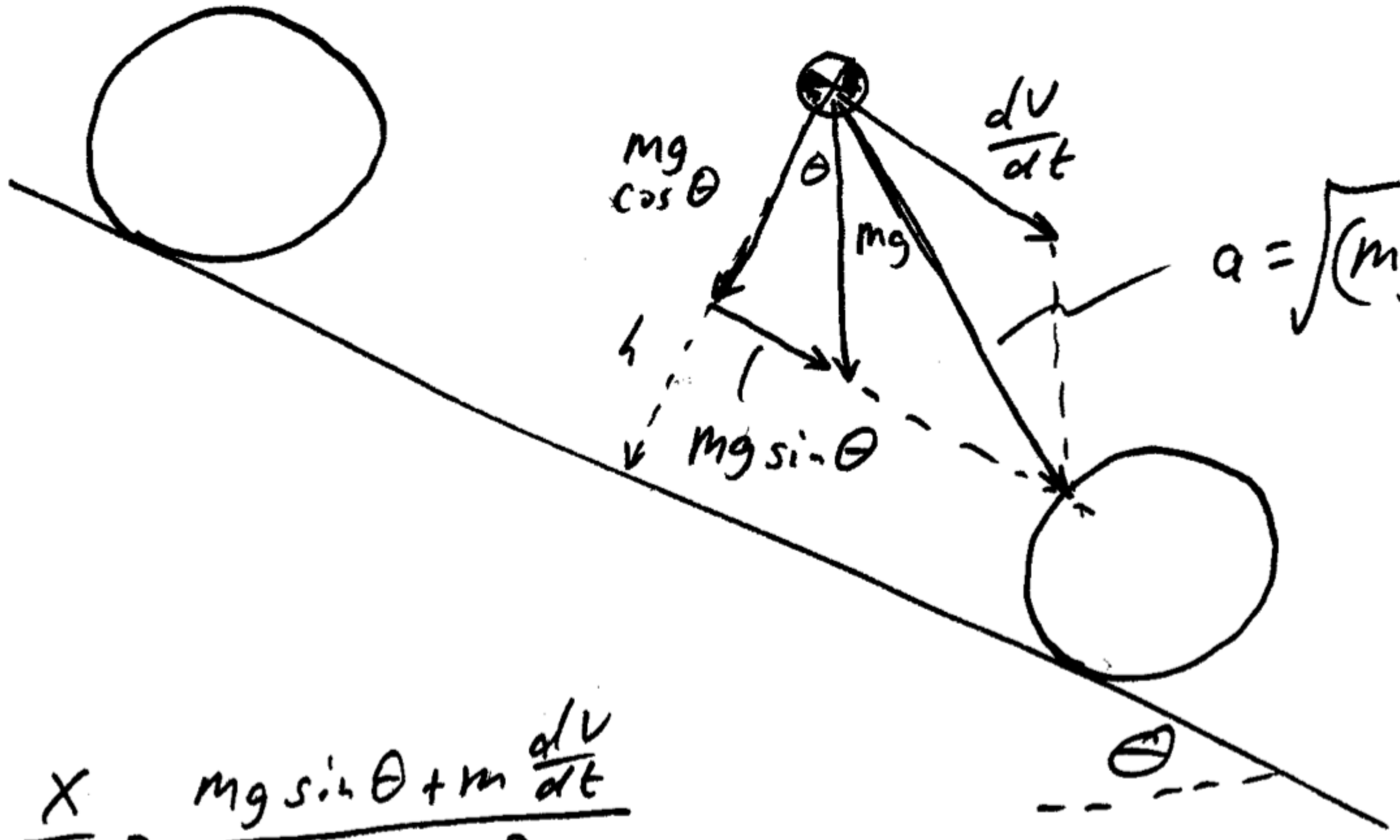
Slopes and Static Stability

- Stability across and along slopes
- Forces and torques on wheels
- Acceleration/deceleration
- Turning
- Hitting obstacles
- Rigid suspensions and obstacles

Accel/Decel on Slopes



$$a = \sqrt{(mg\cos\theta)^2 + \left(mg\sin\theta + m\frac{dV}{dt}\right)^2}$$

$$\frac{X}{h} = \frac{mg\sin\theta + m\frac{dV}{dt}}{mg\cos\theta}$$

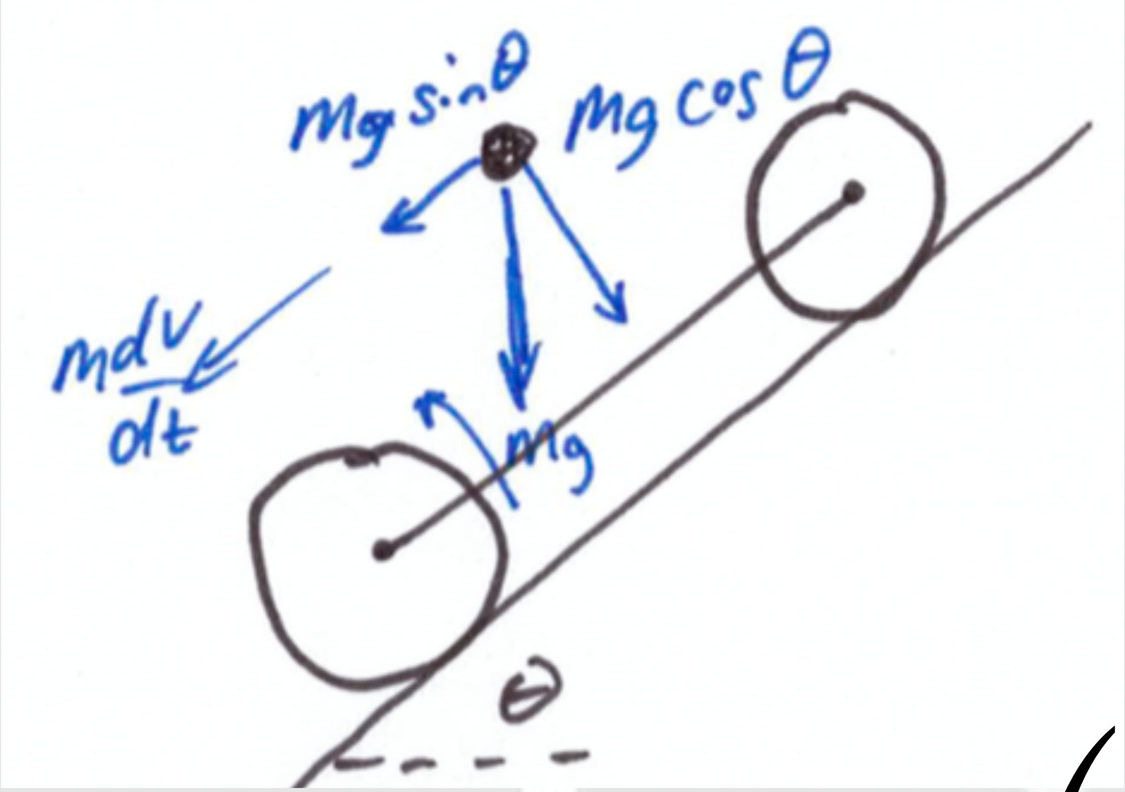
$$= \frac{g\sin\theta + \frac{dV}{dt}}{g\cos\theta} \qquad \text{if } \theta = 0 \Rightarrow \frac{X}{h} = \frac{\frac{dV}{dh}}{g} \ \checkmark$$

$$\frac{X}{h} g\cos\theta = g\sin\theta + \frac{dV}{dt} \Rightarrow \frac{dV}{dt} = g\left(\frac{X}{h}\cos\theta - \sin\theta\right)$$

Moon: $\frac{X}{h} = 1$

$\frac{dV}{dt}\Big\vert_{limit}$	θ	S_{stop} (10 km/hr)
1.6 m/sec²	0	2.42 m
1.3 m/sec²	10°	3.0 m
0.96 m/sec²	20°	4.0 m
0.59 m/sec²	30°	6.5 m

Acceleration Limits Up Slopes



$$\frac{\ell - a}{h + r} = \frac{mg\sin\theta + m\left.\frac{dv}{dt}\right|_{\text{lim}}}{mg\cos\theta}$$

$$\left(\frac{\ell - a}{h + r}\right) g\cos\theta = g\sin\theta + \left.\frac{dv}{dt}\right|_{\text{lim}}$$

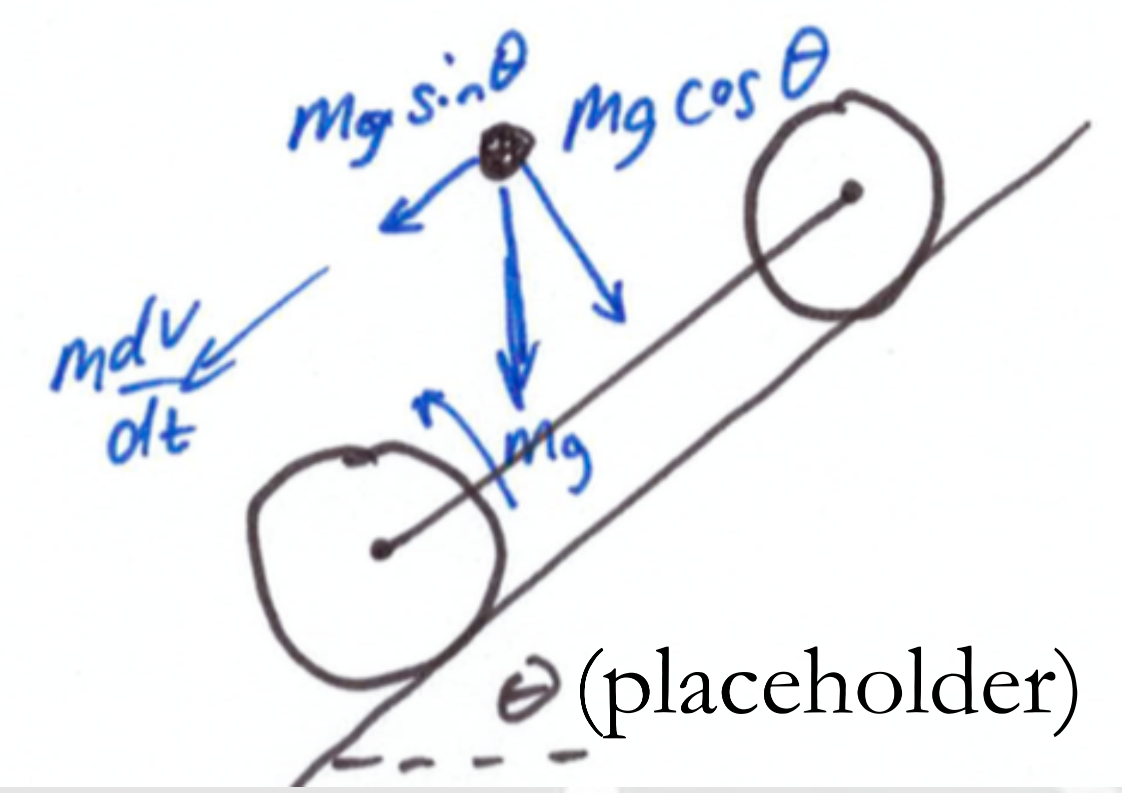
$$\text{LRV: } \frac{\ell - a}{h + r} = 1.251$$

$$\left.\frac{dv}{dt}\right|_{\text{lim}} = g\left[\left(\frac{\ell - a}{h + r}\right)\cos\theta - \sin\theta\right]$$

Slope	$\left.\frac{dv}{dt}\right|_{\text{lim}} \left(\frac{m}{sec^2}\right)$
0°	1.932
10°	1.635
20°	1.288
30°	0.901

Limiting slope for acceleration = 51.4°

Deceleration Limits Up Slopes



$$\frac{a}{h+r} = \frac{-\left(mg\sin\theta + m\left.\frac{dv}{dt}\right|_{\lim}\right)}{mg\cos\theta}$$

$$\left(\frac{a}{h+r}\right) g\cos\theta = -g\sin\theta - \left.\frac{dv}{dt}\right|_{\lim}$$

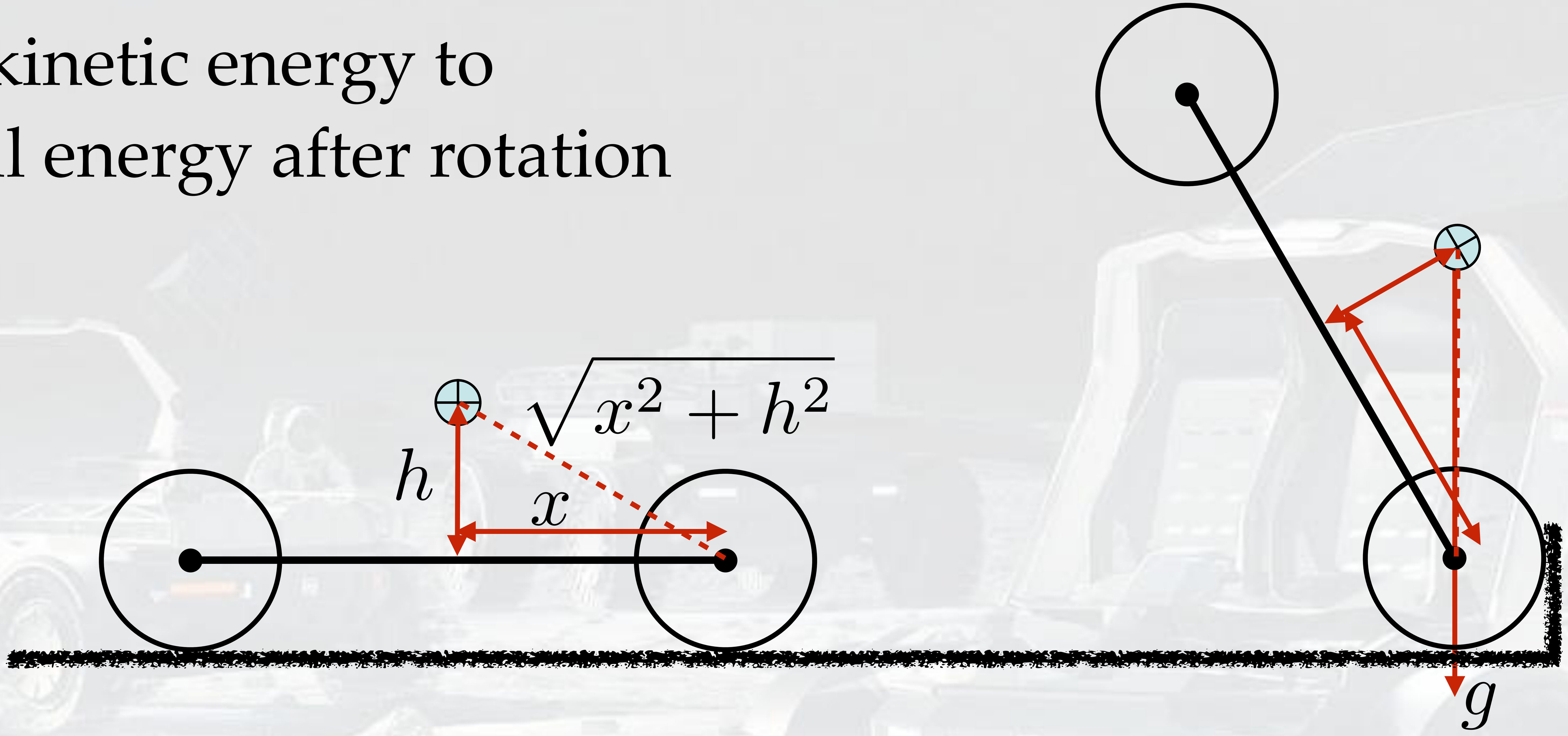
LRV: $\frac{a}{h+r} = 1.251$

$$\left.\frac{dv}{dt}\right|_{\lim} = -g\left[\left(\frac{a}{h+r}\right)\cos\theta + \sin\theta\right]$$

Slope	$\left.\frac{dv}{dt}\right\|_{\lim} \left(\frac{m}{sec^2}\right)$
0°	-1.932
10°	-2.172
20°	-2.345
30°	-2.446

Impulsive Pitch-Over Criteria

- Equate kinetic energy to potential energy after rotation



- Point mass assumption $\implies$ conservative estimate

Impulsive Pitch-Over Criteria

Non-pitchover criterion: $\Delta K.E. \leq \Delta P.E.$

$$\frac{1}{2}mV^2 \leq mg\left(\sqrt{x^2+h^2}-h\right)$$

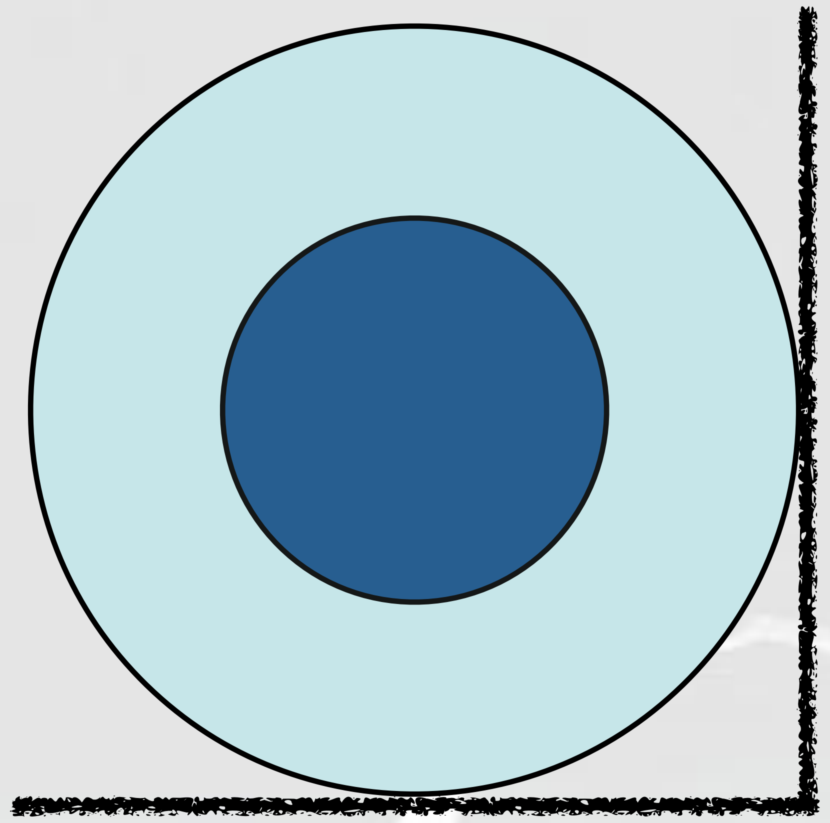
$$\frac{V^2}{2g}+h \geq \sqrt{x^2+h^2}$$

$$V^2 \leq 2g\left(\sqrt{x^2+h^2}-h\right)$$

$$V_{limit} \leq \sqrt{2g\left(\sqrt{x^2+h^2}-h\right)}$$

$$x=2m,\;\; h=0.5m \Longrightarrow V_{lim,Earth}=5.53\frac{m}{sec},\;\; V_{lim,Moon}=2.24\frac{m}{sec}$$

Hitting a Wall



$$s_{stop} \sim \text{tire thickness} \approx 0.3\ m$$

Example: pressurized rover @ 4000 kg

$$10\ km/hr = 2.78\ m/sec\ (\approx 6\ mph)$$

$$\text{Kinetic Energy } KE = \frac{1}{2}mv^2 = 15.4\ KJ$$

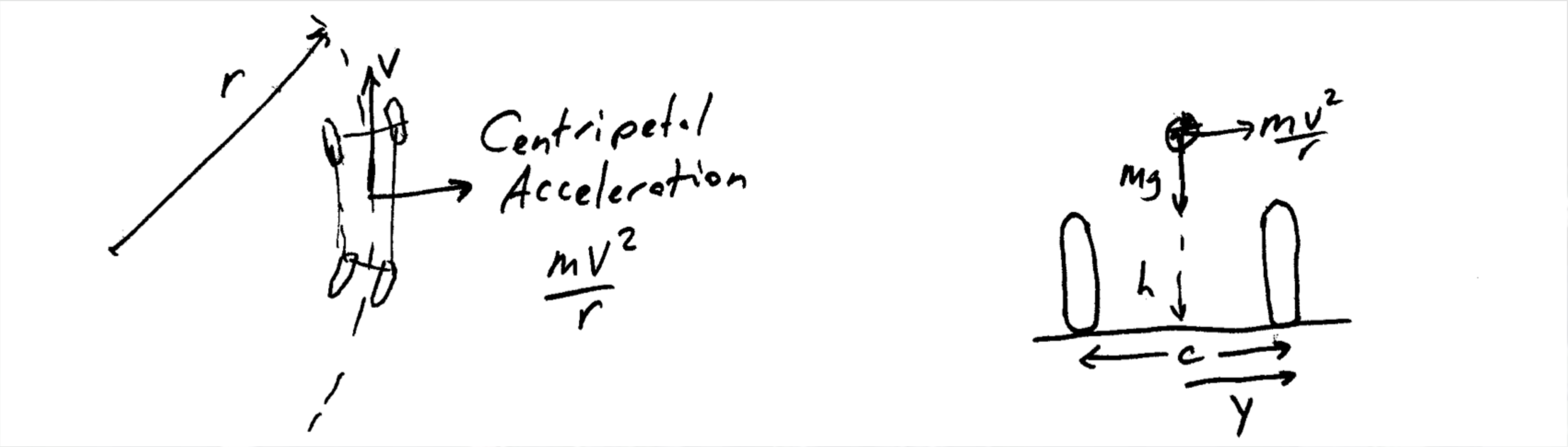
$$a = \frac{v^2}{s} = 12.9\ m/sec^2$$

$$\text{Moon: } \frac{x}{h} = \frac{\frac{dv}{dt}}{g} = \frac{12.9}{1.6} = 8.1$$

Assuming constant deceleration,

$$(4000\ kg)\left(12.9\ m/sec^2\right) = 51.6\ kN\ (= 11{,}600\ lbs)$$

Lateral Stability



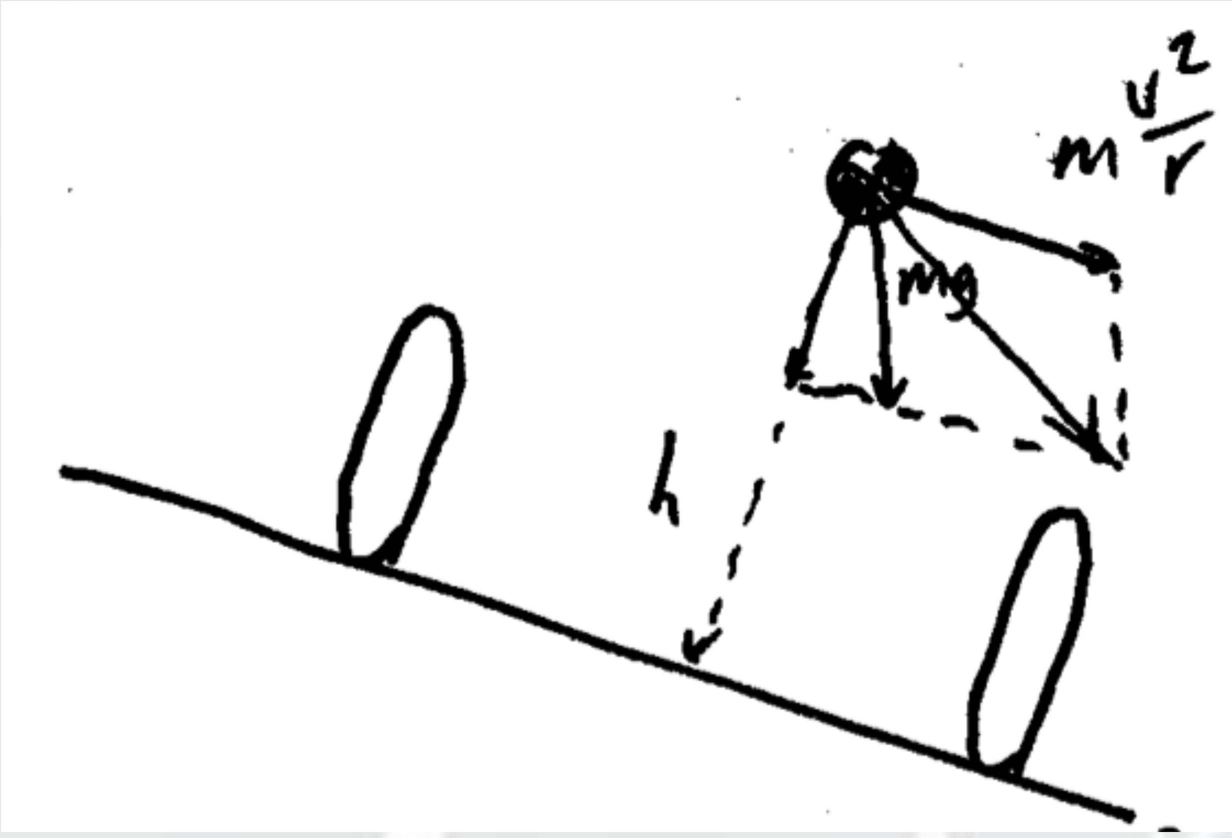
$$\frac{y}{h} = \frac{m\frac{v^2}{r}}{mg} = \frac{v^2}{gr}$$

$$\text{set } \frac{y}{h}, g \implies \frac{v^2}{r} = \text{constant} \left(= g\frac{y}{h} \right)$$

$$\text{Doubling } V \implies \text{increasing } r \times 4 \text{ !!}$$

$$r \propto \frac{1}{g} \text{ at the same velocity}$$

Turning on a Slope – Minimum Radius



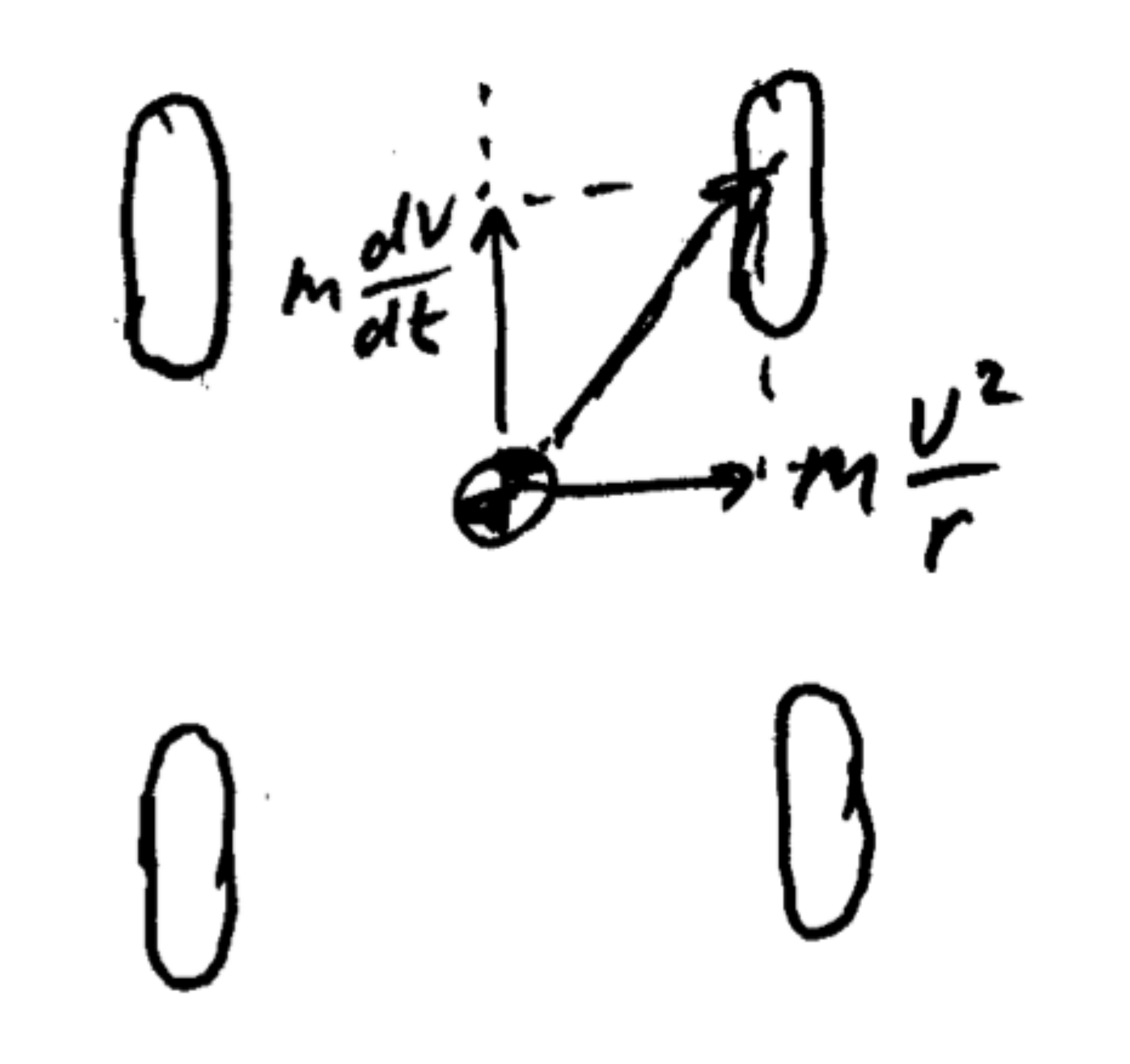
Worst case is driving cross-slope, turning upslope

$$\frac{y}{h} = \frac{mg\sin\theta + m\frac{v^2}{r}}{mg\cos\theta} = \frac{\sin\theta + \frac{v^2}{gr}}{\cos\theta}$$

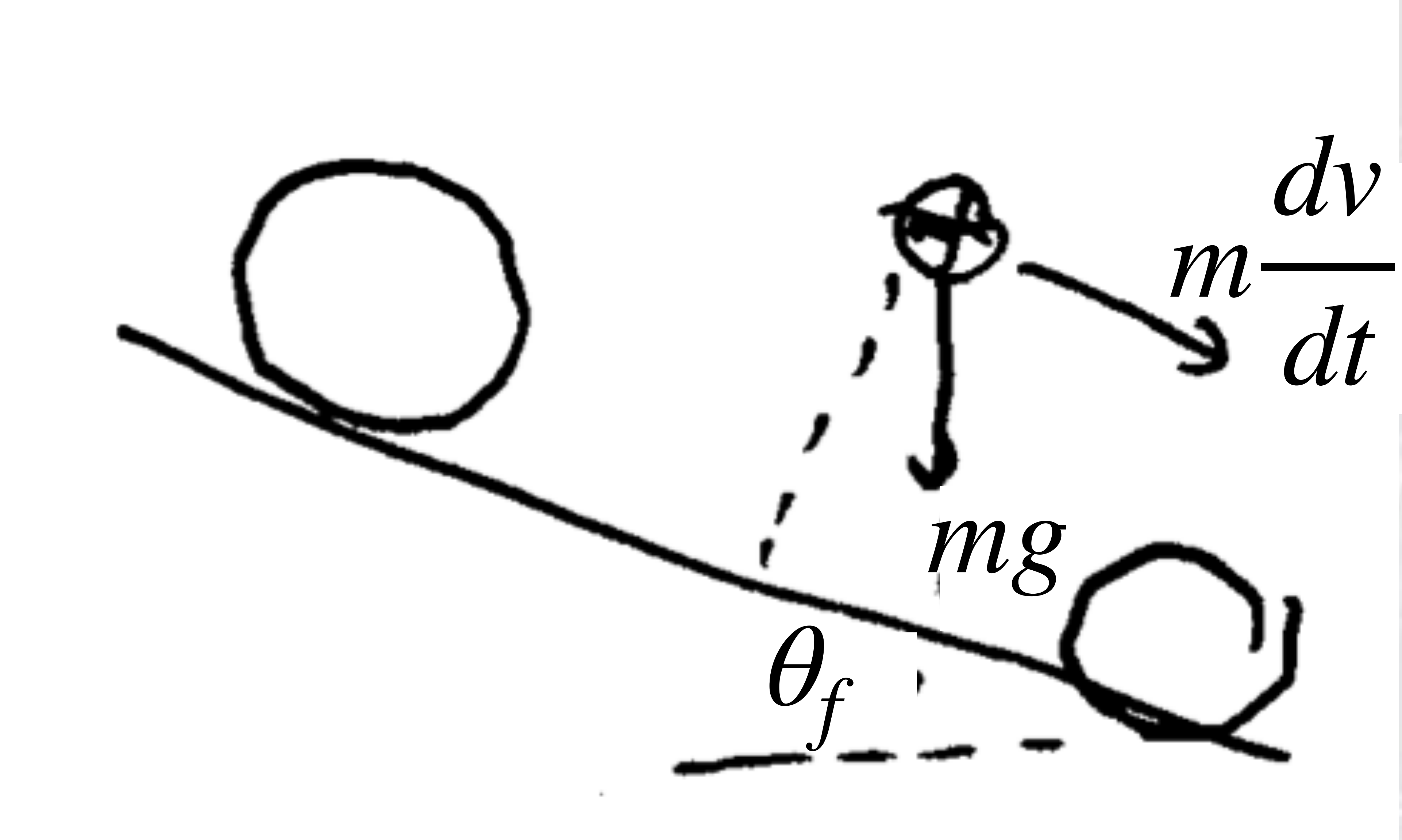
$$\frac{y}{h}\cos\theta - \sin\theta = \frac{v^2}{gr}$$

$$r = \frac{v^2}{g}\left[\frac{y}{h}\cos\theta - \sin\theta\right]^{-1}$$

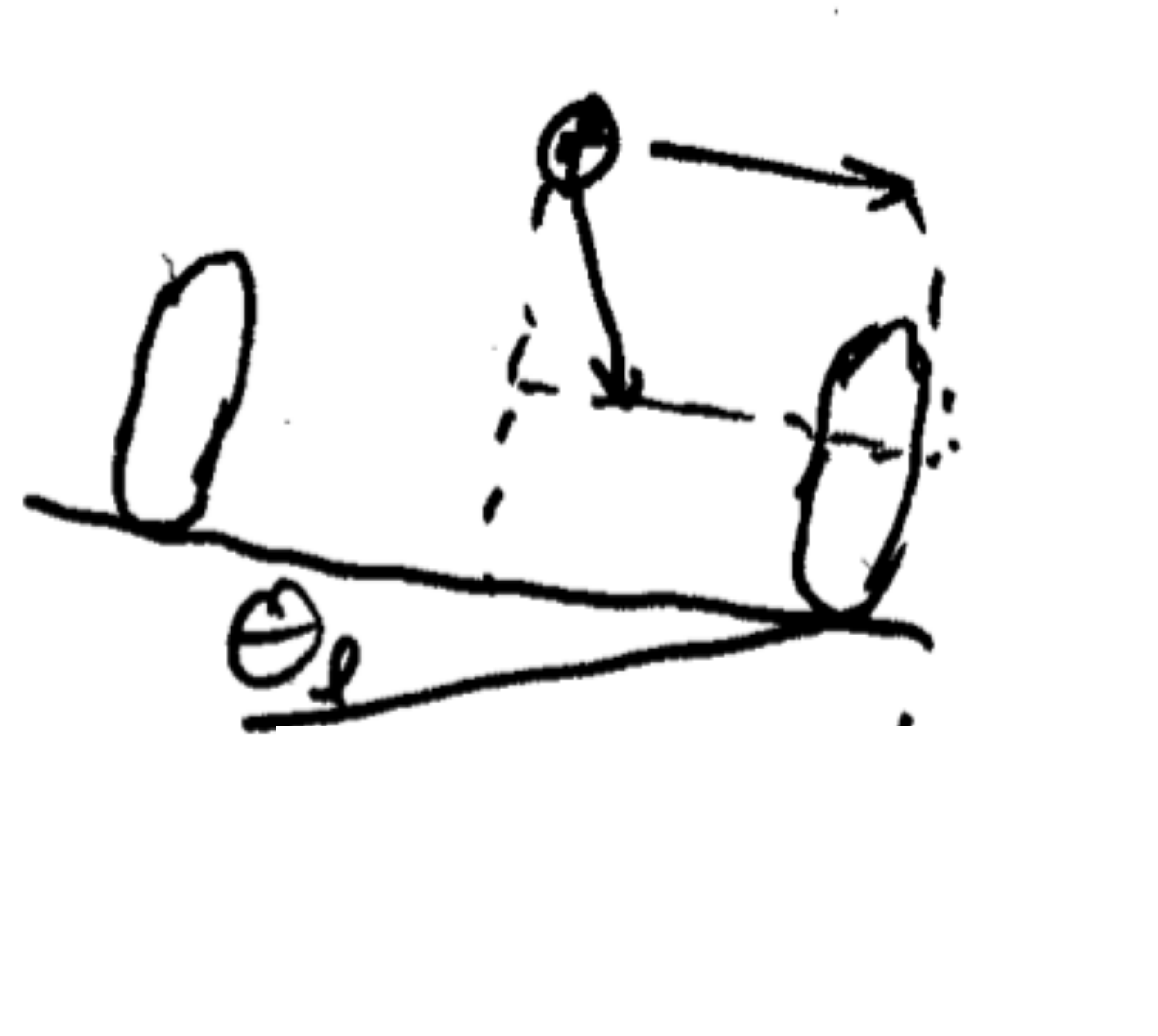
Composite Stability



Top view



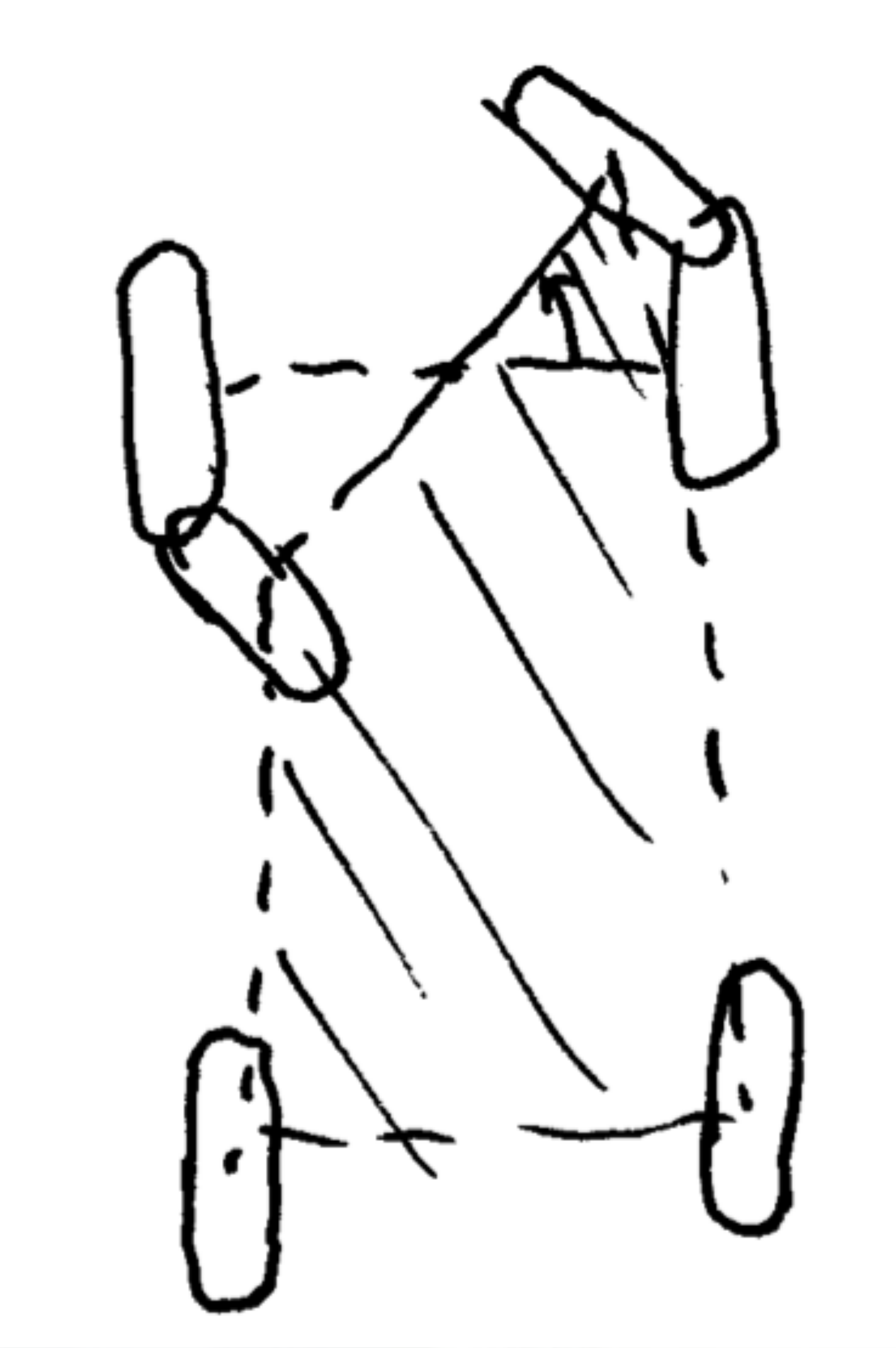
Fore-aft slope



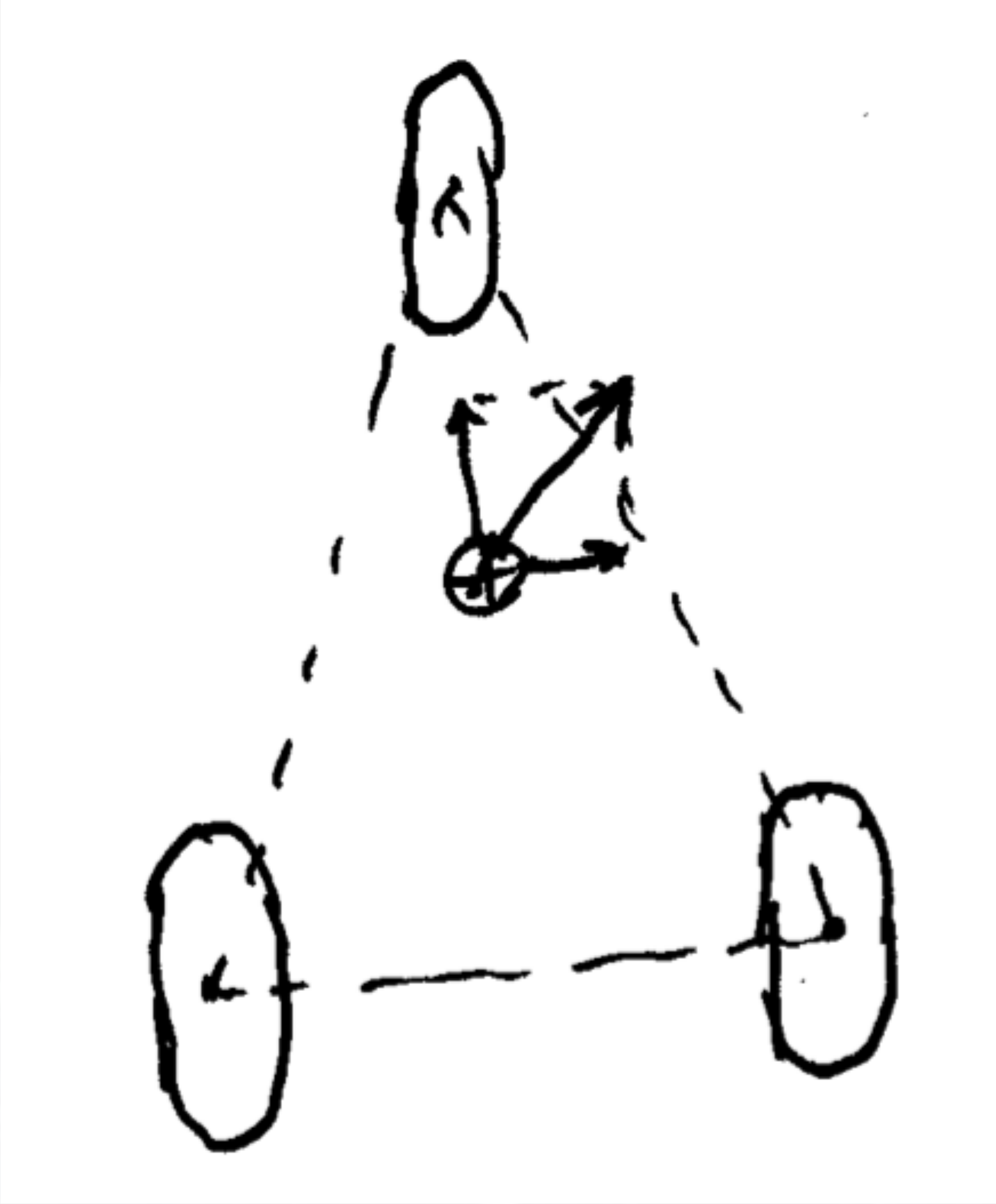
Lateral slope

For conventional configurations (rectangular stability region), lateral and longitudinal stability are decoupled

Stability of Alternative Configurations



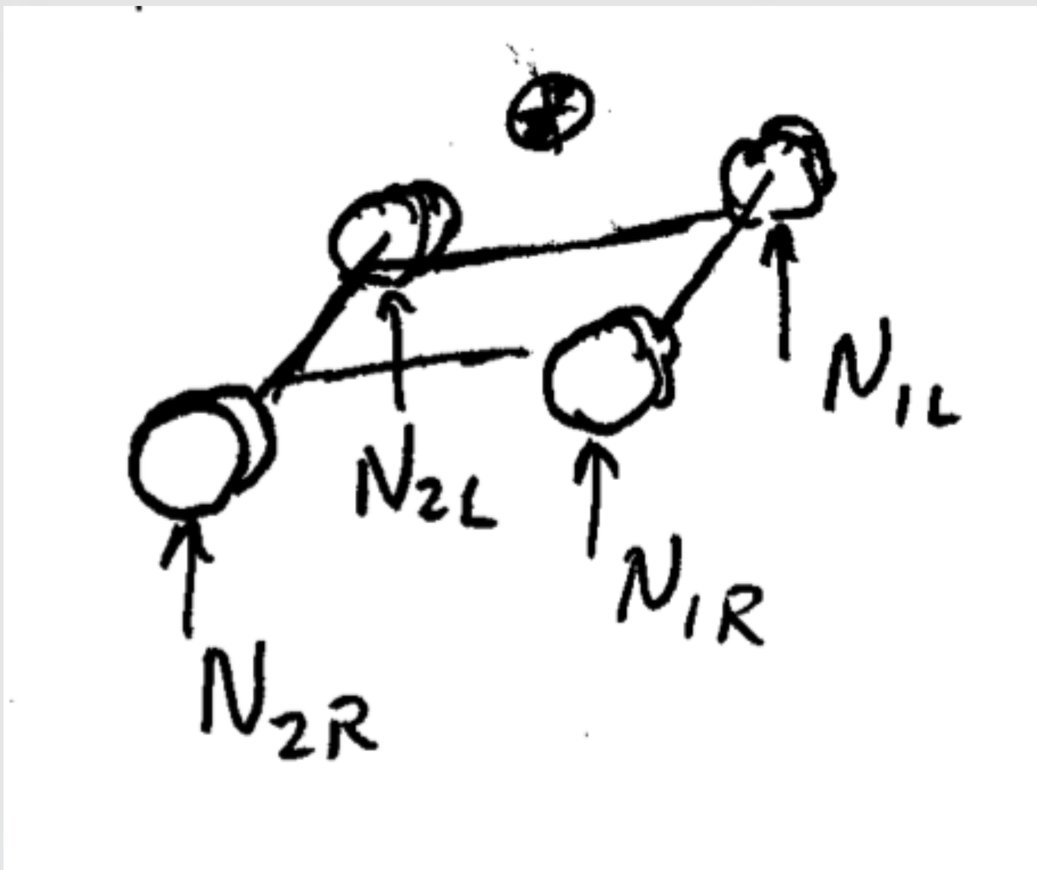
Wagon Steering



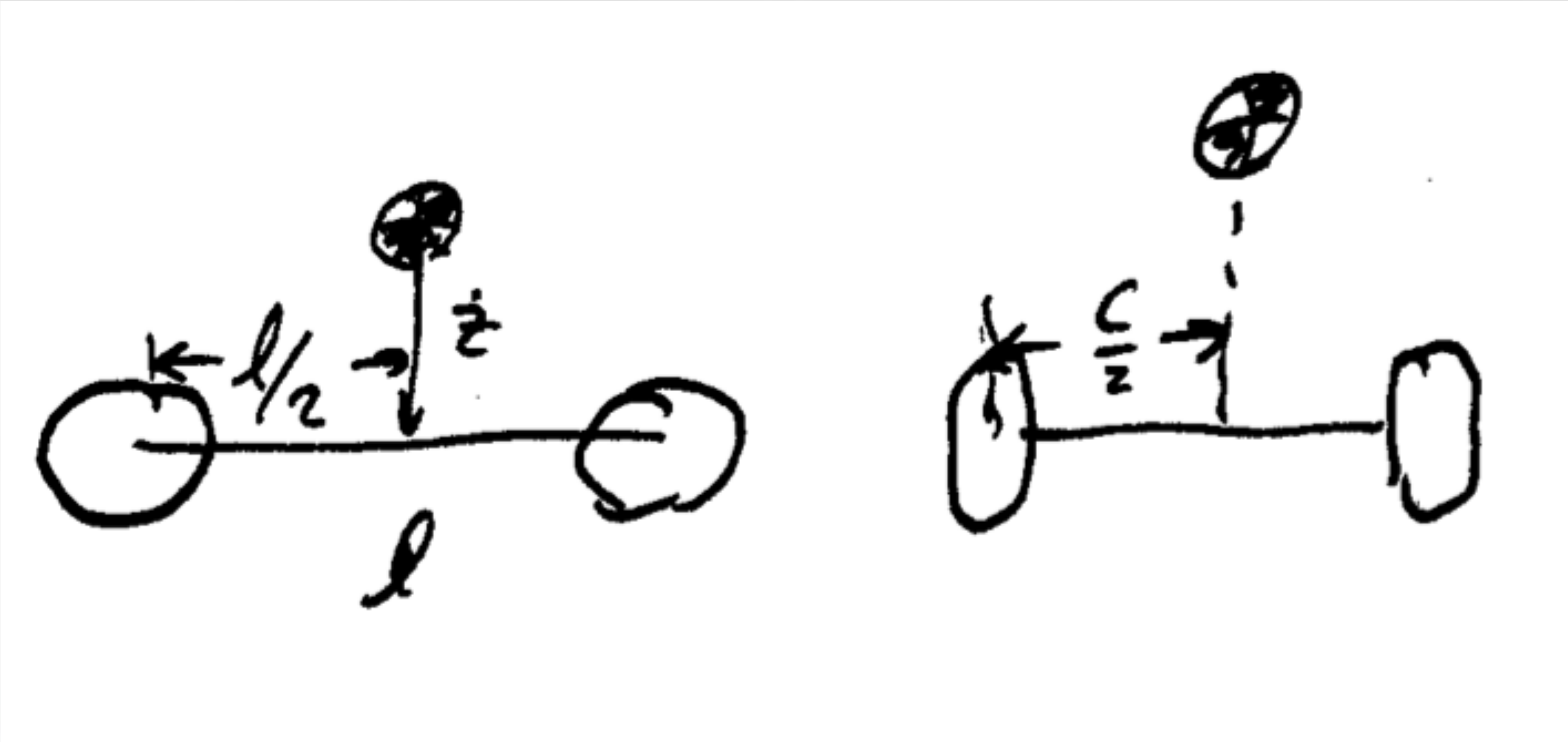
Tricycle Configuration

Lateral and longitudinal forces are stable alone, but unstable together

Static Forces on Rigid Suspension

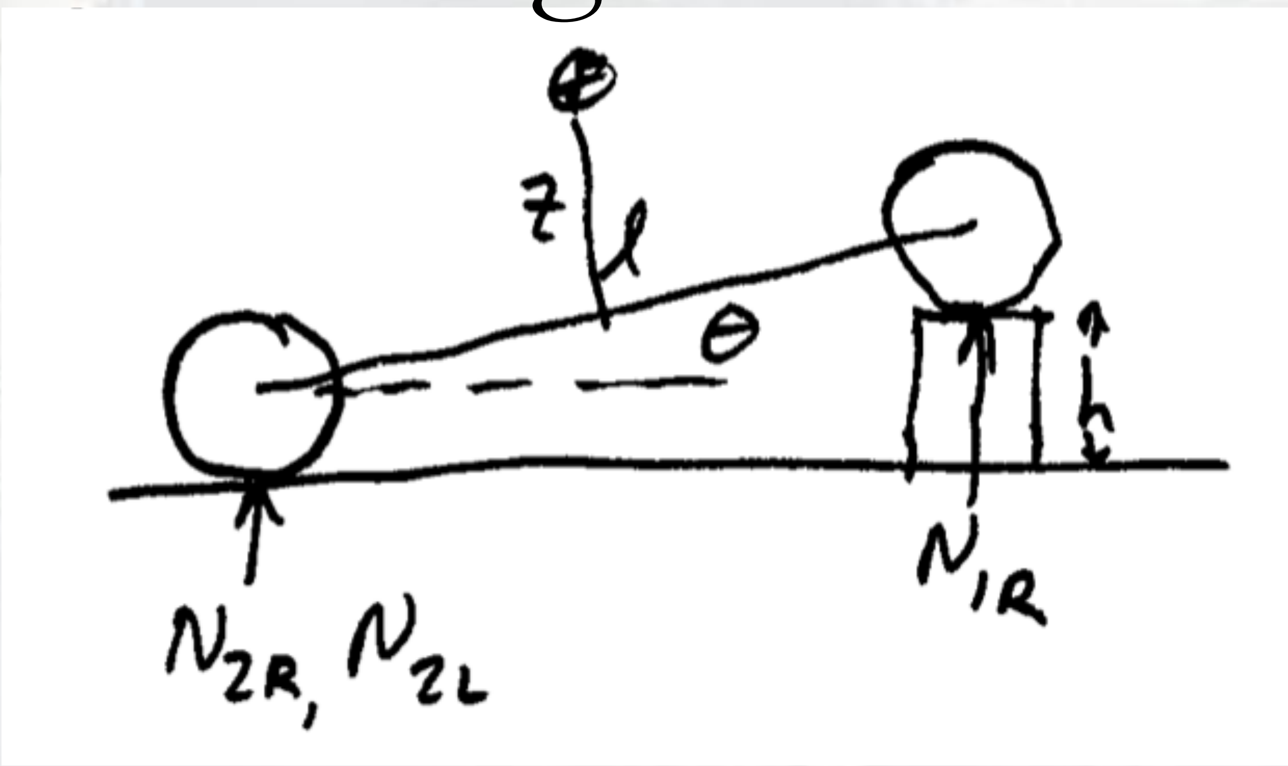


Four-wheeled vehicle



Level Ground: $N_{1R} = N_{2R} = N_{1L} = N_{2L} = \frac{W}{4}$

Obstacle under right front wheel



$N_{1L} = 0$

(Left front wheel is dangling in the air)

Force Equilibrium Over an Obstacle

$$\sum M_{back\ axle} = N_{1R}\, l \cos\theta = W\left(\frac{l}{2}\cos\theta - z\sin\theta\right)$$

$$\sin\theta = \frac{h}{l} \qquad \cos\theta = \sqrt{1 - \frac{h^2}{l^2}}$$

$$Say,\ h = \frac{3}{5}l \quad (arbitrary)$$

$$\sin\theta = \frac{3}{5} \quad \cos\theta = \frac{4}{5} \quad z = \frac{l}{2}$$

$$N_{1R}\, l\left(\frac{4}{3}\right) = W\left(\frac{l}{2}\frac{4}{5} - \frac{l}{2}\frac{3}{5}\right)$$

Force Equilibrium Over an Obstacle

$$N_{1R} = \frac{5}{4}W\left(\frac{4}{10} - \frac{3}{10}\right) = \frac{W}{8} \quad \left(\text{was } \frac{W}{4}\right)$$

$$N_{2R} + N_{2L} = \frac{7}{8}W$$

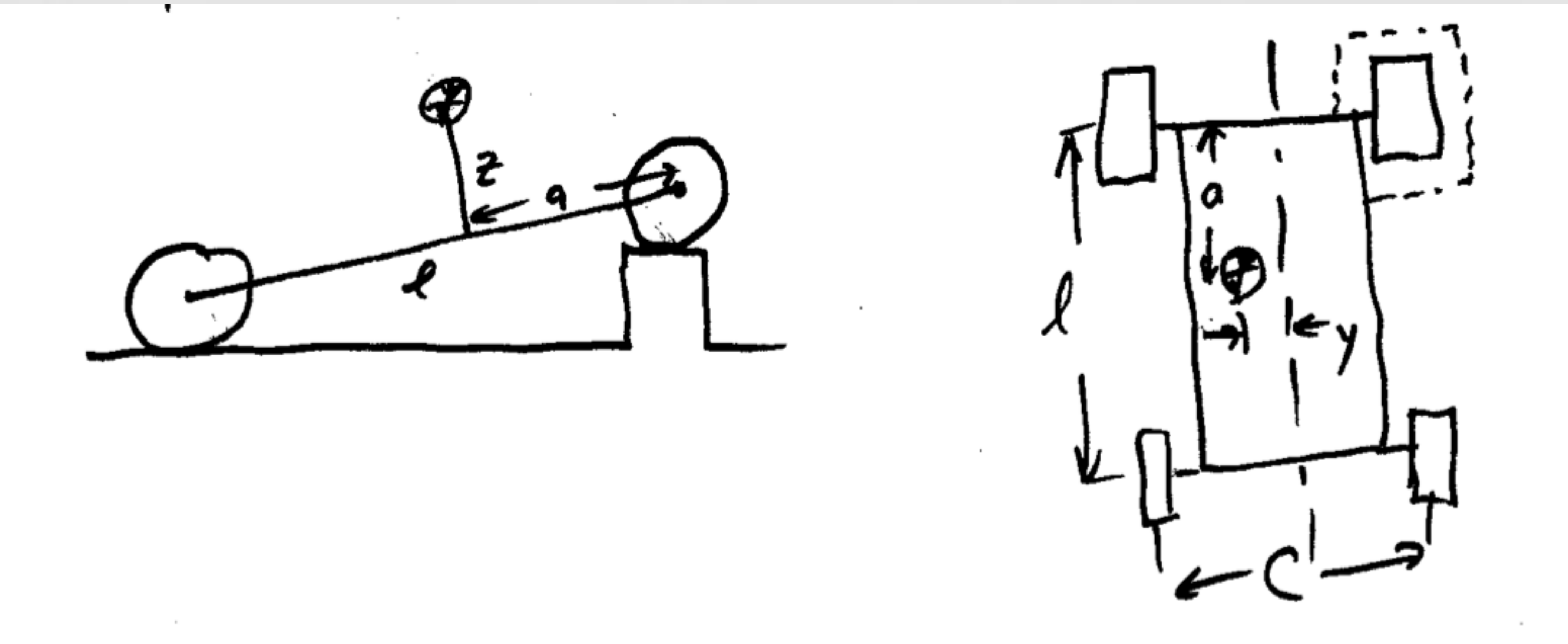
Lateral equilibrium:

$$N_{2L} = N_{2R} + N_{1R} = \frac{1}{8}W + N_{2R} = \frac{W}{2}$$

$$N_{1R} = \frac{W}{8}; \quad N_{2R} = \frac{3W}{8}; \quad N_{2L} = \frac{W}{2}$$

Rigid Suspension – General Case

(Still has front right wheel on obstacle; Front left wheel off the ground)



$\sum$ moments about rear axle

$$N_{1R}\, l \cos\theta = W[(l-a)\cos\theta - z\sin\theta]$$

$$N_{1R} = W\left(\frac{l-a}{l} - \frac{z}{l}\tan\theta\right)$$

$\sum$ moments about centerline

$$N_{2L}\frac{c}{2} = \left(N_{1R} + N_{2R}\right)\frac{c}{2} + Wy$$

$$N_{2L} = N_{1R} + N_{2R} + \frac{2wy}{c}$$

Rigid Suspension – General Solutions

$$N_{2L} = W\left(\frac{l-a}{l} - \frac{z}{l}\tan\theta\right) + N_{2R} + \frac{2Wy}{c}$$

$$\sum \text{Forces:} \quad N_{1R} + N_{2R} + N_{2L} = W$$

$$2W\left(\frac{l-a}{l} - \frac{z}{l}\tan\theta\right) + 2N_{2R} + \frac{2Wy}{c} = W$$

$$N_{2R} = \frac{W}{2} - W\left(\frac{l-a}{l} - \frac{z}{l}\tan\theta\right) - \frac{Wy}{c}$$

$$N_{2L} = \frac{W}{2} + \frac{Wy}{c} \qquad N_{1R} = W\left(\frac{l-a}{l} - \frac{z}{l}\tan\theta\right)$$

Rigid Suspension – Special Cases

$$a = \frac{l}{2} \quad z = 0 \quad y = 0 \quad (CG \text{ in plane of axes})$$

$$N_{2R} = \frac{W}{2} - \frac{W}{2} = 0 \quad N_{2L} = \frac{W}{2} \quad N_{1R} = \frac{W}{2} \quad N_{1L} = 0$$

$y \neq 0$

$$N_{2R} = -W\frac{y}{c} \quad N_{2L} = \frac{W}{2} + \frac{Wy}{c} \quad N_{1R} = \frac{W}{2}$$

$$\text{if } y > 0, \quad N_{2R} \rightarrow 0$$