} - h_2)}\right)$$

$$d = v_h\left(\sqrt{\frac{2h_{peak}}{g}} + \sqrt{\frac{2}{g}(h_{peak} - h_2)}\right)$$

$$d\sqrt{g} = v_h\left(\sqrt{2h_{peak}} + \sqrt{2(h_{peak} - h_2)}\right)$$

$$v_h = \frac{d\sqrt{g}}{\sqrt{2h_{peak}} + \sqrt{2(h_{peak} - h_2)}}$$

$$\Delta v = \left(\sqrt{v_h^2 + v_{v1}^2} + \sqrt{v_h^2 + v_{v2}^2}\right)$$

Nondimensional Form of Equations

$$\text{Remember that } \nu \equiv \frac{v}{\sqrt{dg}};\ \eta \equiv \frac{h_{peak}}{d};\ \lambda \equiv \frac{h_2}{d}$$

$$\Delta\nu = \left(\sqrt{\nu_h^2 + \nu_{v1}^2} + \sqrt{\nu_h^2 + \nu_{v2}^2}\right)$$

$$\nu_{v1} = \sqrt{2\eta} \qquad \nu_{v2} = \sqrt{2(\eta - \lambda)}$$

$$\nu_h = \frac{1}{\sqrt{2\eta} + \sqrt{2(\eta - \lambda)}}$$

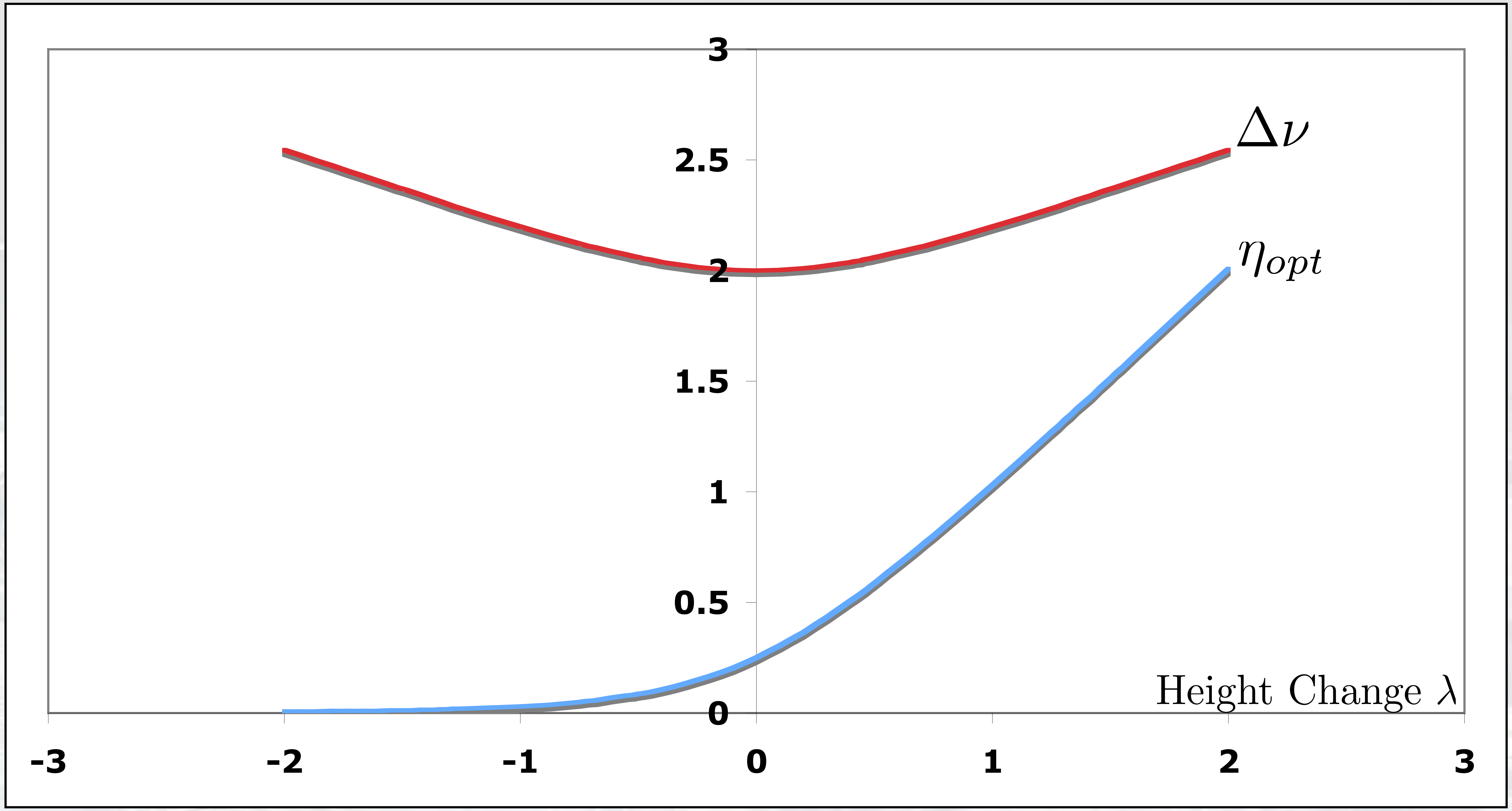
$$\Delta\nu = \sqrt{\left(\frac{1}{\sqrt{2\eta} + \sqrt{2(\eta - \lambda)}}\right)^2 + 2\eta} + \sqrt{\left(\frac{1}{\sqrt{2\eta} + \sqrt{2(\eta - \lambda)}}\right)^2 + 2(\eta - \lambda)}$$

Optimization of Height-Changing Hop

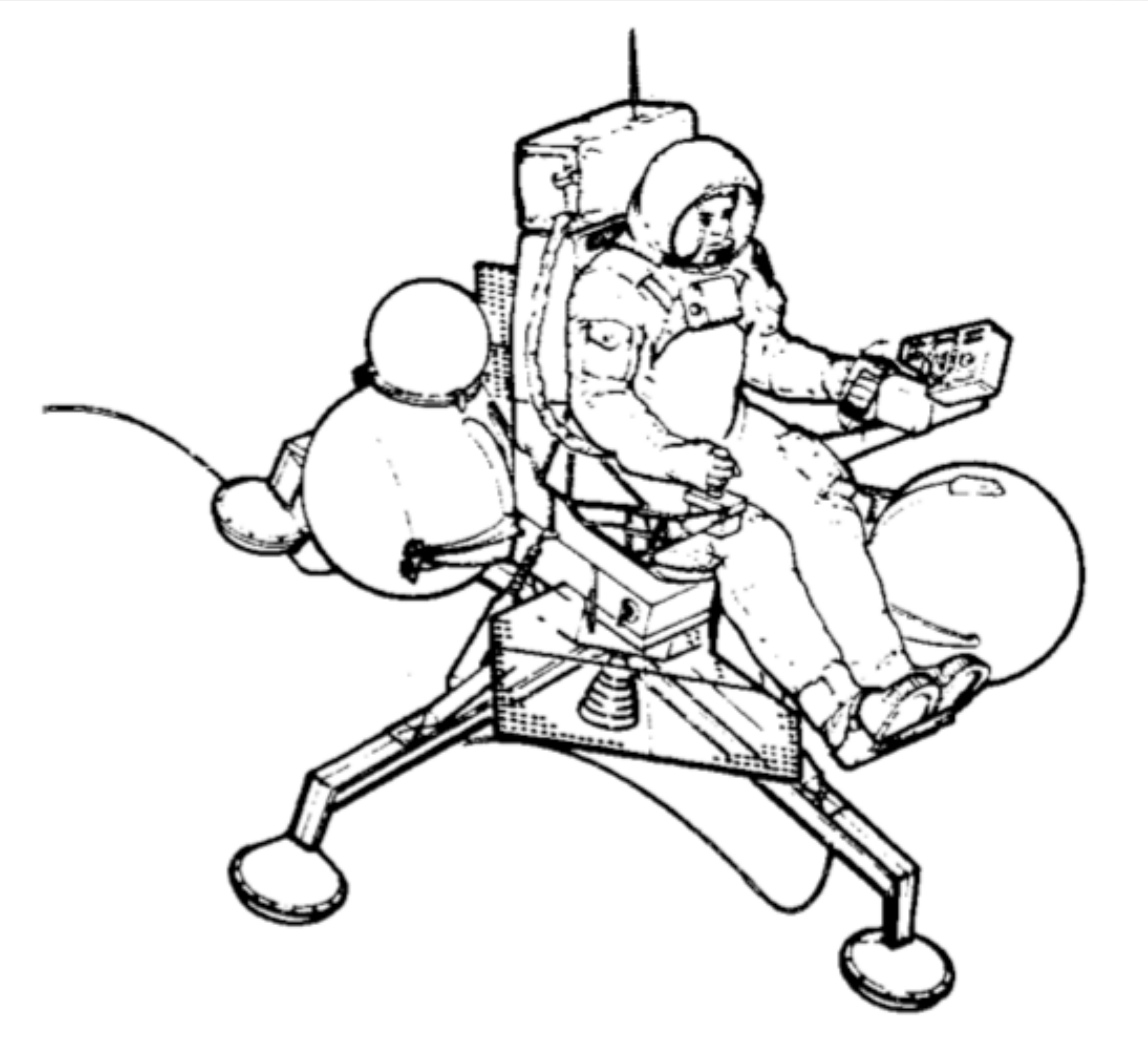
- This is not going to be one where you can take the derivative and set equal to zero, so use the equation to find a numerical optimization
- Set $\lambda = 0$ to check for plain hop solution

$$\Delta \nu = 2\sqrt{\frac{1}{8\eta} + 2\eta} \Rightarrow \eta_{opt} = \frac{1}{4}$$

Trajectory Design for Height Change



Apollo Concept of Lunar Flying Vehicle

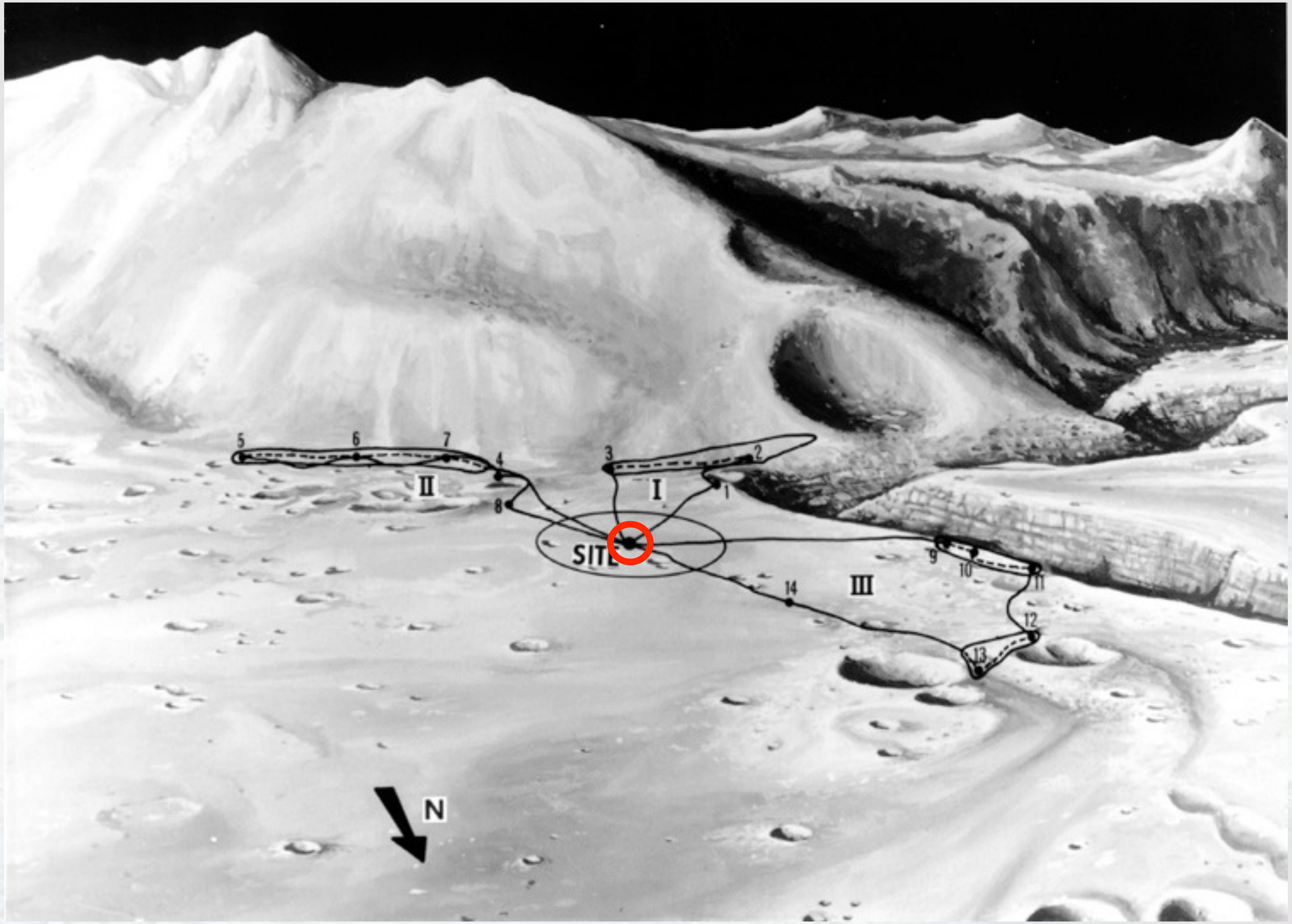


from "Study of One-Man Lunar Flying Vehicle - Final Report Volume 1: Summary" North American Rockwell, NASA CR-101922, August 1969

Apollo 15 Revisited: Lunar Flying Vehicle Sortie

Basic assumptions

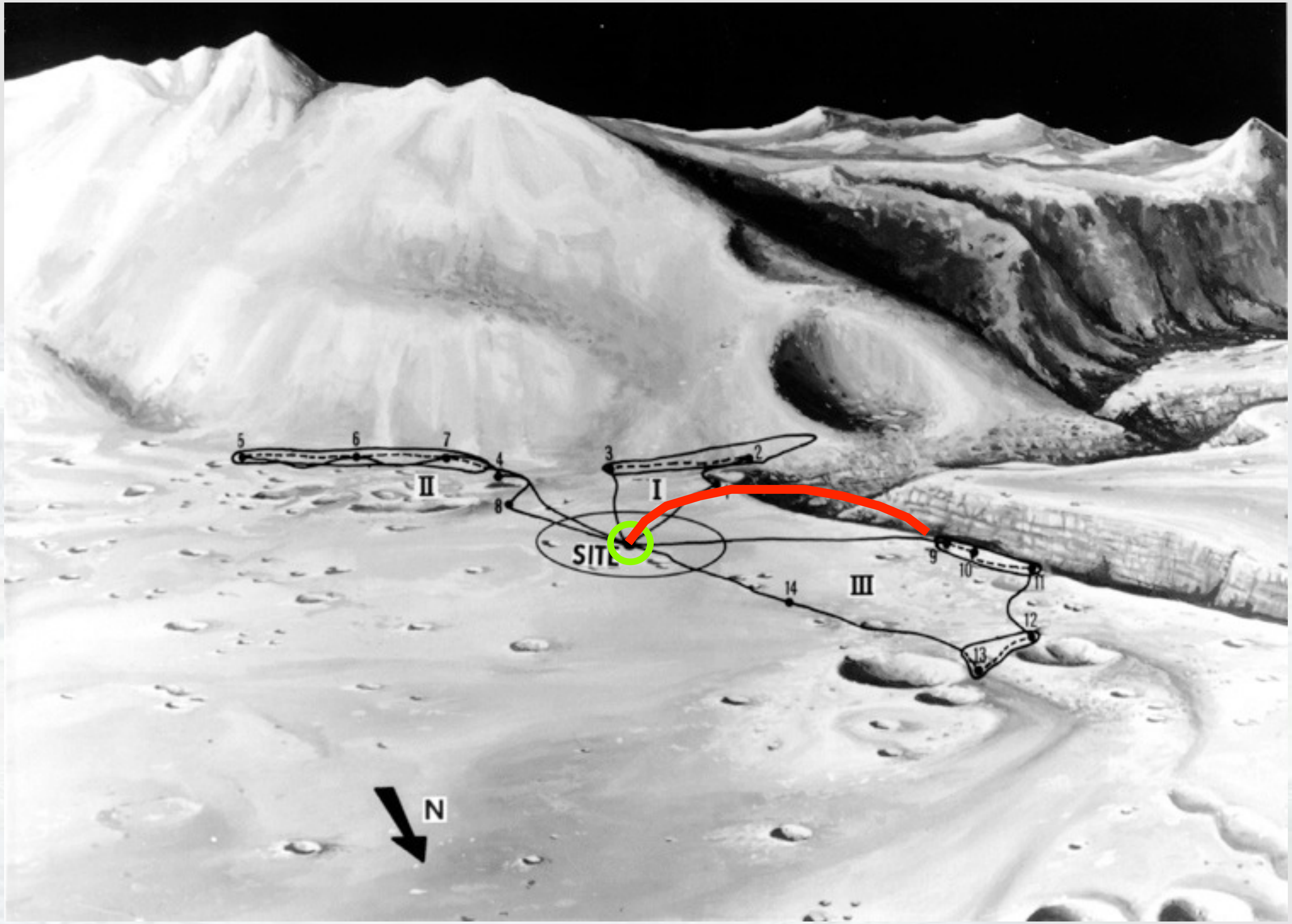
- Vehicle inert mass=300 kg
- Crew mass=150 kg
- Science package=100 kg
- Total propellant=130 kg
- V_e=4200 m/sec



Apollo 15 Revisited: Leg 1

Base camp to bottom of rille

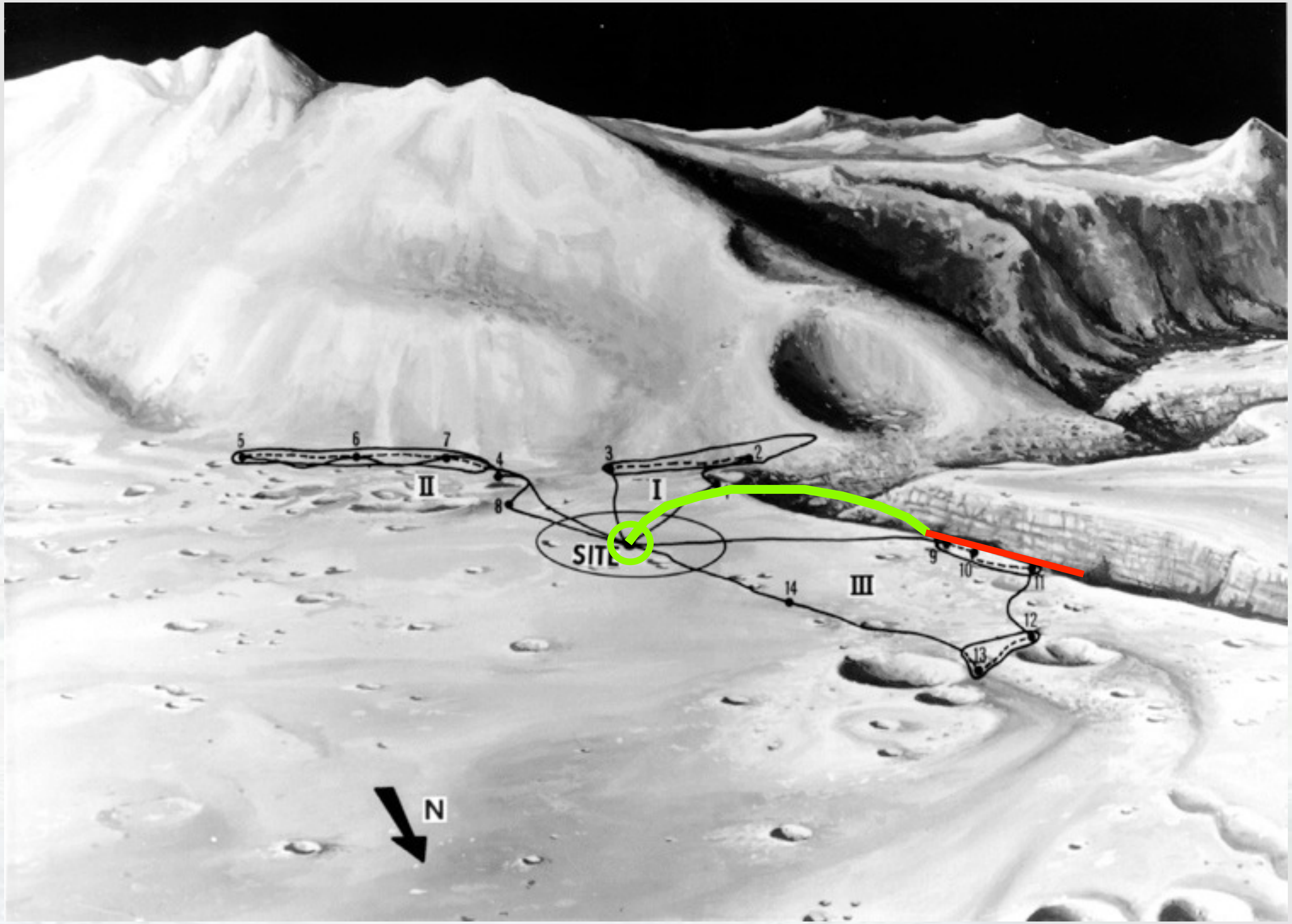
- Distance 3 km
- Altitude change -150 m
- ΔV=139 m/sec
- Propellant used=22 kg
- Collect 20 kg of samples at landing site



Apollo 15 Revisited: Leg 2

Propulsive glide along bottom of rille

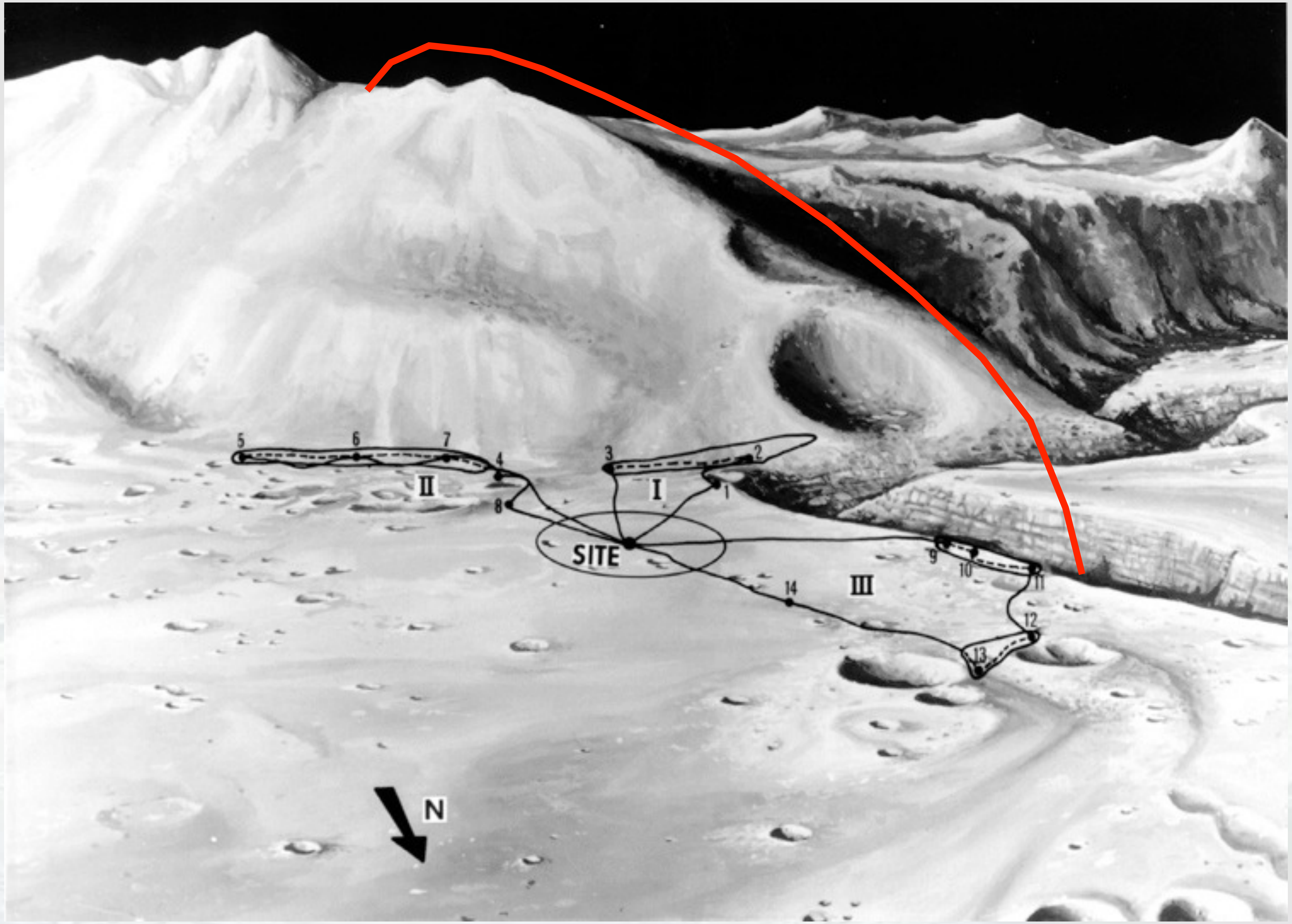
- Distance 2 km (=88 mph!)
- No net altitude change
- ΔV=160 m/sec
- Propellant used=25 kg
- Collect 20 kg of samples at landing site; leave 25 kg science package



Apollo 15 Revisited: Leg 3

Hop to top of mountain

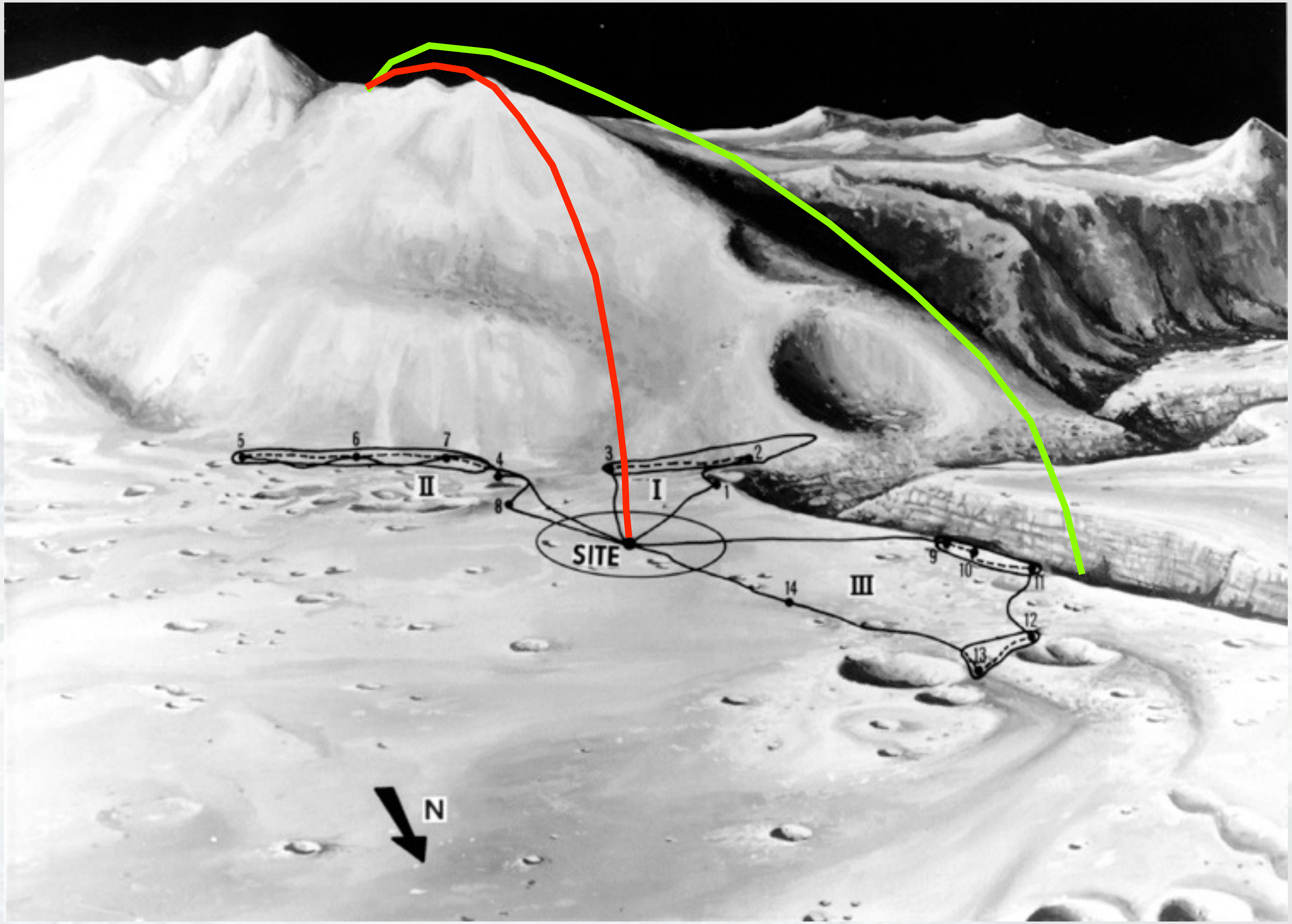
- Distance 15 km
- Altitude change 1600 m
- ΔV=310 m/sec
- Propellant used=46 kg
- Collect 30 kg of samples at landing site; leave 50 kg science package



Apollo 15 Revisited: Leg 4

Return to base

- Distance 12 km
- Altitude change -1450 m
- ΔV=278 m/sec
- Propellant used=37 kg
- Return with 25 kg of science equipment and 70 kg of samples
- Total propellant used 130 kg



Apollo 15 Revisited: Discussion

- Current minimum estimates are for 400 kg of residual propellants (LOX/LH2) in lunar lander at landing - would support three equivalent sorties
- Presence of water ice or ISRU propellant production at outpost would easily support moderate flier mission requirements
- Challenges in routine refueling of cryogenic propellants on the lunar surface, reliable flight and landing control system

Landing Impact Attenuation

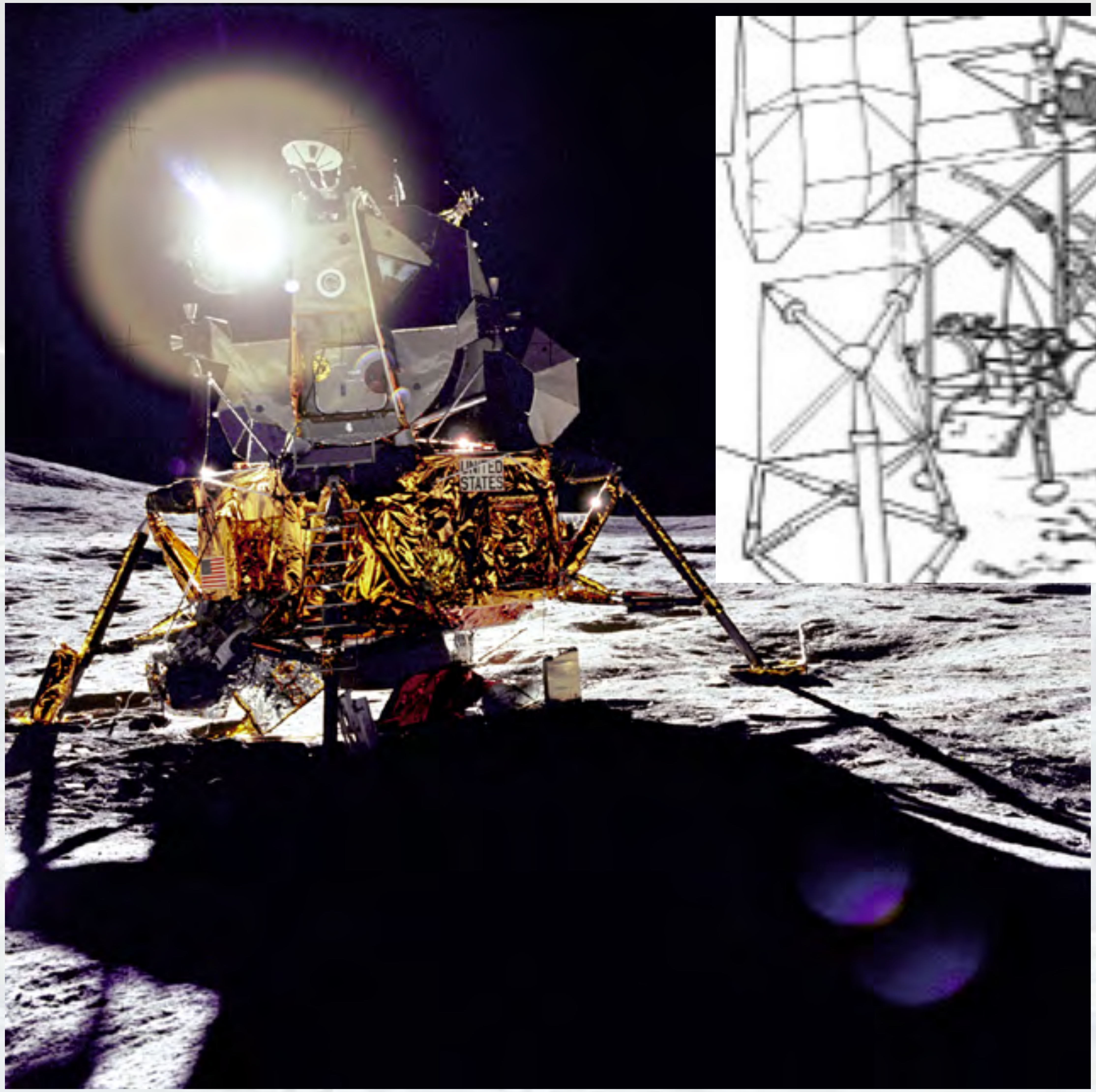
- Cannot rely on achieving perfect zero velocity at touchdown
- Specifications for landing conditions
  - Vertical velocity ≤ 3 m/sec
  - Horizontal velocity ≤ 1 m/sec

$$\text{Kinetic Energy} = \frac{1}{2}mv^2 = \frac{1}{2}m(v_h^2 + v_v^2)$$

$$\text{Max case 500 kg vehicle} \implies E = 2500Nm$$

Mars Phoenix Lander

Apollo Lunar Module



Landing Deceleration

- Look at 3 m/sec vertical velocity
- Constant force deceleration

$$\frac{1}{2}mv^2 = Fd \qquad \frac{1}{2}v^2 = \frac{F}{m}d = a_{desired}d \qquad d = \frac{1}{2}\frac{v^2}{a_{desired}}$$

$$t_{decel} = \frac{v}{a_{desired}}$$

$a_{desired}$	$d\langle cm\rangle$	$t_d\langle sec\rangle$
1/6 g	281	1.88
1/2 g	92	0.61
1 g	46	0.31
2 g	23	0.15
3 g	15	0.10

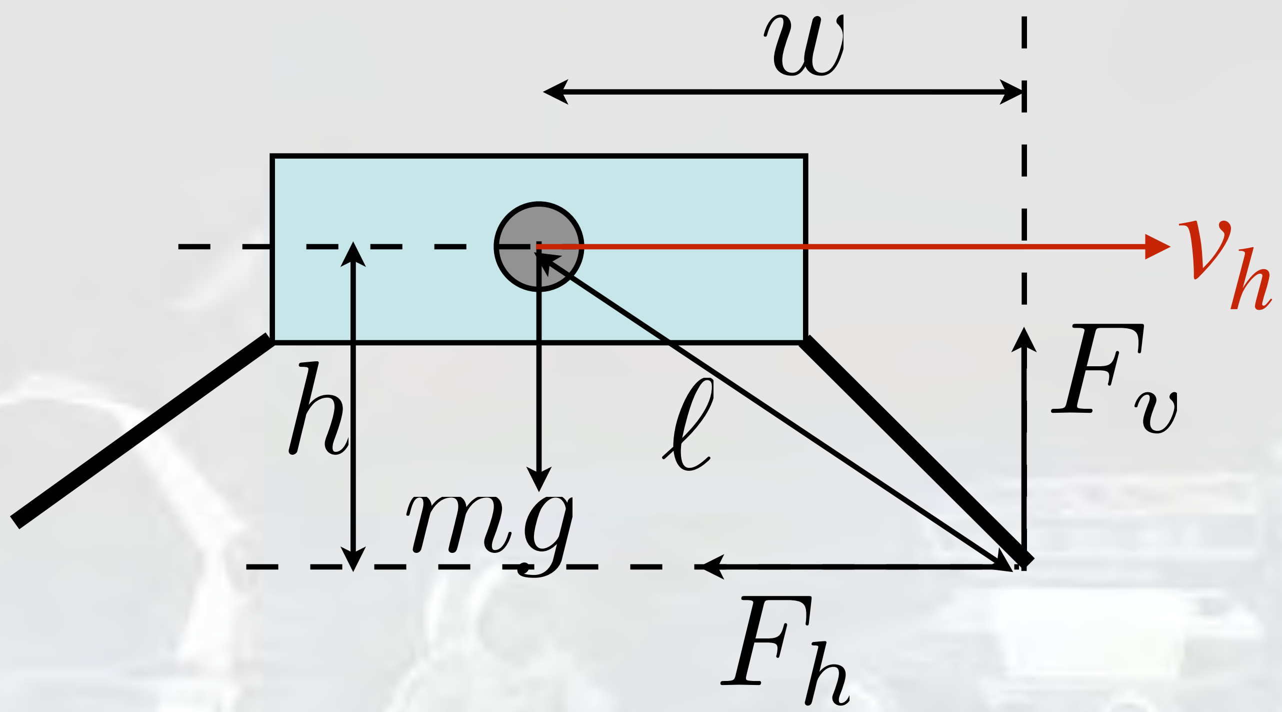
- Spring deceleration

$$F = kx \qquad \int F dx = \frac{1}{2}mv^2$$

$$k = \frac{mv^2}{d^2} \qquad a_{peak} = \frac{kd}{m}$$

Effect of Lateral Velocity at Touchdown

- Resolve torques around landing gear footpad

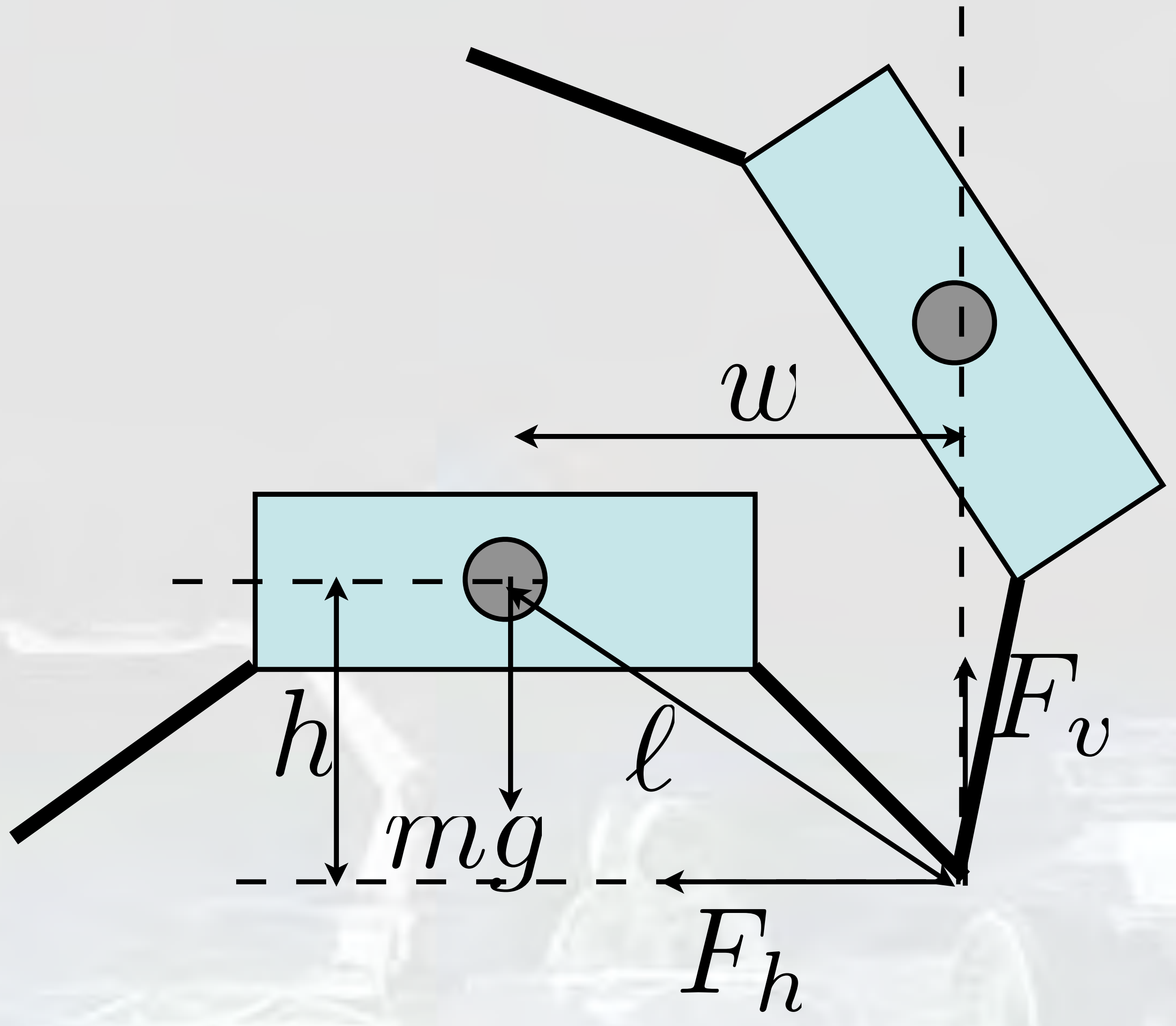


$$\ddot{\theta} = \frac{\tau_{tot}}{I_{tot}}$$

$$\ddot{\theta} = \frac{F_h h - F_v w - mgw}{I_{cg} + m\ell^2}$$

- Worst cases - hit obstacle (high force), landing downhill
- Issue: rotational velocity induced is counteracted by vehicle weight
- Will vehicle rotation stop before overturn limit?

Simple Approach to Landing Stability



Kinetic energy at landing

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m(v_v^2 + v_h^2)$$

Dissipated by potential energy of raising C.G. by rotation around impact point

$$P.E. = mg\Delta h = mg(\ell - h)$$

$$v_{crit} = \sqrt{2g(\ell - h)} \quad \text{or} \quad w_{req} = \sqrt{\left(\frac{v^2}{2g} + h\right)^2 - h^2}$$