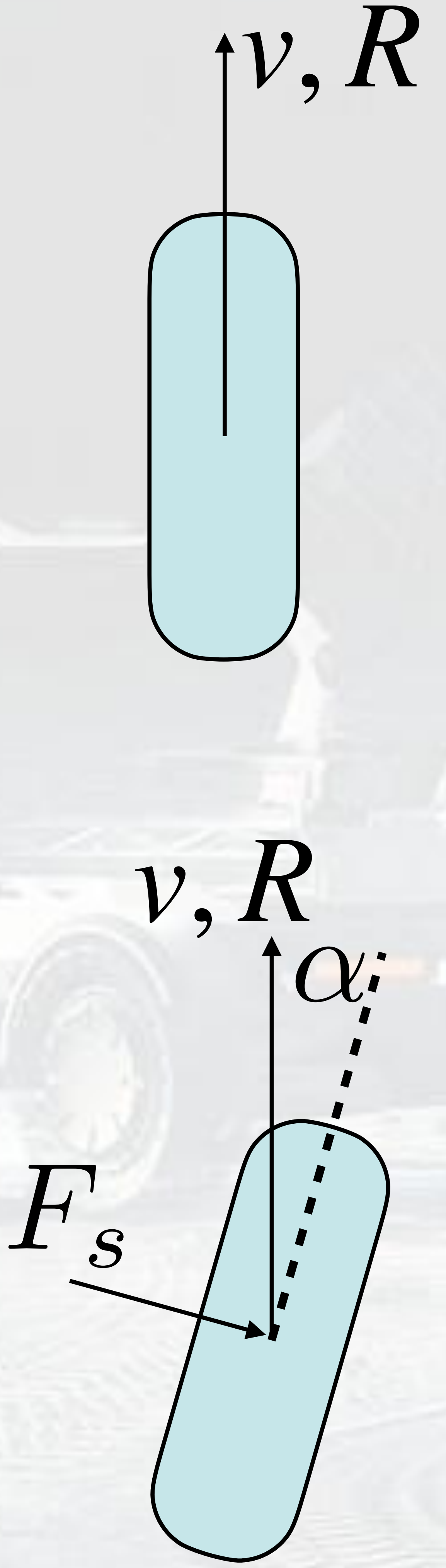
Steering Forces/Slopes and Static Stability

- Side forces on wheel
- Power comparison between skid-steer and ideally steered
- Stability across and along slopes
- Forces and torques on wheels
- Acceleration/deceleration
- Turning
- Hitting obstacles
- Rigid suspensions and obstacles

Skidding Forces and Power



$$R = \text{ total wheel resistance } < N >$$

$$\text{Drive power } P_r = Rv \qquad < \frac{Nm}{\text{sec}} > = < W >$$

$$\text{Side force} \equiv F_s = \mu_s N$$

$$\text{Normal force into soil} \equiv N$$

$$\text{Skid power } P_s = F_s v_s = \mu_s N v \sin \alpha$$

$$\text{Total wheel power} \equiv P_w = v(R + \mu_s N \sin \alpha)$$

Wheel Drive Power

We can define $R = \mu_r N$

$$P_w = vN(\mu_r + \mu_s \sin\alpha)$$

$$P_w = v\mu_r N(1 + \frac{\mu_s}{\mu_r}\sin\alpha)$$

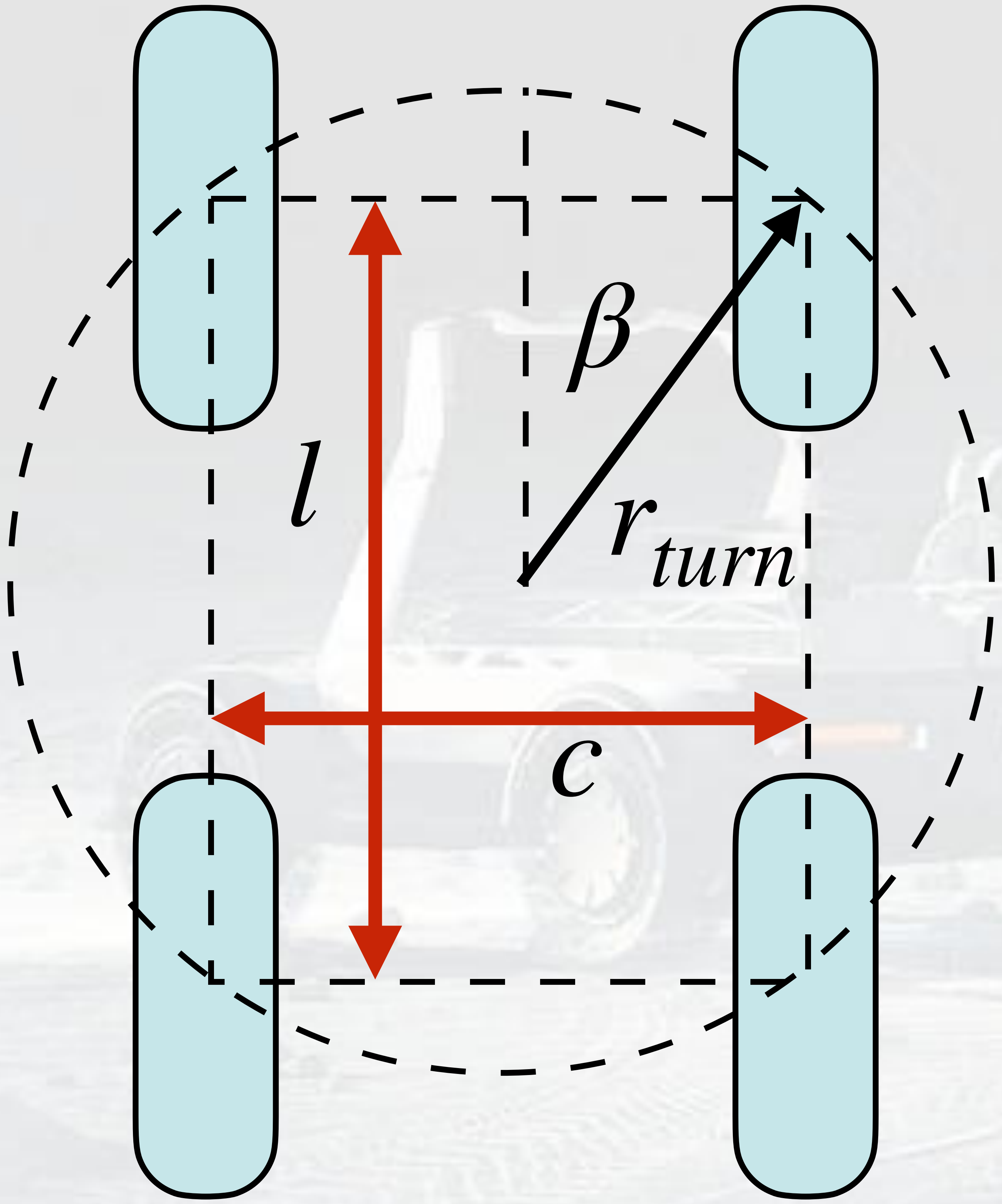
$$P_{roll} \equiv v\mu_r N$$

$$P_w = P_{roll}(1 + \frac{\mu_s}{\mu_r}\sin\alpha)$$

For roads $\implies \mu_r \approx 0.05$ or less

Off-road $\implies \mu_r \approx 0.2; \;\; \mu_s \approx 1$

Turn-in-Place (Skid Steering)



$$P_{skid} = P_{roll} \quad \text{if} \quad \frac{\mu_s}{\mu_r} \sin\alpha = 1$$

$$\frac{\mu_s}{\mu_r} \sim 5 \Longrightarrow \sin\alpha = 0.2 \Longrightarrow \alpha = 11.5^{\text{o}}$$

Turn in place (skid steer)

$$r_{turn} = \sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$\cos\beta = \frac{c/2}{r_{turn}} \qquad \sin\beta = \frac{l/2}{r_{turn}}$$

Power Required for TIP Skid Steering

$$V_r = \omega r_{turn} \cos\beta = \frac{\omega c}{2}$$

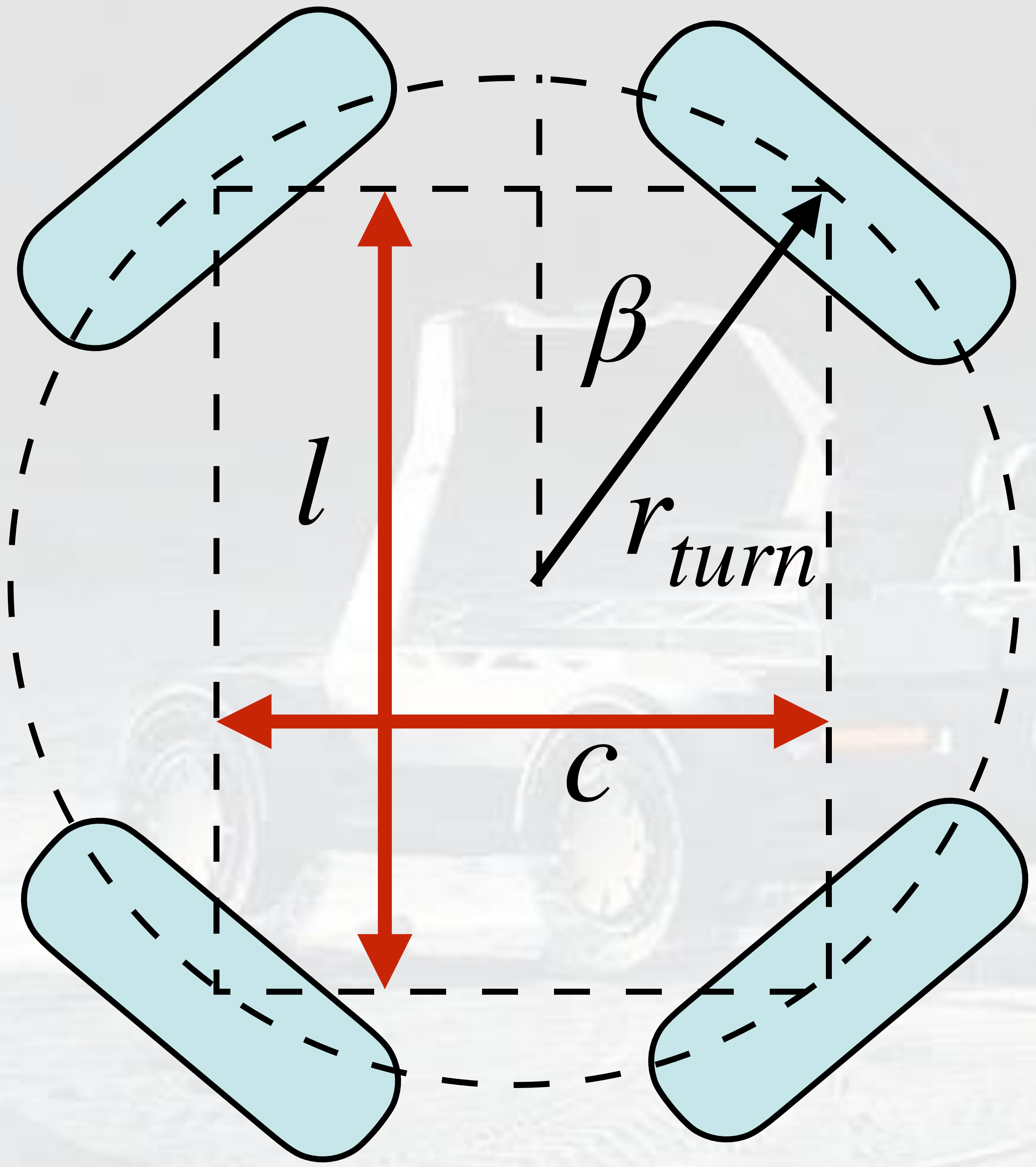
$$V_s = \omega r_{turn} \sin\beta = \frac{\omega \ell}{2}$$

$$P_{roll} = V_r \mu_r N = \frac{\omega c}{2} \mu_r N$$

$$P_{skid} = V_s \mu_s N = \frac{\omega l}{2} \mu_s N$$

$$P_w = \omega N \left(\frac{c}{2} \mu_r + \frac{l}{2} \mu_s \right) = \frac{\omega N}{2} \left(c\mu_r + l\mu_s \right)$$

Turn in Place (Skid vs. Steered)



Turn in place (steered)

$$P_w = \omega r_{turn} \mu_r N$$

$$\frac{P_{skid}}{P_{steer}} = \frac{\frac{\omega N}{2}\left(c\mu_r + l\mu_s\right)}{\omega r_{turn}\mu_r N}$$

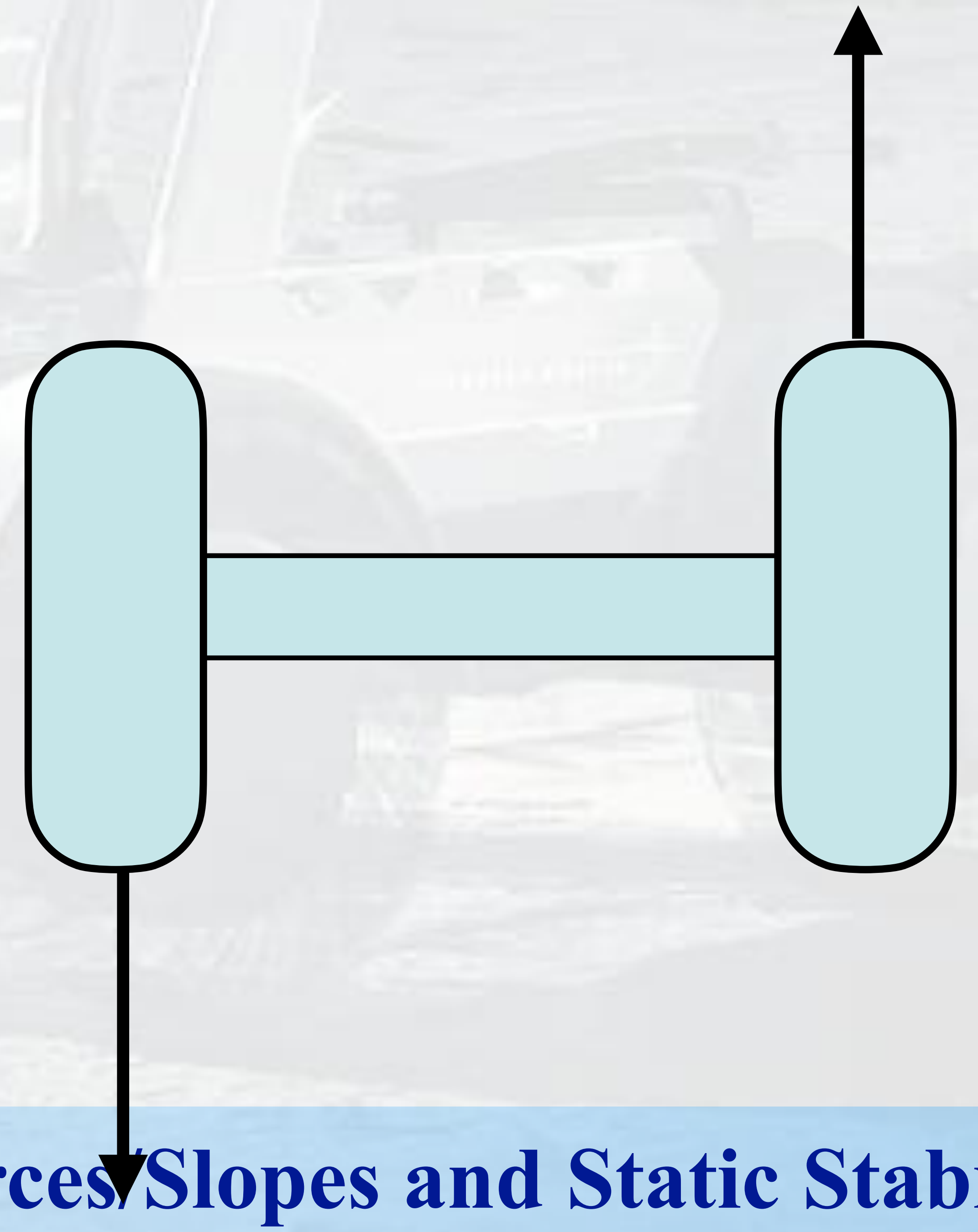
$$= \frac{1}{2}\frac{c + l\frac{\mu_s}{\mu_r}}{r_{turn}} = \frac{1}{2}\frac{c + l\frac{\mu_s}{\mu_r}}{\sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}}$$

Effect of Wheelbase on Skid Steering Power

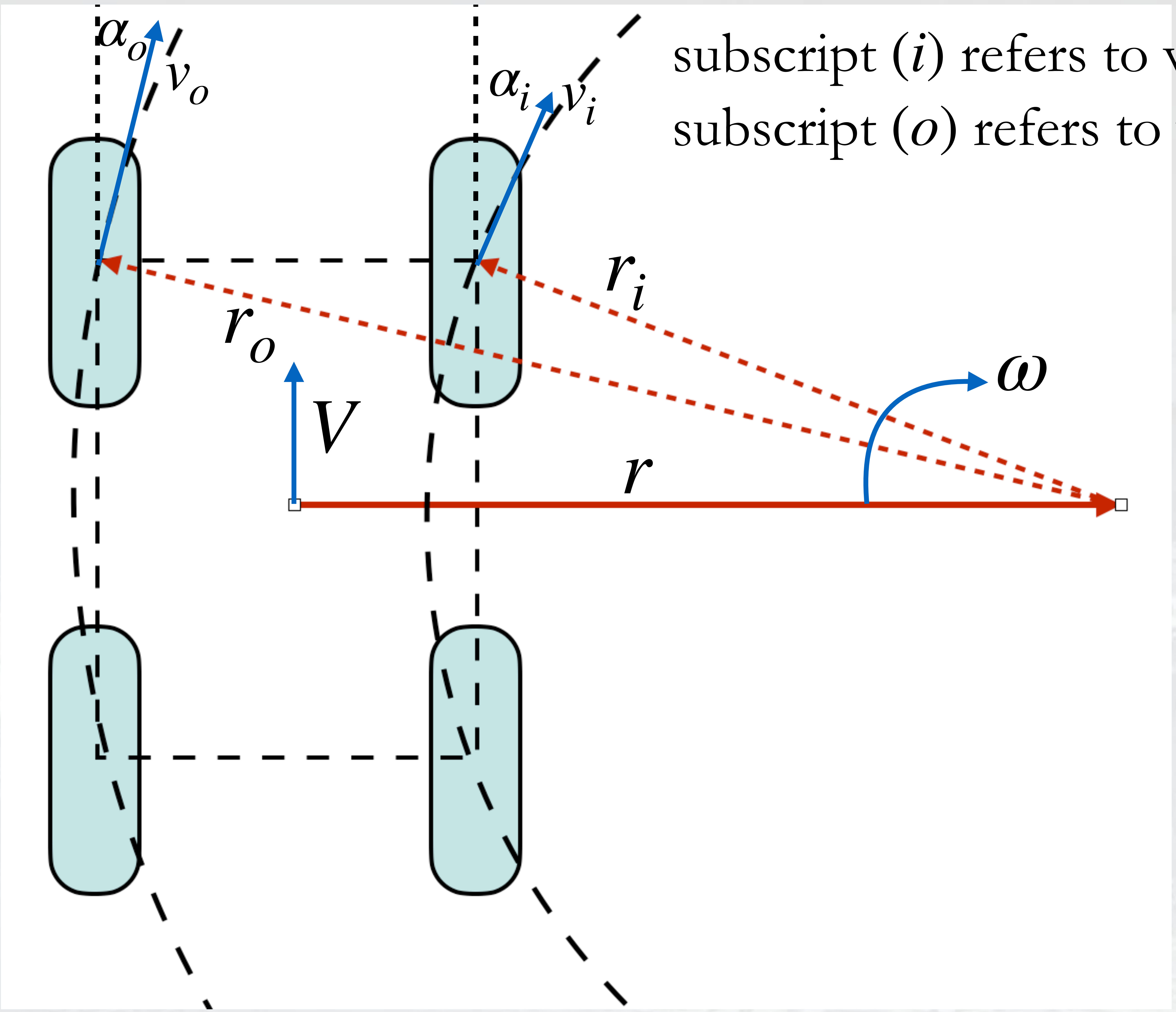
$$\frac{P_{skid}}{P_{steer}} = \frac{c + \frac{\mu_s}{\mu_r} l}{\sqrt{c^2 + l^2}}$$

$$\frac{\mu_s}{\mu_r} \sim 5 \implies \text{Skid power goes up with } l$$

$$\frac{P_{skid}}{P_{steered}} \longrightarrow 1 \text{ for } l \longrightarrow 0$$



Skid Steering around a Turn



subscript (i) refers to wheel on inside of turn

subscript (o) refers to wheel on outside of turn

$$V = \omega r$$

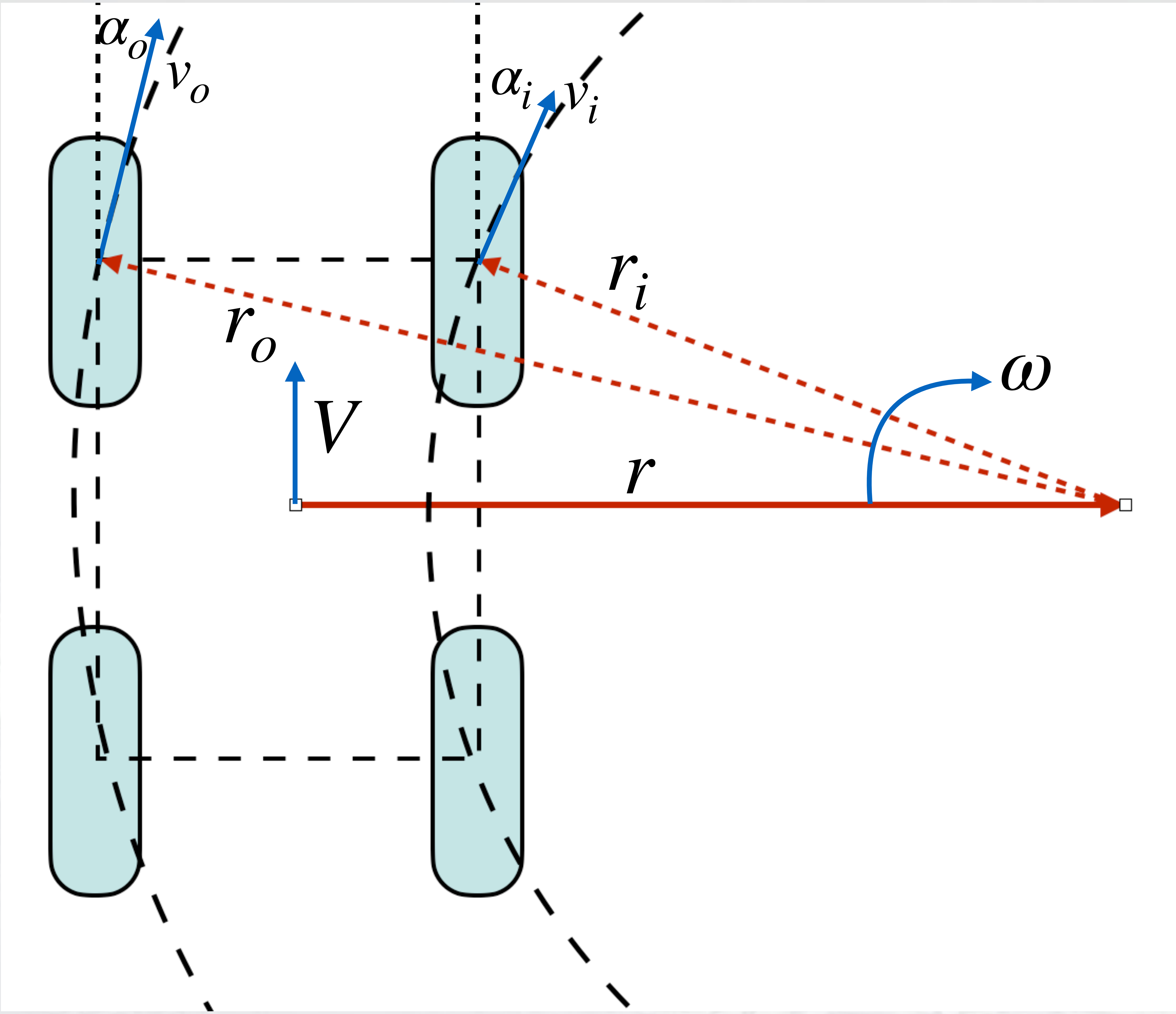
$$r_o = \sqrt{\left(r + \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$r_i = \sqrt{\left(r - \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$v_o = \omega r_o = \omega\sqrt{\left(r + \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$v_i = \omega r_i = \omega\sqrt{\left(r + \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

Skid Steering around a Turn



$$\cos\alpha_o = \frac{r+\frac{c}{2}}{r_o} \qquad \sin\alpha_o = \frac{l/2}{r_o}$$

$$\cos\alpha_i = \frac{r-\frac{c}{2}}{r_i} \qquad \sin\alpha_i = \frac{l/2}{r_i}$$

$$v_{r,o} = \omega r_o \cos\alpha_o = \omega r_o \frac{r+\frac{c}{2}}{r_o} = \omega\left(r+\frac{c}{2}\right)$$

$$v_{s,o} = \omega r_o \sin\alpha_o = \omega r_o \frac{l/2}{r_o} = \omega\frac{l}{2}$$

$$v_{r,i} = \omega r_i \cos\alpha_i = \omega r_i \frac{r-\frac{c}{2}}{r_i} = \omega\left(r-\frac{c}{2}\right)$$

$$v_{s,i} = \omega r_i \sin\alpha_i = \omega r_i \frac{l/2}{r_i} = \omega\frac{l}{2}$$

Power Required for Turning Skid Steer

$$P_{r,o} = v_{r,o}\,\mu_r\,N_o = \omega\left(r + \frac{c}{2}\right)\mu_r\,N_o = V\left(1 + \frac{c}{2r}\right)\mu_r\,N_o$$

$$P_{s,o} = v_{s,o}\,\mu_s\,N_o = \frac{\omega l}{2}\,\mu_s\,N_o = \frac{Vl}{2r}\,\mu_s\,N_o$$

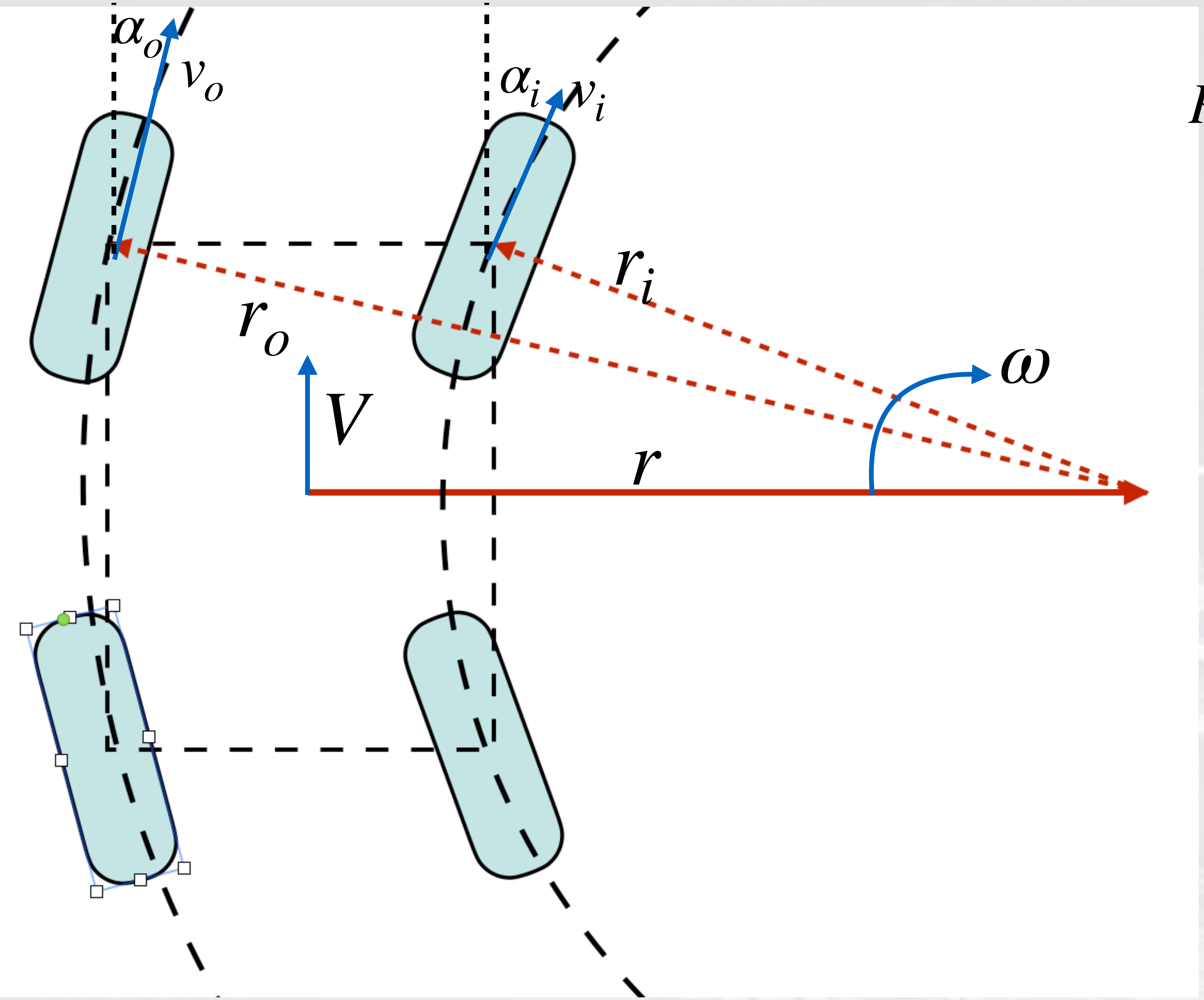
$$P_{w,o} = \omega N_o\left[\left(r + \frac{c}{2}\right)\mu_r + \frac{l}{2}\mu_s\right] = V\left[\left(1 + \frac{c}{2r}\right)\mu_r + \frac{l}{2r}\mu_s\right]N_o$$

$$P_{r,i} = V\left(1 - \frac{c}{2r}\right)\mu_r\,N_i$$

$$P_{s,i} = \frac{Vl}{2r}\,\mu_s\,N_i$$

$$P_{w,i} = V\left[\left(1 - \frac{c}{2r}\right)\mu_r + \frac{l}{2r}\mu_s\right]N_i$$

Double Ackermann Steering around a Turn



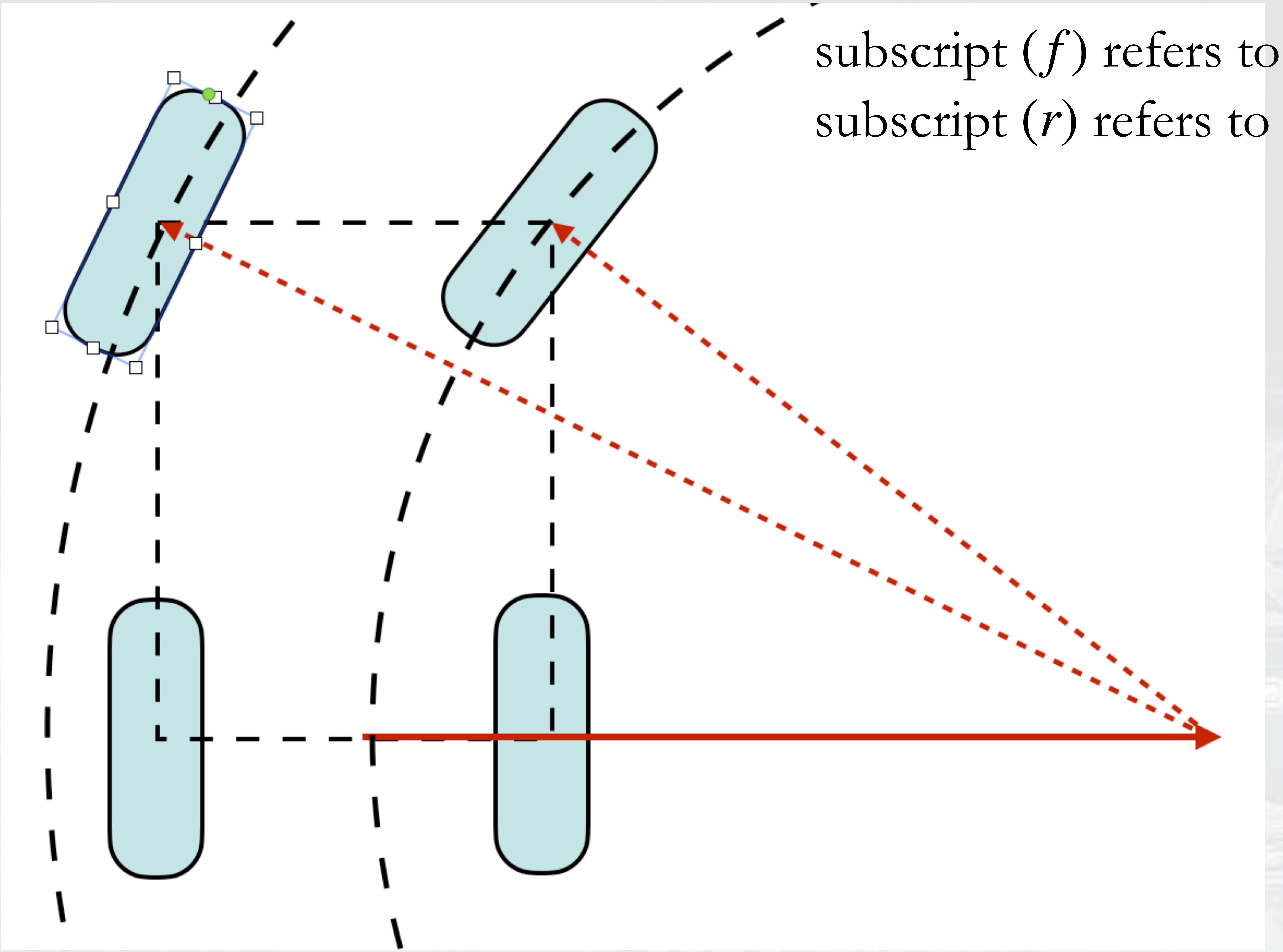
$$P_{w,o} = \omega r_0 \mu_r N_o = \frac{V}{r}\sqrt{\left(r+\frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}\mu_r N_o$$

$$= V\sqrt{\left(1+\frac{c}{2r}\right)^2 + \left(\frac{l}{2r}\right)^2}\mu_r N_o$$

$$P_{w,i} = V\sqrt{\left(1-\frac{c}{2r}\right)^2 + \left(\frac{l}{2r}\right)^2}\mu_r N_i$$

$$P_{total} = 2\left(P_{w,o} + P_{w,i}\right)$$

Single Ackermann Steering



subscript (f) refers to front wheel

subscript (r) refers to rear wheel

$$v_{o,f} = \omega r_{o,f} = \omega\sqrt{\left(r + \frac{c}{2}\right)^2 + l^2}$$

$$v_{i,f} = \omega r_{i,f} = \omega\sqrt{\left(r - \frac{c}{2}\right)^2 + l^2}$$

$$v_{o,r} = \omega r_{o,r} = \omega\left(r + \frac{c}{2}\right)$$

$$v_{i,r} = \omega r_{i,r} = \omega\left(r - \frac{c}{2}\right)$$

Turning Power Required with Single Ackermann

$$P_{w,of} = v_{r,of}\ \mu_r\ N_{of} = \omega\sqrt{\left(r+\frac{c}{2}\right)^2 + l^2}\ \mu_r\ N_{of} = V\sqrt{\left(1+\frac{c}{2r}\right)^2 + \left(\frac{l}{r}\right)^2}\ \mu_r\ N_{of}$$

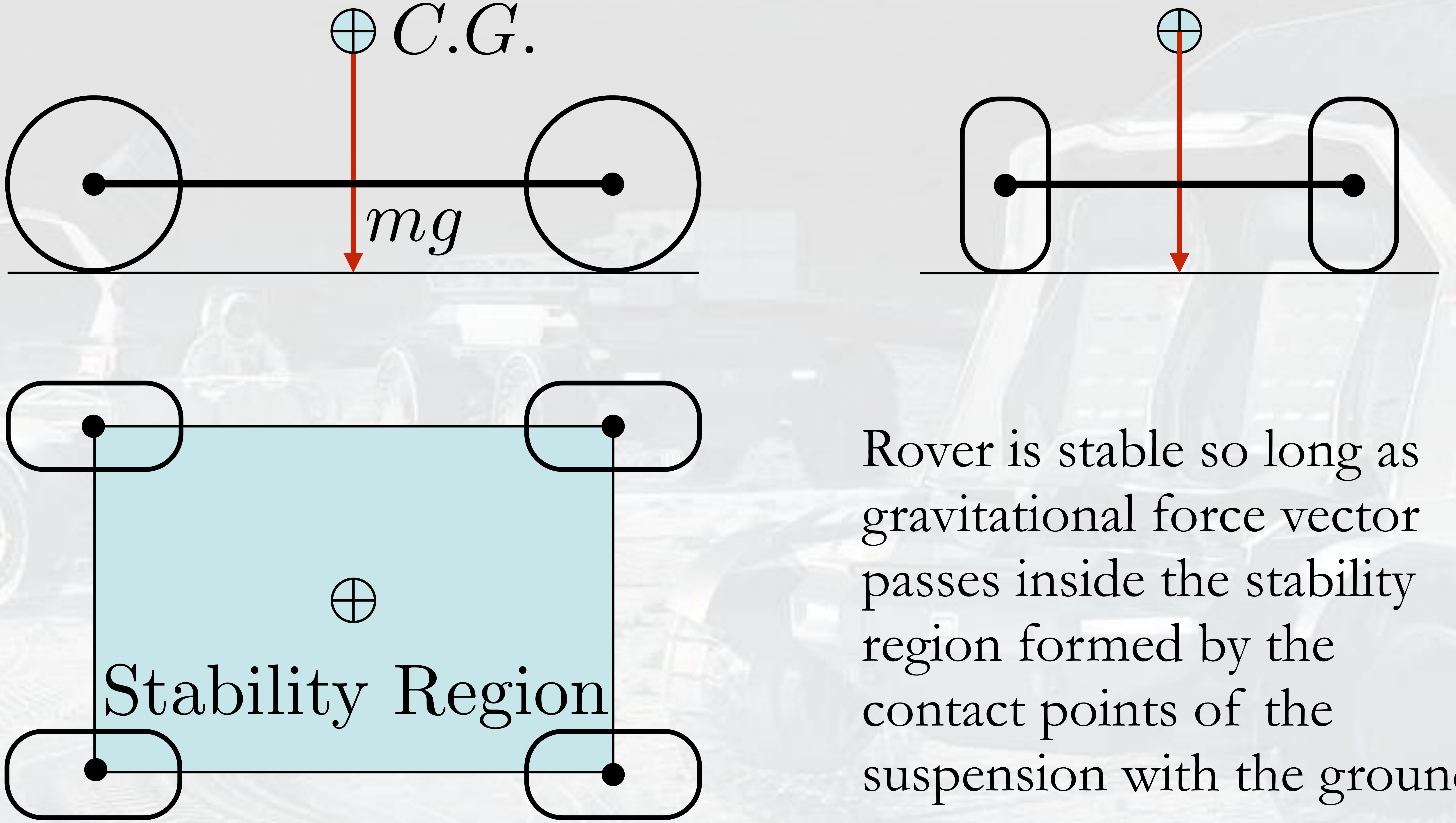
$$P_{w,if} = V\sqrt{\left(1-\frac{c}{2r}\right)^2 + \left(\frac{l}{r}\right)^2}\ \mu_r\ N_{if}$$

$$P_{w,or} = V\left(1+\frac{c}{2r}\right)\mu_r N_{or}$$

$$P_{w,ir} = V\left(1-\frac{c}{2r}\right)\mu_r N_{ir}$$

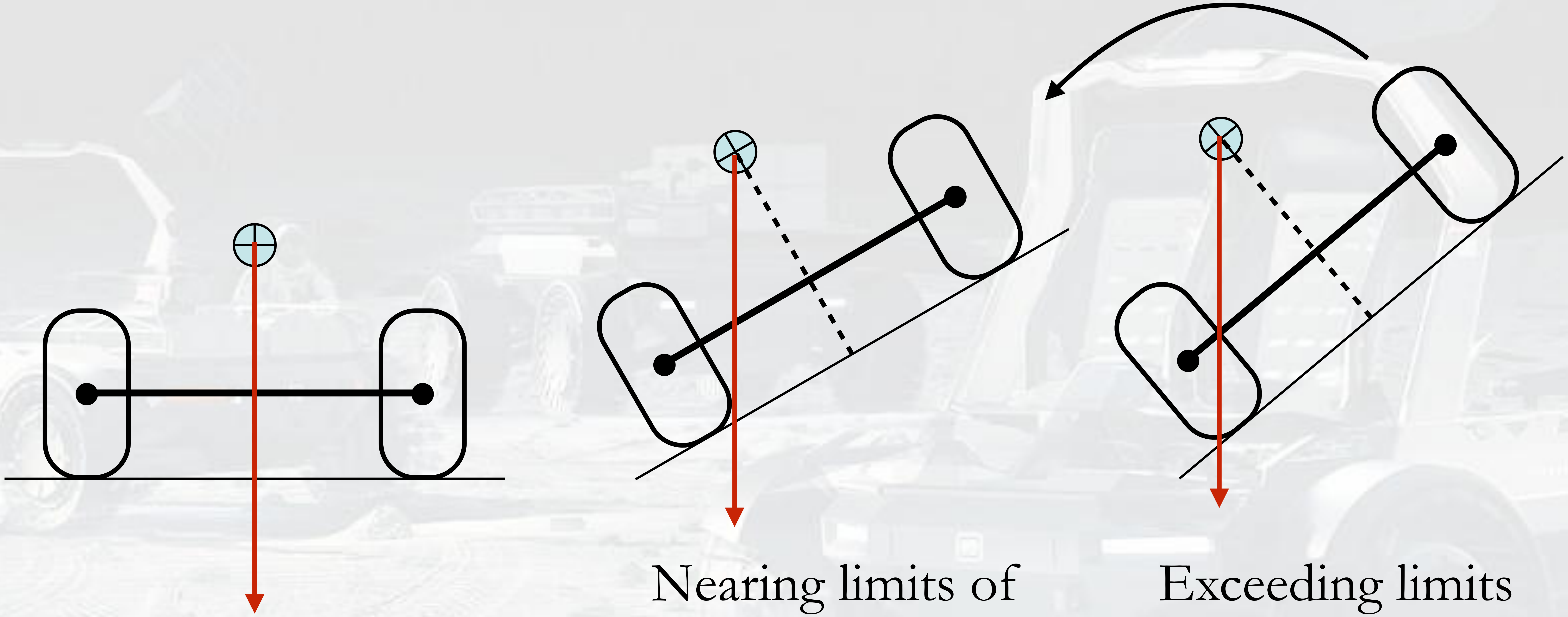
$$P_{total} = P_{w,of} + P_{w,if} + P_{w,or} + P_{w,ir}$$

Rover with CG and Force Vector



Rover is stable so long as gravitational force vector passes inside the stability region formed by the contact points of the suspension with the ground

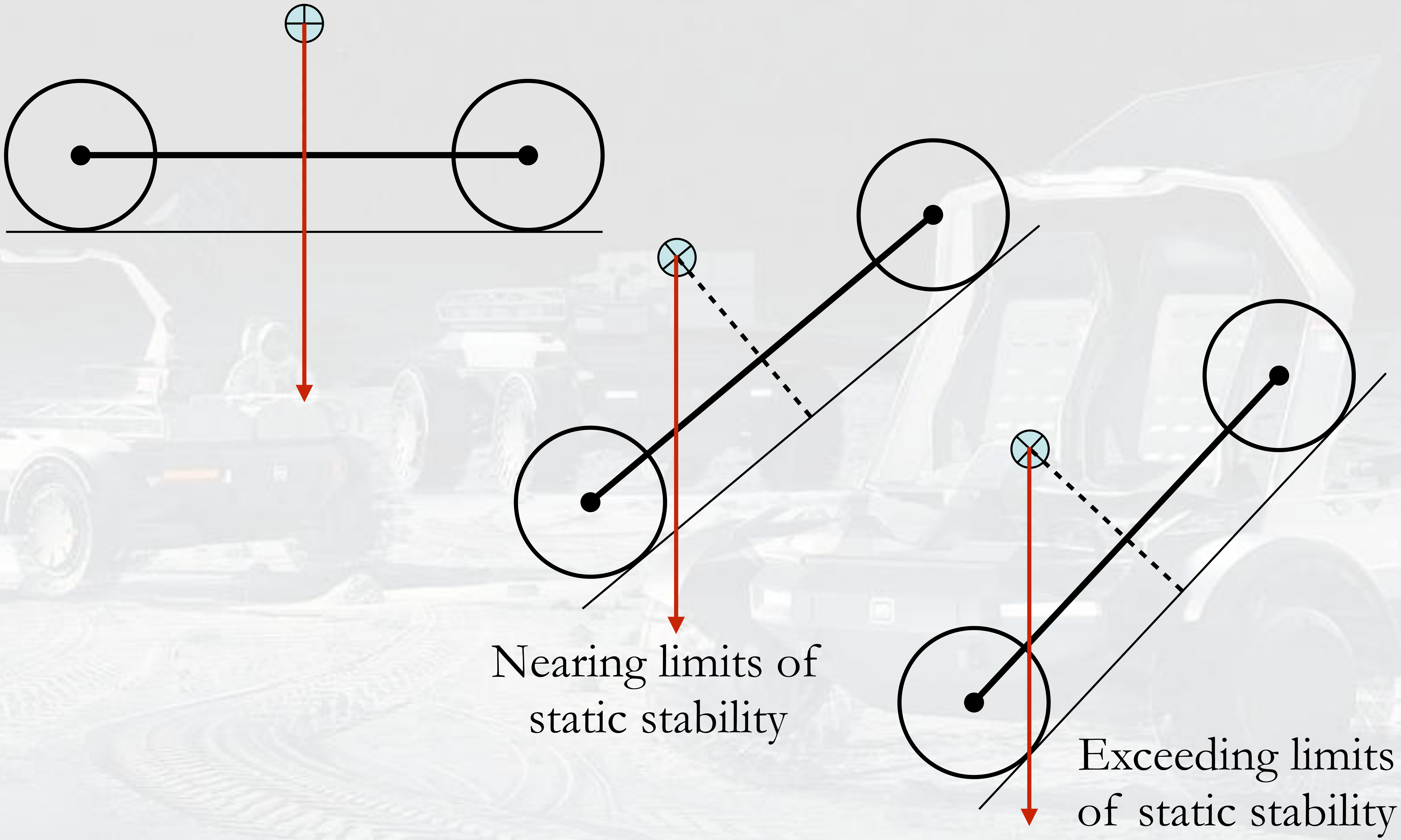
Rover on Cross Slope



Nearing limits of static stability

Exceeding limits of static stability

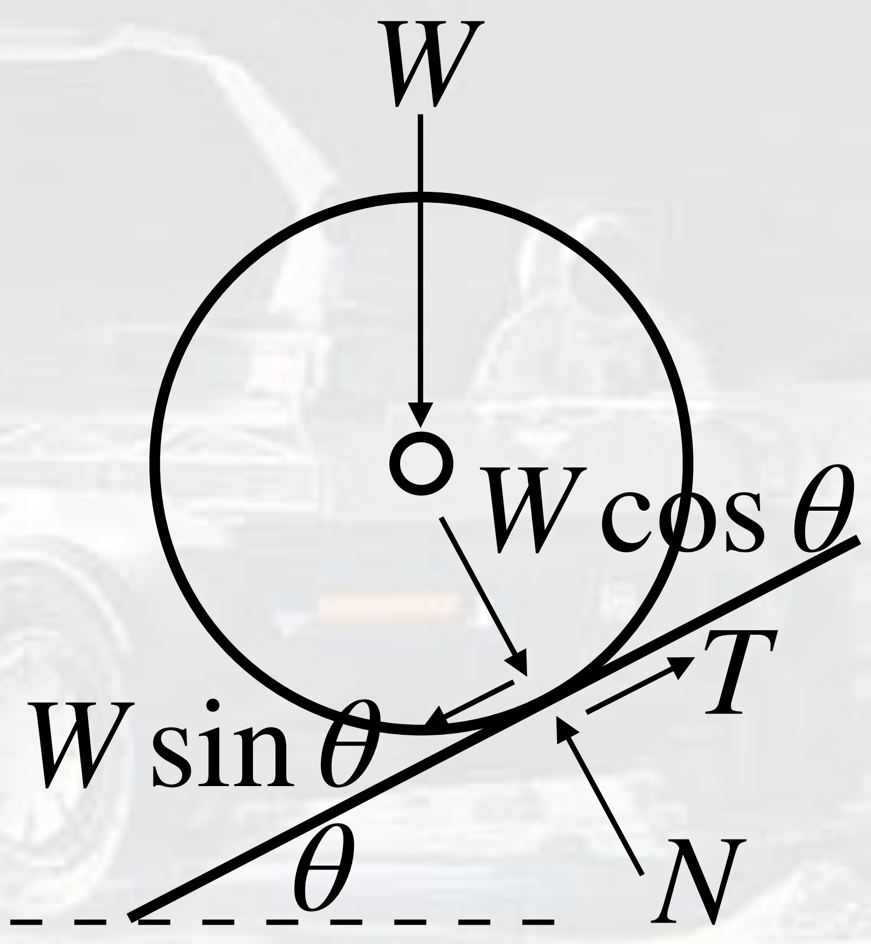
Rover Climbing/Descending Slope



Slopes and Obstacles

$$\mu \equiv \text{Wheel coefficient of friction with ground}$$

$$N \equiv \text{normal force to surface}$$



$$\tau = \mu r N = \mu r W \sin\theta$$

$$T \equiv \text{wheel thrust} = \mu N$$

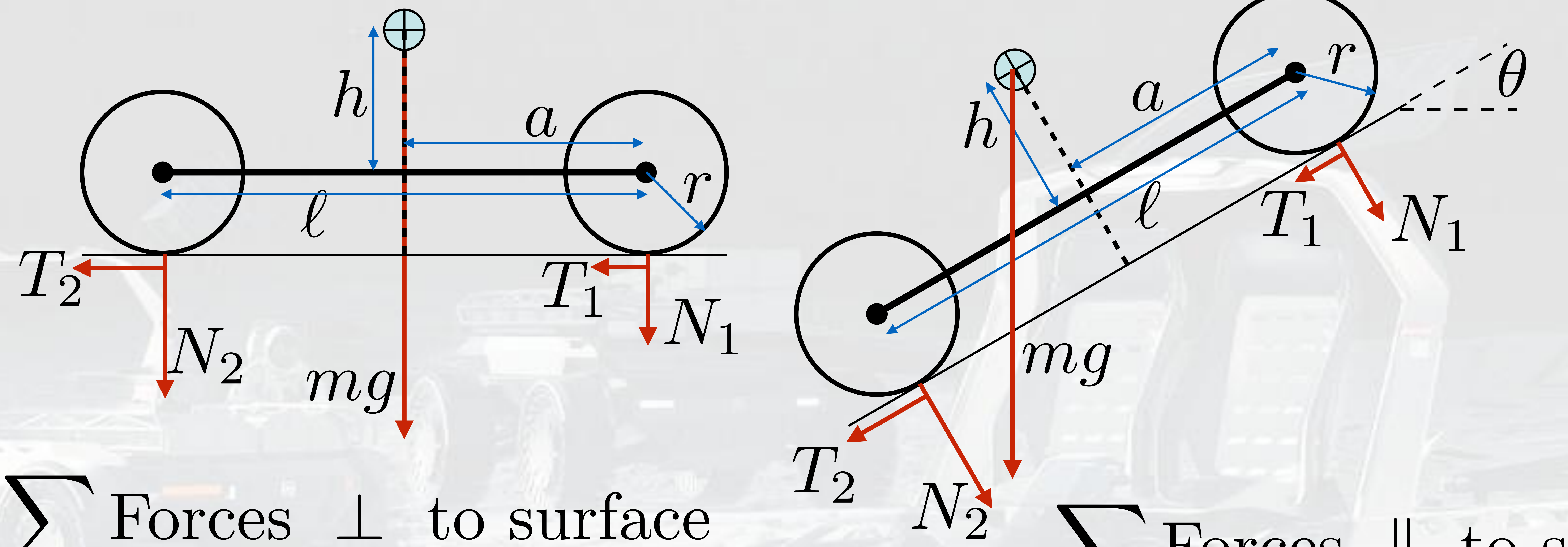
$$T = \frac{\tau}{r} = W \sin\theta$$

$$\mu W \cos\theta = W \sin\theta$$

$$\tan\theta = \mu$$

Assume $\tau > \mu_{limit} N r$ (friction limited, not torque limited)

Rover Climbing/Descending Slope



$$\sum \text{Forces} \perp \text{to surface}$$

$$N_1 + N_2 = mg\cos\theta$$

$$\sum \text{Forces} \parallel \text{to surface}$$

$$T_1 + T_2 = mg\sin\theta$$

$$\sum \text{Torques about rear axle}$$

$$T_1 r + T_2 r + N_1 \ell = mg\left[(\ell - a)\cos\theta - h\sin\theta\right]$$

Static Equilibrium Conditions

$$\sum \text{Forces} \perp \text{to surface} \qquad \sum \text{Forces} \parallel \text{to surface}$$

$$N_1 + N_2 = mg\cos\theta \qquad T_1 + T_2 = mg\sin\theta$$

$$\sum \text{Torques about rear axle}$$

$$T_1 r + T_2 r + N_1 \ell = mg\left[(\ell - a)\cos\theta - h\sin\theta\right]$$

Friction forces proportional to force into surface

$$\frac{T_1}{N_1} = \frac{T_2}{N_2}$$

Four equations, four unknowns

Static Equilibrium Solutions

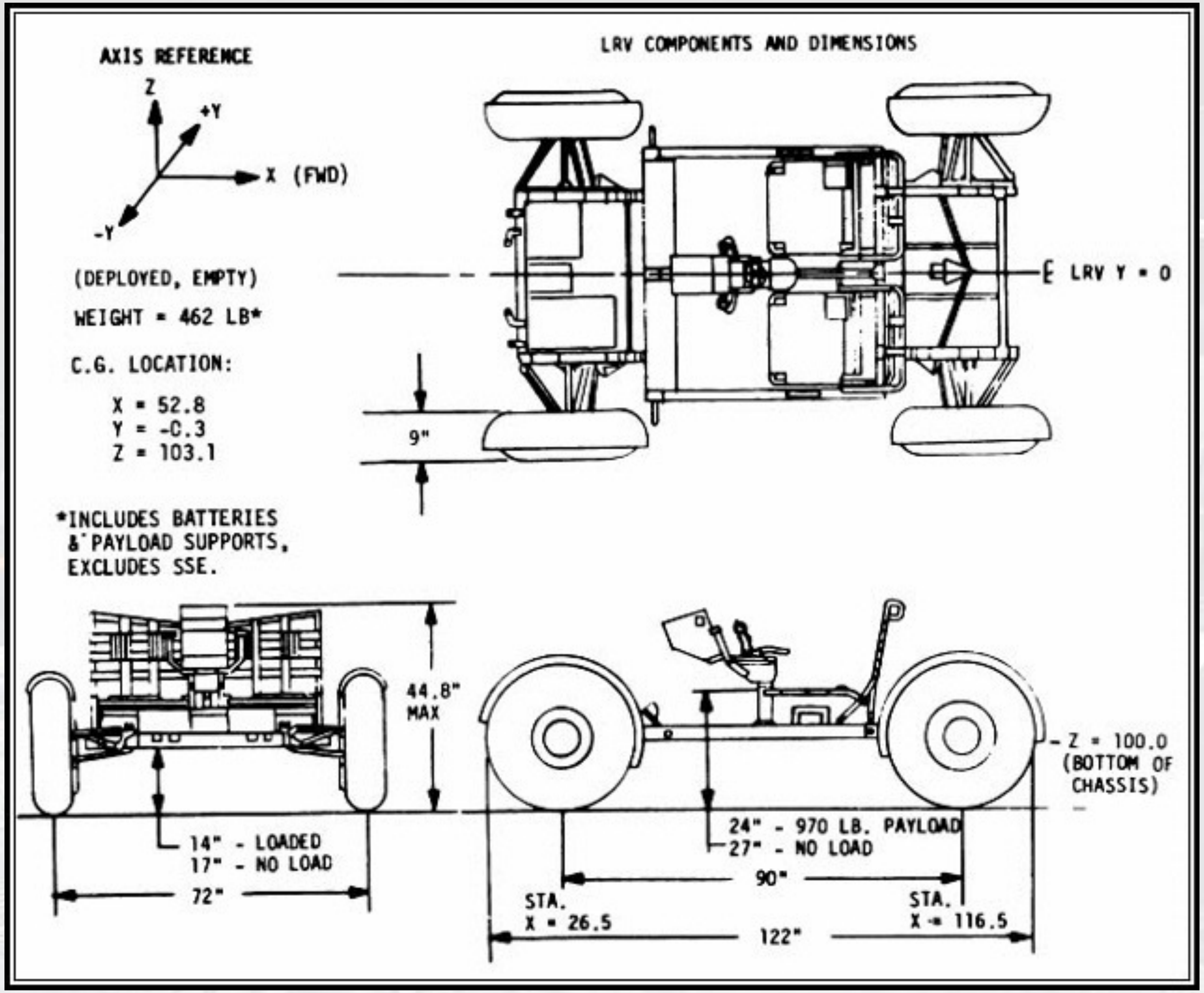
$$N_1 = mg\left[\left(1 - \frac{a}{\ell}\right)\cos\theta - \left(\frac{h}{\ell} + \frac{r}{\ell}\right)\sin\theta\right]$$

$$N_2 = mg\left[\frac{a}{\ell}\cos\theta + \left(\frac{h}{\ell} + \frac{r}{\ell}\right)\sin\theta\right]$$

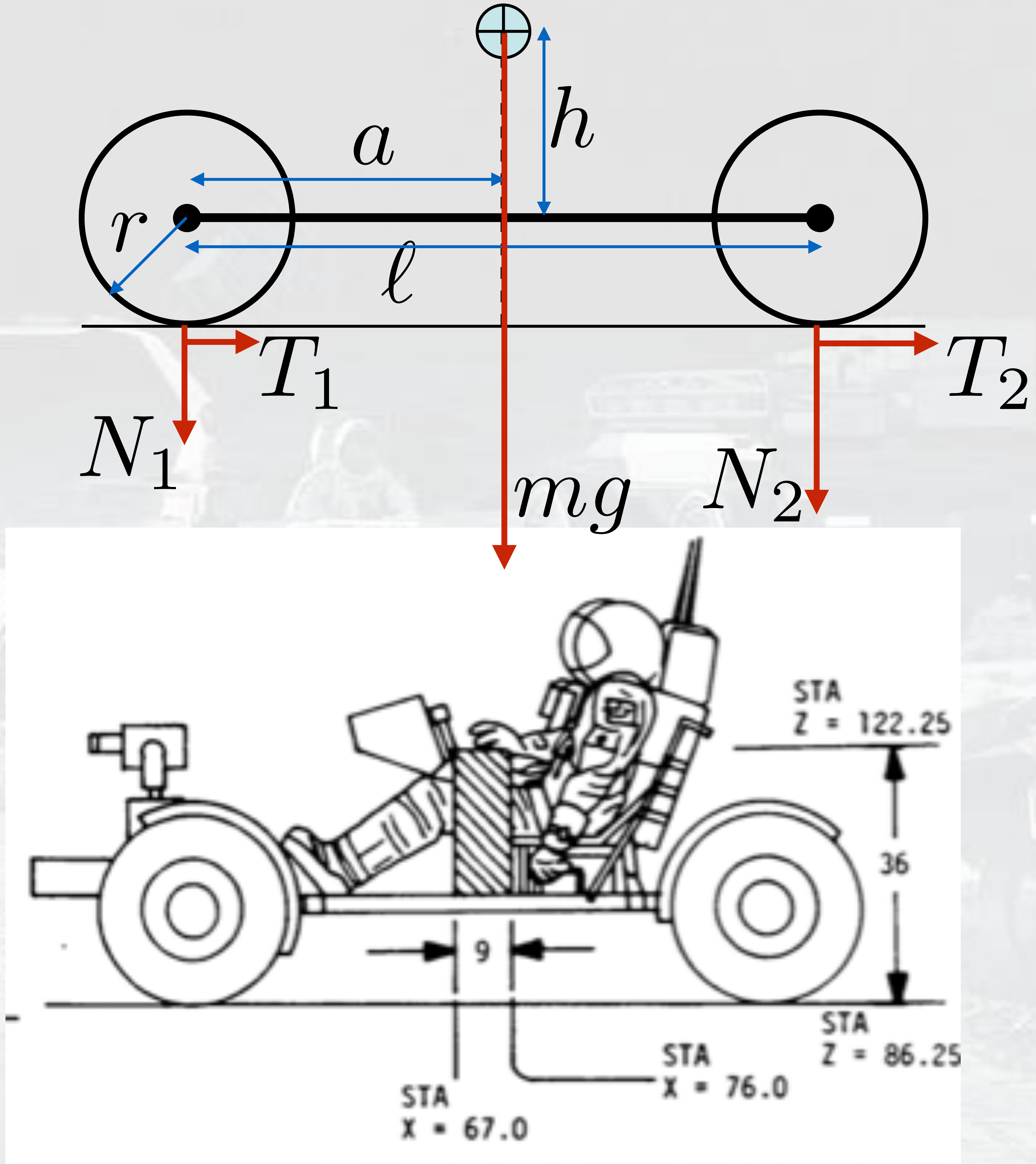
$$T_2 = \frac{N_2}{N_1 + N_2} mg\sin\theta$$

$$T_1 = \frac{N_1}{N_1 + N_2} mg\sin\theta$$

LRV Three-View and Dimensions

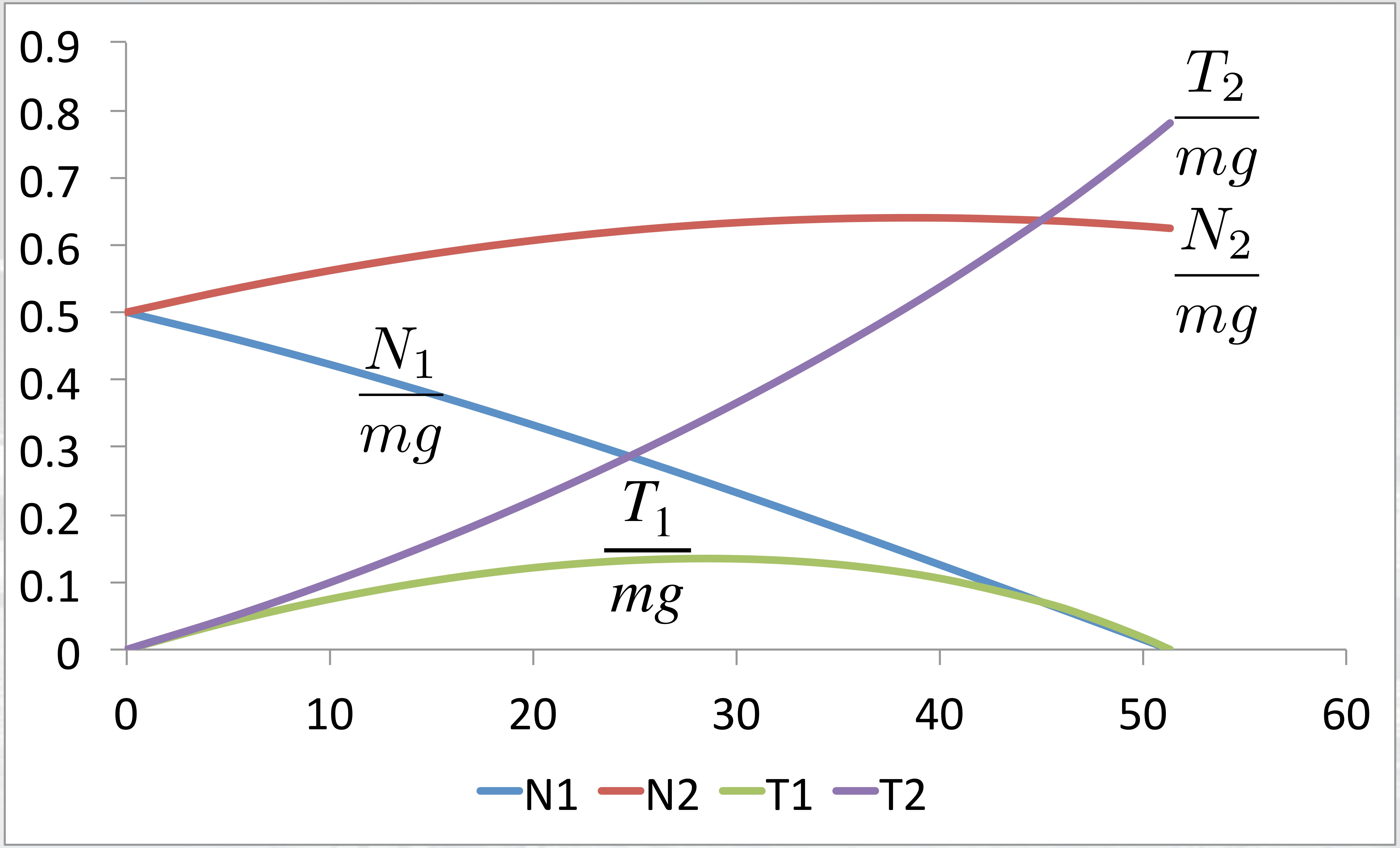


Loaded LRV Weight Distribution

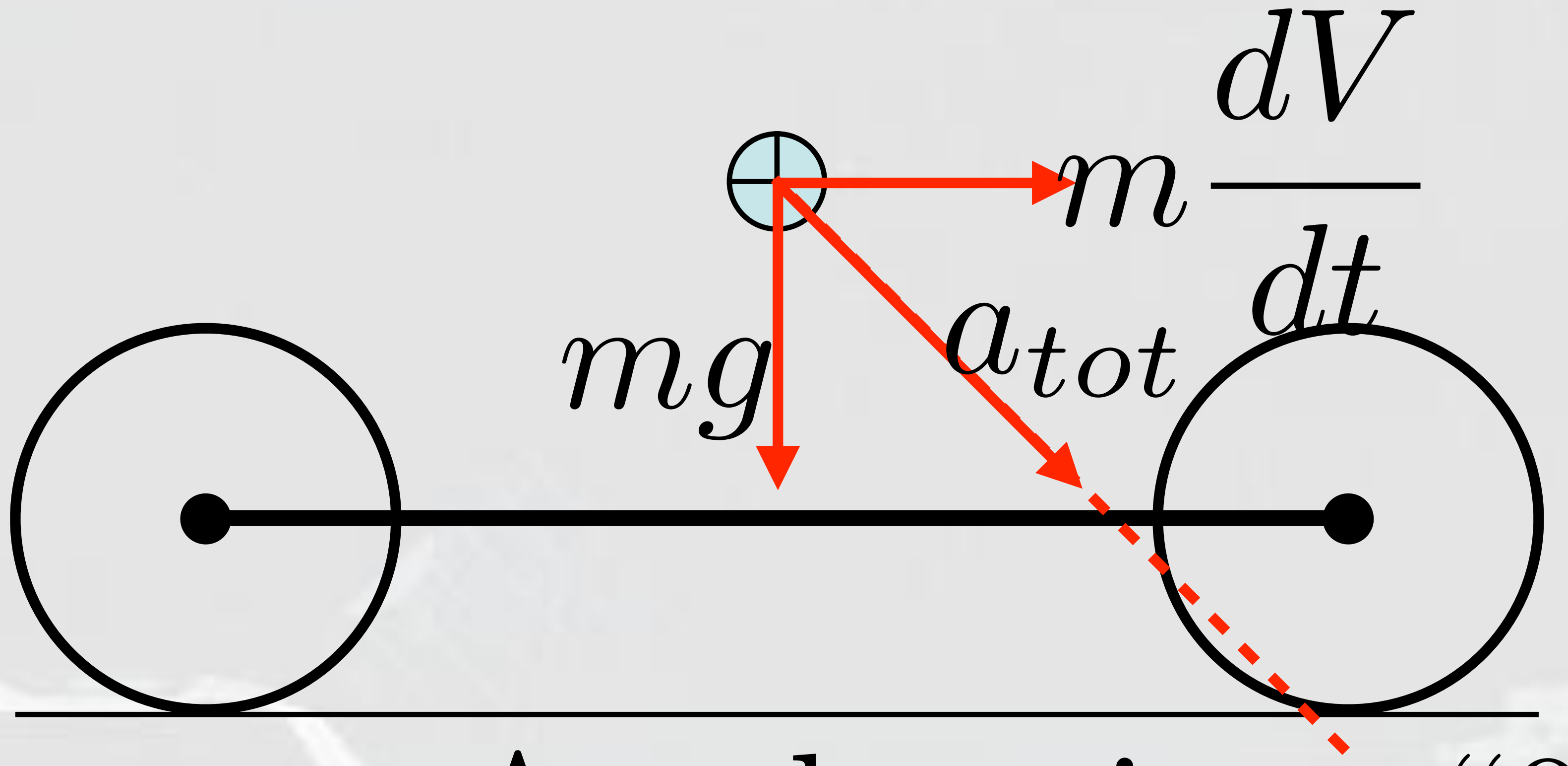


$$m = 690\ kg$$
$$mg = 1120\ N$$
$$\ell = 228.6\ cm$$
$$c = 183\ cm$$
$$r = 40.6\ cm$$
$$a = 114.3\ cm$$
$$h = 50.8\ cm$$
$$N_1 = N_2 = 279.8\ N$$
$$b = 22.9\ cm$$

Normal and Shear Wheel Force w/Slope



Acceleration



$$a_{tot} = \sqrt{g^2 + \left(\frac{dV}{dt}\right)^2}$$

Acceleration: "0-60 mph in X seconds"

60 mph = 88 ft/sec = 26.8 m/sec

Time (sec)	$\frac{dV}{dt}\left(\frac{m}{sec^2}\right)$	Total accel (g's)
7	**3.83**	**1.07**
6	**4.47**	**1.10**
5	**5.36**	**1.14**
4	**6.70**	**1.21**

Longitudinal Dynamic Conditions

$\sum$ Forces $\perp$ to surface

$$N_1 + N_2 = mg\cos\theta$$

$\sum$ Forces $||$ to surface

$$T_1 + T_2 = mg\sin\theta + ma_x$$

$\sum$ Torques about rear axle

$$T_1 r + T_2 r + N_1 \ell + ma_x h = mg[(\ell - a)\cos\theta - h\sin\theta]$$

Friction forces proportional to force into surface

$$\frac{T_1}{N_1} = \frac{T_2}{N_2}$$

Four equations, four unknowns

Longitudinal Dynamic Solutions

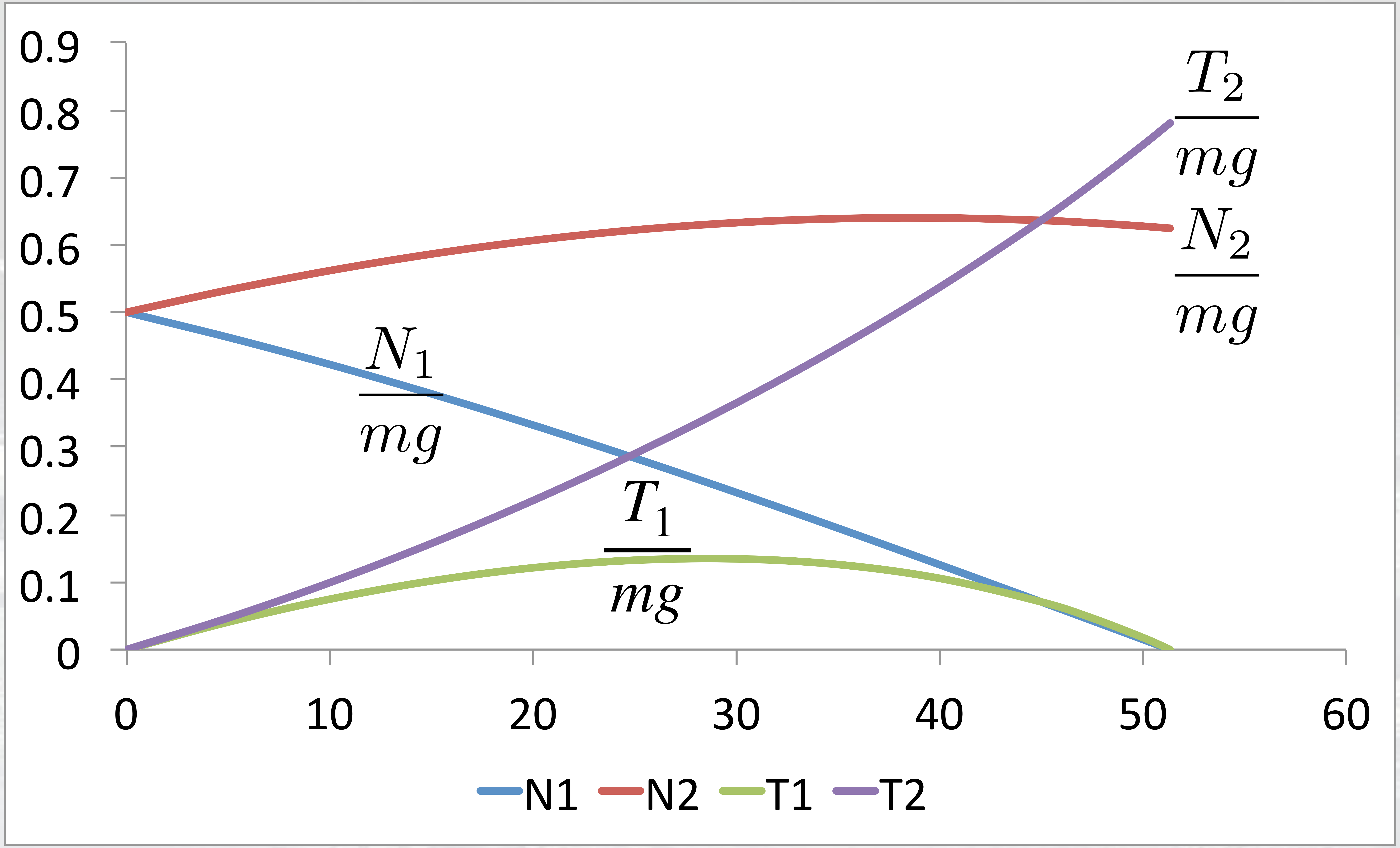
$$N_1 = mg\left[\left(1-\frac{a}{\ell}\right)\cos\theta - \left(\frac{h}{\ell}+\frac{r}{\ell}\right)\sin\theta - \left(\frac{h}{\ell}-\frac{r}{\ell}\right)\frac{a_x}{g}\right]$$

$$N_2 = mg\left[\frac{a}{\ell}\cos\theta + \left(\frac{h}{\ell}+\frac{r}{\ell}\right)\sin\theta + \left(\frac{h}{\ell}-\frac{r}{\ell}\right)\frac{a_x}{g}\right]$$

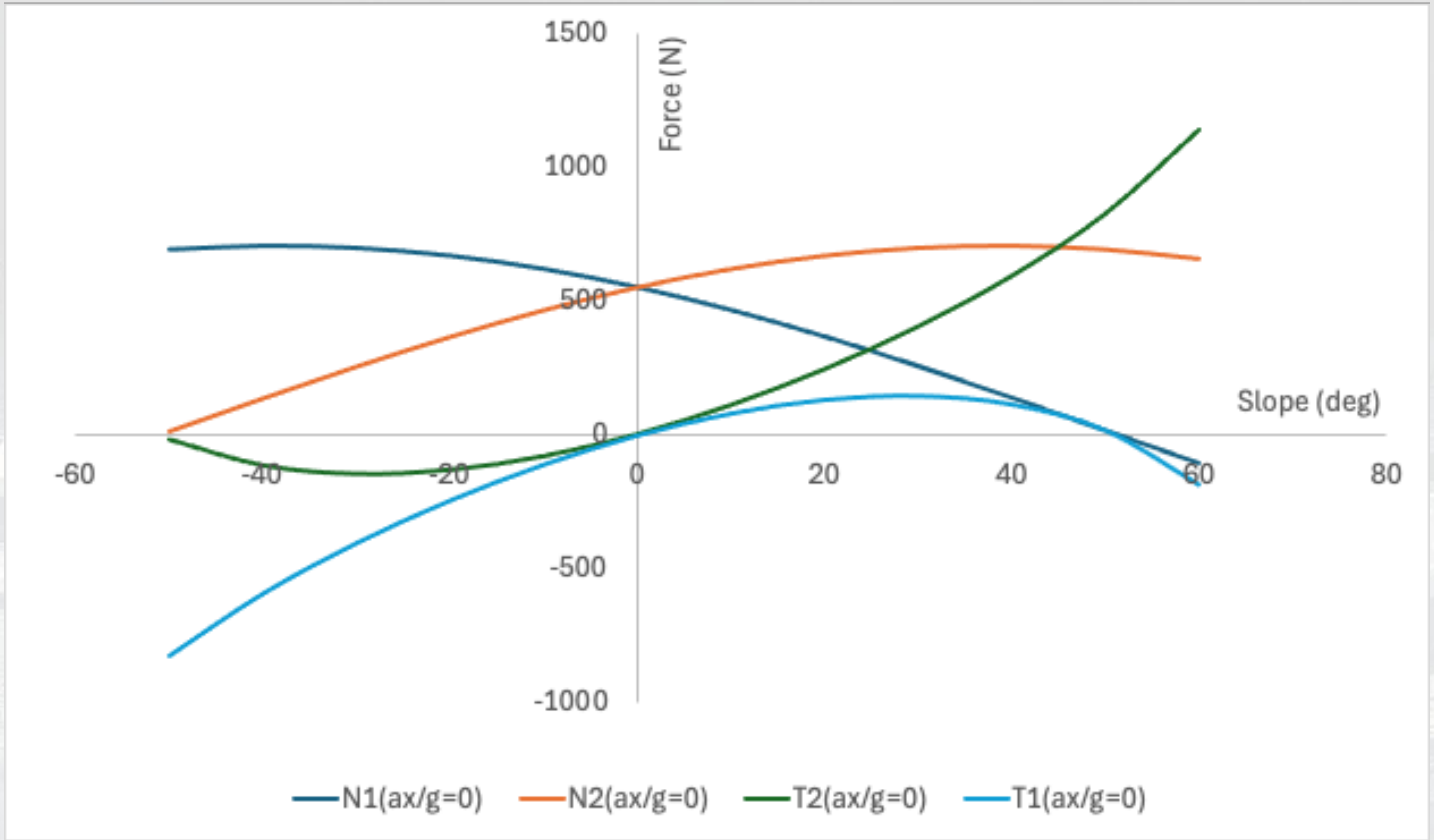
$$T_2 = \frac{N_2}{N_1+N_2}\left(mg\sin\theta + ma_x\right)$$

$$T_1 = \frac{N_1}{N_1+N_2}\left(mg\sin\theta + ma_x\right)$$

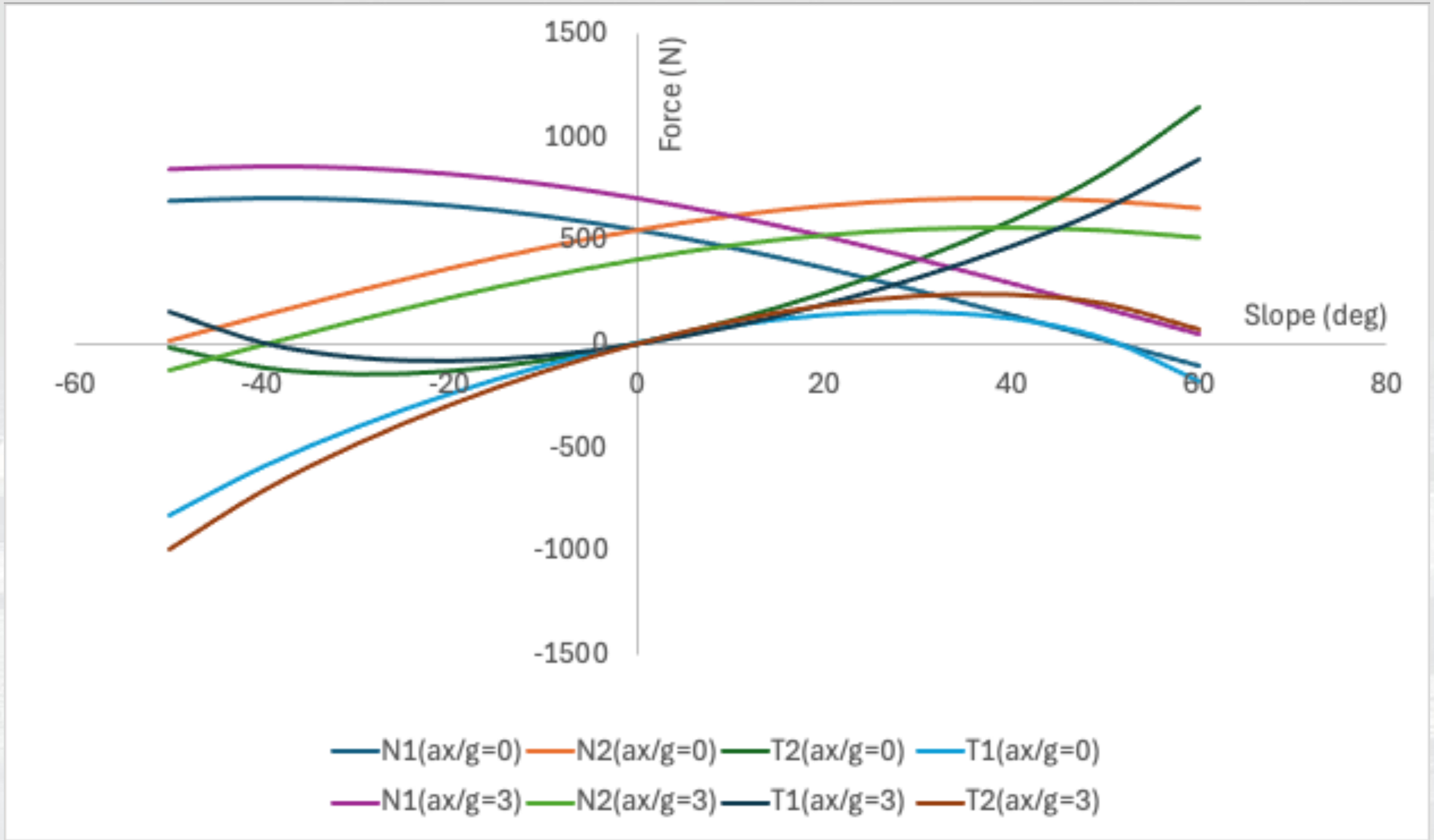
Normal and Shear Wheel Force w/Slope



Apollo LRV Full Slope Range, Static Case



Apollo LRV Slope Forces



Overturn Limits

$$\left(1-\frac{a}{\ell}\right)\cos\theta_{limit}-\left(\frac{h}{\ell}+\frac{r}{\ell}\right)\sin\theta_{limit}=\frac{h-r}{\ell}\frac{a_x}{g}$$

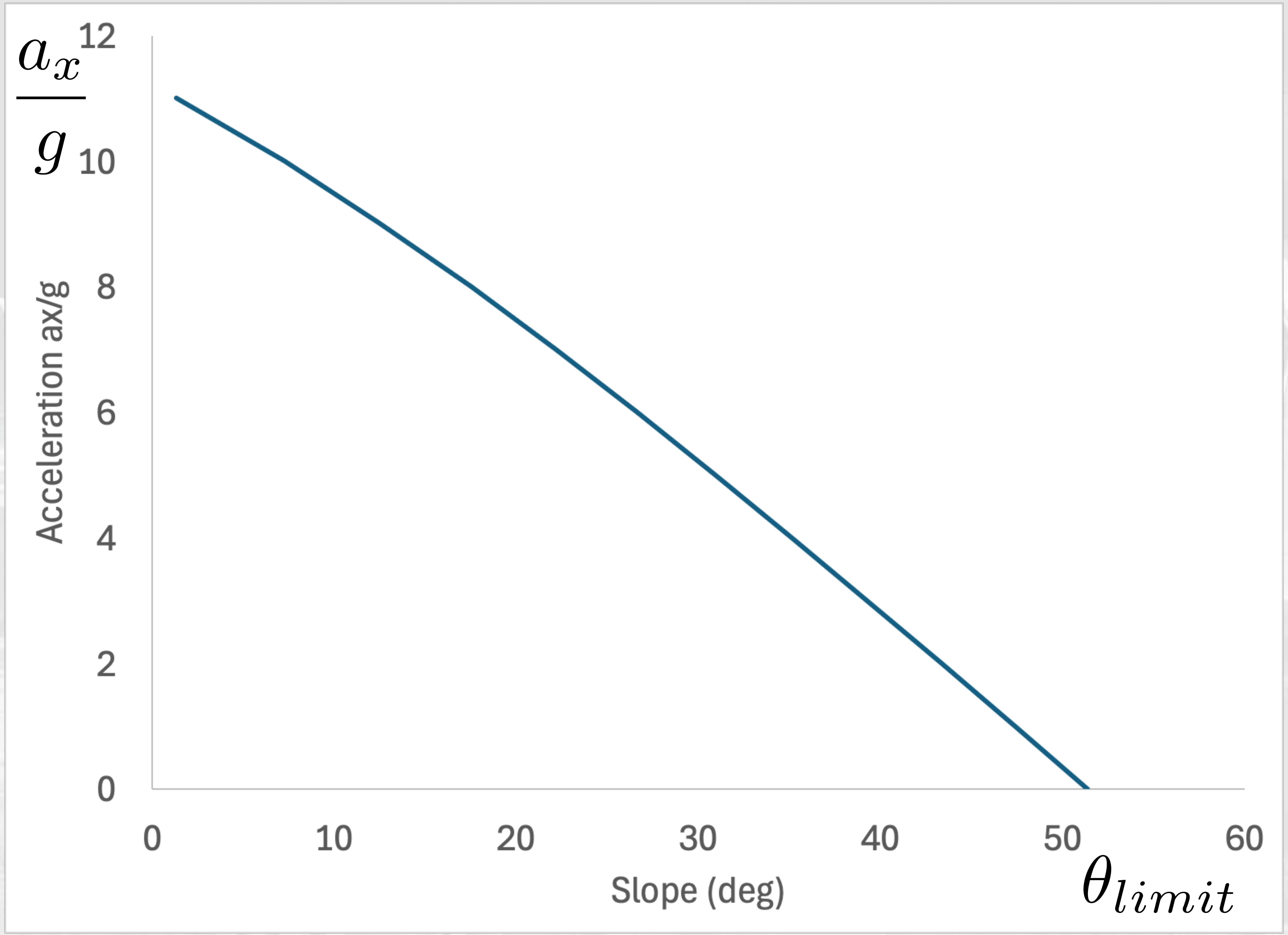
For the static case $(a_x = 0)$

$$\left(1-\frac{a}{\ell}\right)\cos\theta_{limit}=\left(\frac{h}{\ell}+\frac{r}{\ell}\right)\sin\theta_{limit}$$

$$\tan\theta_{limit}=\frac{\left(1-\frac{a}{\ell}\right)}{\left(\frac{h}{\ell}+\frac{r}{\ell}\right)}=\frac{\ell-a}{h+r}$$

Limiting accel on flat ground $\Rightarrow a_{x,limit}=\dfrac{g(\ell-a)}{r+h}$

Limiting Slope Under Acceleration



Deceleration

Deceleration: "Leave one car length per 10 mph"

$$V = 60\ mph,\ \ell = 4m \implies 24\text{ m to stop}$$

$$s = \frac{1}{2}at^2;\ v = at \implies s = \frac{1}{2}\frac{v^2}{a} \implies a = \frac{1}{2}\frac{v^2}{s}$$

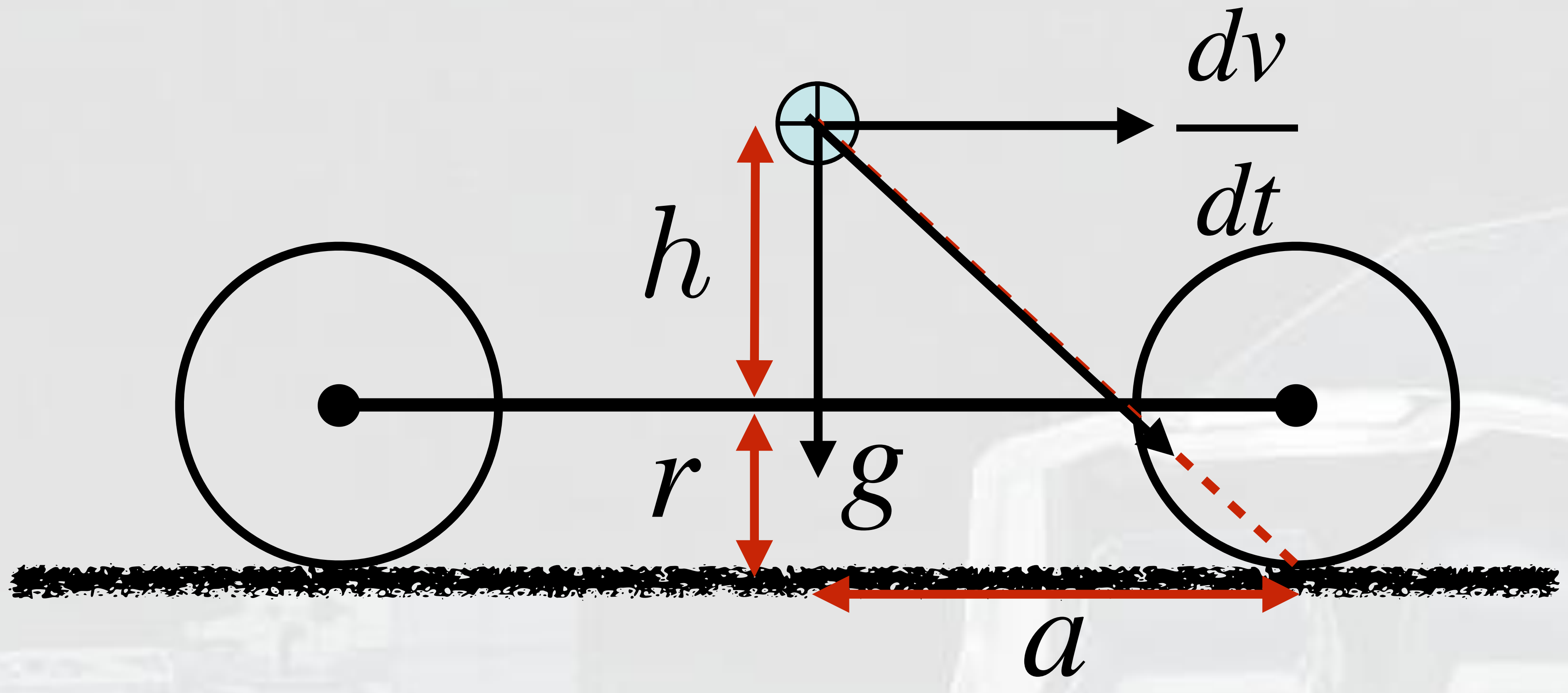
$$a_{decel} = \frac{(26.8\ m/sec)^2}{2(24m)} = 14.96\ \frac{m}{sec}$$

$$\text{for Earth, } g = 9.8\ \frac{m}{sec^2} \implies a_{tot} = 1.84\ g$$

Deceleration Stability Limits (Level Ground)

Distance – CG to front wheel

	$v = 60\ mph$ $\frac{dv}{dt} = 15 \frac{m}{sec^2}$ $\left(\frac{a}{h+r}\right)_{limit}$	$v = 10\ mph$ $\frac{dv}{dt} = 2.5 \frac{m}{sec^2}$ $\left(\frac{a}{h+r}\right)_{limit}$
Earth	1.53	0.255
Mars	4.03	0.658
Moon	9.38	1.56



$$\frac{a}{h+r} = \frac{1}{g}\frac{dv}{dt}$$

Less stable in lower gravity

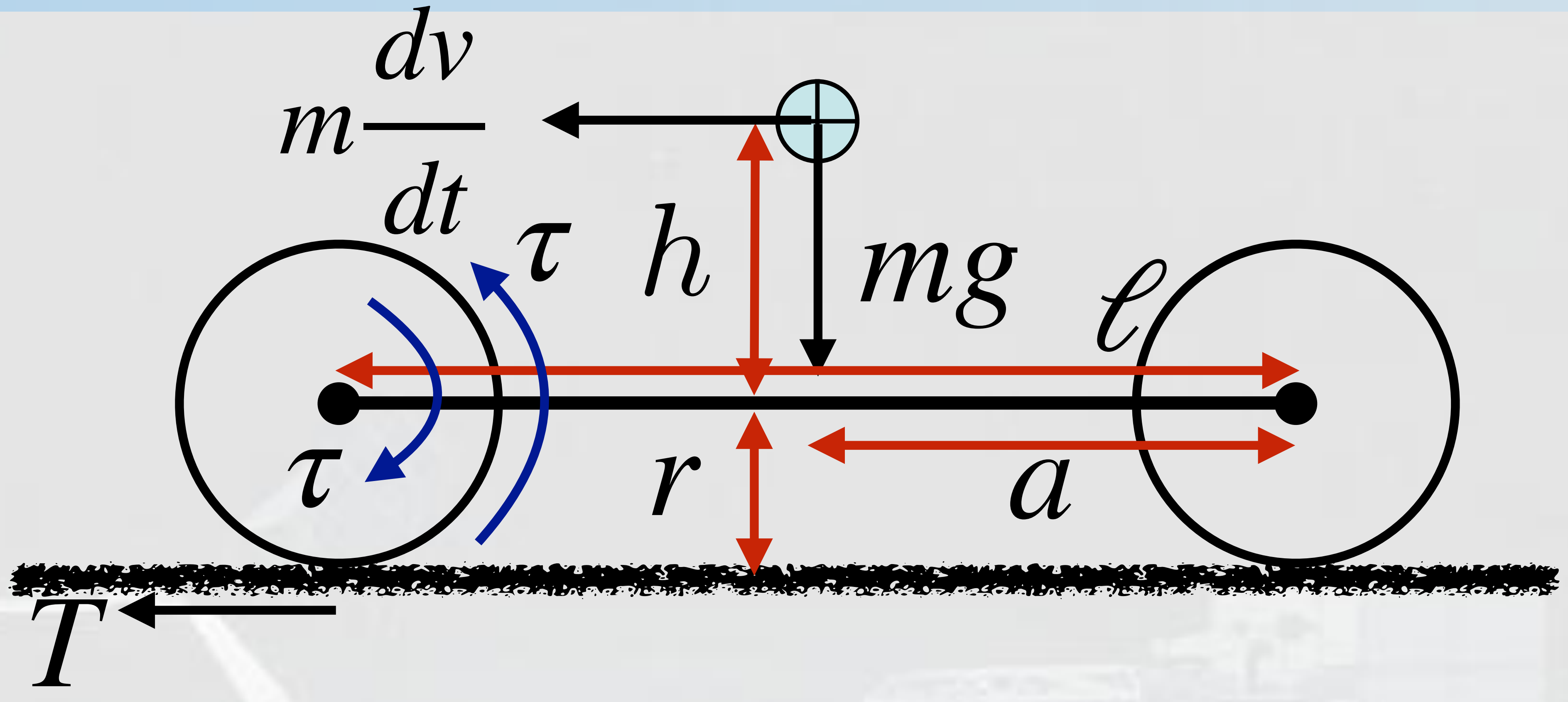
60 mph panic stop on Earth=
10 mph panic stop on Moon

$$\text{For given } \frac{a}{h+r}: \quad \left.\frac{a}{h+r}\right|_{limit} \propto g$$

$$S_{min} \propto \frac{1}{g}$$

60 mph panic stop on Moon requires 147 m

Inertially-Limited Acceleration



$$\tau = Tr$$

$$T = m\frac{dv}{dt}$$

$$\tau = rm\frac{dv}{dt}$$

$\sum$ moments about rear axle at torque limit

$$\tau + hm\frac{dv}{dt} = mg(\ell - a)$$

$$rm\frac{dv}{dt} + hm\frac{dv}{dt} = mg(\ell - a)$$

$$\left.\frac{dv}{dt}\right|_{\lim} = g\frac{\ell - a}{r + h}$$

Inertially-Limited Acceleration

Lunar Roving Vehicle:

$$\ell = 2.29\ m$$
$$r = 0.41\ m$$
$$h = 0.51\ m$$
$$a = 1.14\ m$$

$$\left.\frac{dv}{dt}\right|_{\lim} = 1.25g$$
$$= 2.0\ \frac{m}{sec^2}\ \text{(Moon)}$$
$$= 12.3\ \frac{m}{sec^2}\ \text{(Earth)}$$

$$\text{Moon: } 0 \rightarrow 10\ kph\ (2.78\ m/sec) \Rightarrow 1.4\ sec$$
$$0 \rightarrow 60\ mph\ (26.8\ m/sec) \Rightarrow 13.4\ sec$$