Tracked Vehicles

- Overview of tracked propulsion
- Terramechanics of tracks
- Effects of track loading patterns
- Steering with tracks
- Tracked vehicle kinematics

Tracked Vehicles

Tracked Vehicles with Space Applications

Tracked Vehicles (Lost) in Space

Overview of Tracked Vehicles

- Typically used for specific applications
  - Wet or loose soils
  - Low bearing pressure limitations
  - Requirement for high tractive forces
  - Obstacle-rich terrains
- Generally come with significant drawbacks
  - Low propulsive efficiency
  - High weight
  - Large number of moving parts
  - High maintenance requirements

Tracked Vehicles

Bekker soil sinkage equation

$$P = kz^n = \left(\frac{k_c}{b} + k_\phi\right) z^n \Rightarrow z = \left(\frac{P}{\frac{k_c}{b} + k_\phi}\right)^{\frac{1}{n}}$$

$$\text{Uniform pressure } \Rightarrow P = \frac{W}{A} = \frac{W}{bl} \qquad (A = bl)$$

$$z = \left(\frac{\frac{W}{bl}}{\frac{k_c}{b} + k_\phi}\right)^{\frac{1}{n}} = \left(\frac{W}{l\left(k_c + bk_\phi\right)}\right)^{\frac{1}{n}}$$

$$\text{Work} = bl\int_0^z P\,dz = bl\int_0^z \left(\frac{k_c}{b} + k_\phi\right) z^n dz$$

Tracked Vehicles

$$\text{Work} = bl\int_0^z P dz = bl\left(\frac{k_c}{b} + k_\phi\right)\frac{z^{n+1}}{n+1}dz$$

$$\text{Substituting, Work} = \frac{bl}{(n+1)\left(\frac{k_c}{b} + k_\phi\right)}\left(\frac{W}{bl}\right)^{\frac{n+1}{n}} = R_c l$$

$$R_c = \frac{b}{(n+1)\left(\frac{k_c}{b} + k_\phi\right)^{\frac{1}{n}}}\left(\frac{W}{bl}\right)^{\frac{n+1}{n}}$$

$$R_c = \frac{1}{(n+1)\left(k_c + bk_\phi\right)^{\frac{1}{n}}}\left(\frac{W}{l}\right)^{\frac{n+1}{n}}$$

Motion resistance due to soil compaction by uniformly loaded tread

Tread Traction

Maximum tractive effort H_{max} determined by terrain shear strength τ_{max} and contact area A

$$H_{max} = A\tau_{max} = a\left(c_0 + P\tan\phi\right) = Ac_0 + W\tan\phi$$

Sandy soil: $c_0 \to 0 \quad H_{max} \approx W\tan\phi$

$$\phi = 35^\circ \quad \Rightarrow \quad \tan\phi \approx 0.7 \qquad H_{max} \approx 0.7W$$

Clay Soil: $\phi \to 0 \quad H_{max} \approx Ac_0$ (Weight has no effect)

Larger contact area ⇒more traction

Slip Definition for Tracks

$$\left(\text{same as slip } s \text{ for wheels}\right) \implies s = 1 - \frac{v}{r\omega} = 1 - \frac{v}{v_t} = \frac{v_t - v}{v_t} = \frac{v_s}{v_t}$$

r, ω refer to drive sprocket $\Rightarrow$ $v_t = r\omega \equiv$ track velocity

v_s is speed of slip $\Rightarrow$ direction is opposite to vehicle motion

(for skidding, v_s is in the same direction as vehicle's motion)

since track cannot stretch, local slip velocity v_s is constant everywhere

Distance of slip $\quad j = v_s t \quad t = \dfrac{x}{x_s} \quad j = \dfrac{v_s x}{v_t} = sx \quad$ where x = vehicle distance traveled

From soil mechanics, $\tau = \tau_{max}\left(1 - e^{-j/k}\right) = (c_0 + P\tan\phi)\left(1 - e^{-j/k}\right)$

τ = shear stress j = shear displacement k = shear deformation modulus

$$\text{Total tractive effort } F = b\int_0^l \tau dx$$

$$F = b\int_0^l (c_0 + P\tan\phi)\left(1 - e^{-j/k}\right)dx$$

$$= (Ac_0 + W\tan\phi)\left[1 - \frac{k}{sl}\left(1 - e^{-sl/k}\right)\right]$$

$$\left(\text{for l/k large and } s \sim 1 \Rightarrow e^{-sl/k} \sim 0 \text{ and } \frac{k}{sl} \sim 0\right)$$

$$100\,\%\ \text{slip} \Rightarrow F \cong A_c + W\tan\phi$$

Example: Two Vehicles @ W=135 kN

Terrain: $n = 1.6 \quad k_c = 4.37 \text{ kN/m}^{2.6} \quad k_\phi = 196.72 \text{ kN/m}^{3.6}$

$k = 5 \text{ cm} \quad c_0 = 1.0 \text{ kP} \quad d = 19.7°$

Vehicles: $A = 7.2 \text{ m}^2$ (both) Unit A: $b = 1 \text{ m} \quad l = 3.6 \text{ m}$

Unit B: $b = 0.8 \text{ m} \quad l = 4.5 \text{ m}$

Unit A: Sinkage $$z_0 = \left(\frac{P}{k_c/b + k_\phi}\right)^{1/n} = \left(\frac{135/7.2}{4.37/1 + 196.n2}\right)^{0.625} = 0.227 \text{ m}$$

$$R_c = 2b\left(\frac{k_c}{b} + k_\phi\right)\frac{z_0^{n+1}}{n+1} = 3.28 \text{ kN}$$

unit B: $z_0 = 0.226 \text{ m} \quad R_c = 2.6 \text{ kN}$

Continued Examples

$$F_{\max}(\text{ unit } A) = 2blc + W\tan\phi = 2 \times 1 \times 3.6 \times 1 + 135 \times 0.358 = 55.54 \text{ kN}$$

Slip	A	B
5 %	40.54	43.32
10 %	47.84	49.37
20 %	51.68	52.46
40 %	53.62	54.00
60 %	54.25	54.51
80 %	54.57	54.77

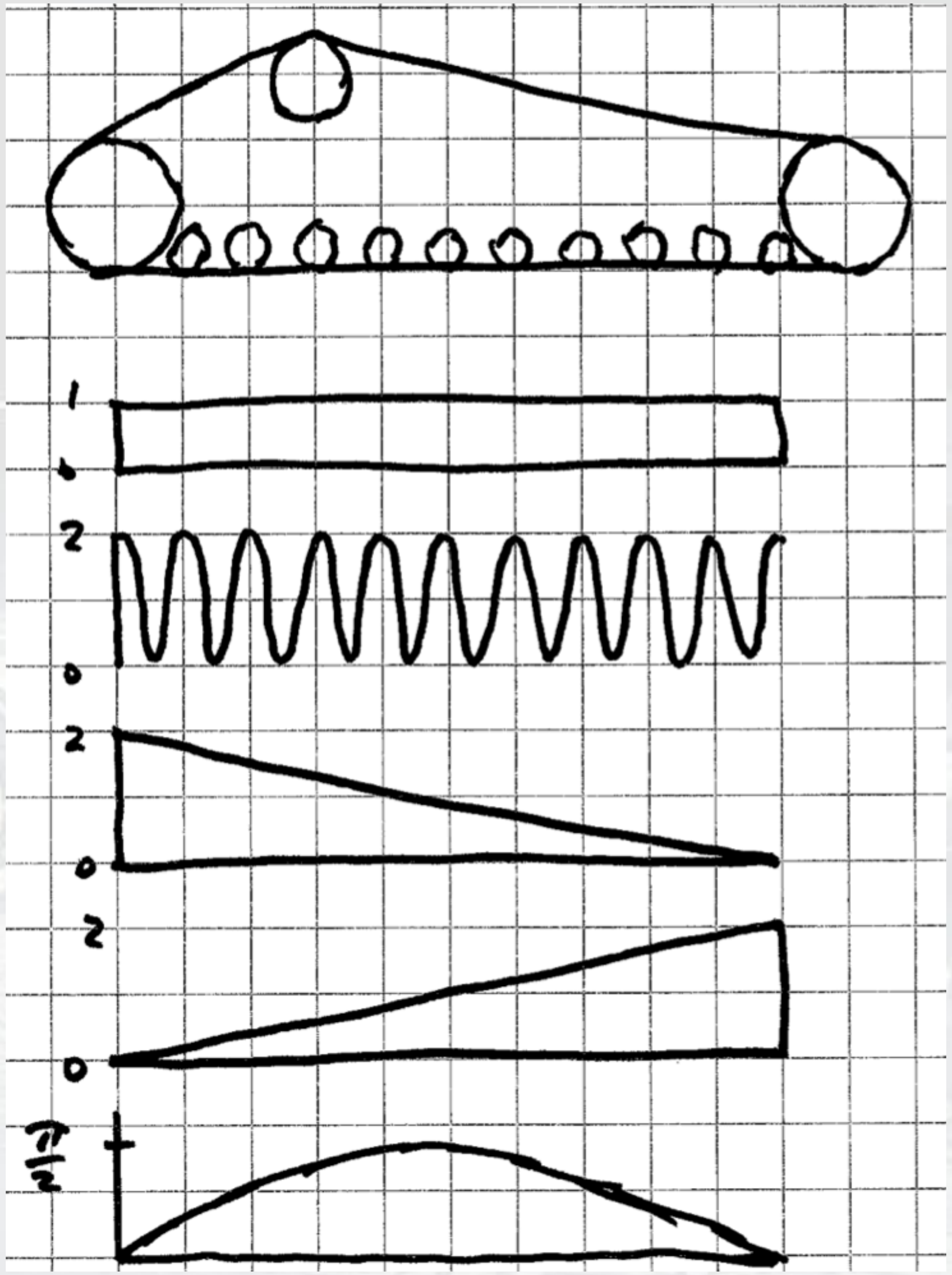
Consider two vehicles: $A_1 = A_2$, $w_1 = w_2$, but $l_1 = 2l_2$ so $b_1 = b_2/2$

Set tractive forces to be the same

$$1 - \frac{k}{s_1 l_1}\left(1 - e^{-s_1 l_1/k}\right) = 1 - \frac{k}{s_2 l_1/2}\left(1 - e^{-s_2 l_2/k}\right)$$

$$e^{-sl/k} \text{ small} \quad \Rightarrow \quad s_1 = \frac{1}{2}s_2 \quad \Rightarrow \quad \text{longer track slips less}$$

Sample Track Loading Patterns



Linear

Multipeak sinusoidal

Linear increasing front → back

Linear increasing back → front

Single sinusoidal

Effect of Track Loading Patterns (1)

Assume frictional soil ($c_0 \rightarrow 0$)

Multipeak sinusoidal pressure

$$P = \frac{W}{bl}\left(1 + \cos\frac{2n\pi x}{l}\right) \qquad n = \text{number of peaks}$$

$$\tau = \frac{W}{bl}\tan\theta\left(1 + \cos\frac{2n\pi x}{l}\right)\left(1 - e^{-\frac{sx}{k}}\right)$$

$$F = b\int_0^l \frac{W}{bl}\tan\phi\left(1 + \cos\frac{2\pi nx}{l}\right)\left(1 - e^{-\frac{sx}{k}}\right)dx$$

$$F = W\tan\phi\left[1 + \frac{k}{sl}\left(e^{-\frac{sl}{k}} - 1\right) + \frac{k\left(e^{-sl/k} - 1\right)}{sl\left(1 + 4n^2k^2\frac{\pi^2}{s^2}l^2\right)}\right]$$

Effect of Track Loading Patterns (2)

Increasing linearly front $\rightarrow$ back

$$P = 2\frac{W}{bl}\frac{x}{l}$$

$$F = W\tan\phi\left[1 - 2\left(\frac{k}{sl}\right)^2\left(1 - e^{-\frac{sl}{k}} - \frac{sl}{k}e^{-\frac{sl}{k}}\right)\right]$$

Increasing linearly back $\rightarrow$ front

$$P = 2\frac{W}{bl}\frac{l-x}{l}$$

$$F = 2W\tan\phi\left[1 - \frac{k}{sl}\left(1 - e^{-sl/k}\right)\right] - W\tan\phi\left[1 - 2\left(\frac{k}{sl}\right)^2\left(1 - e^{-sl/k} - \frac{sl}{k}e^{-sl/k}\right)\right]$$

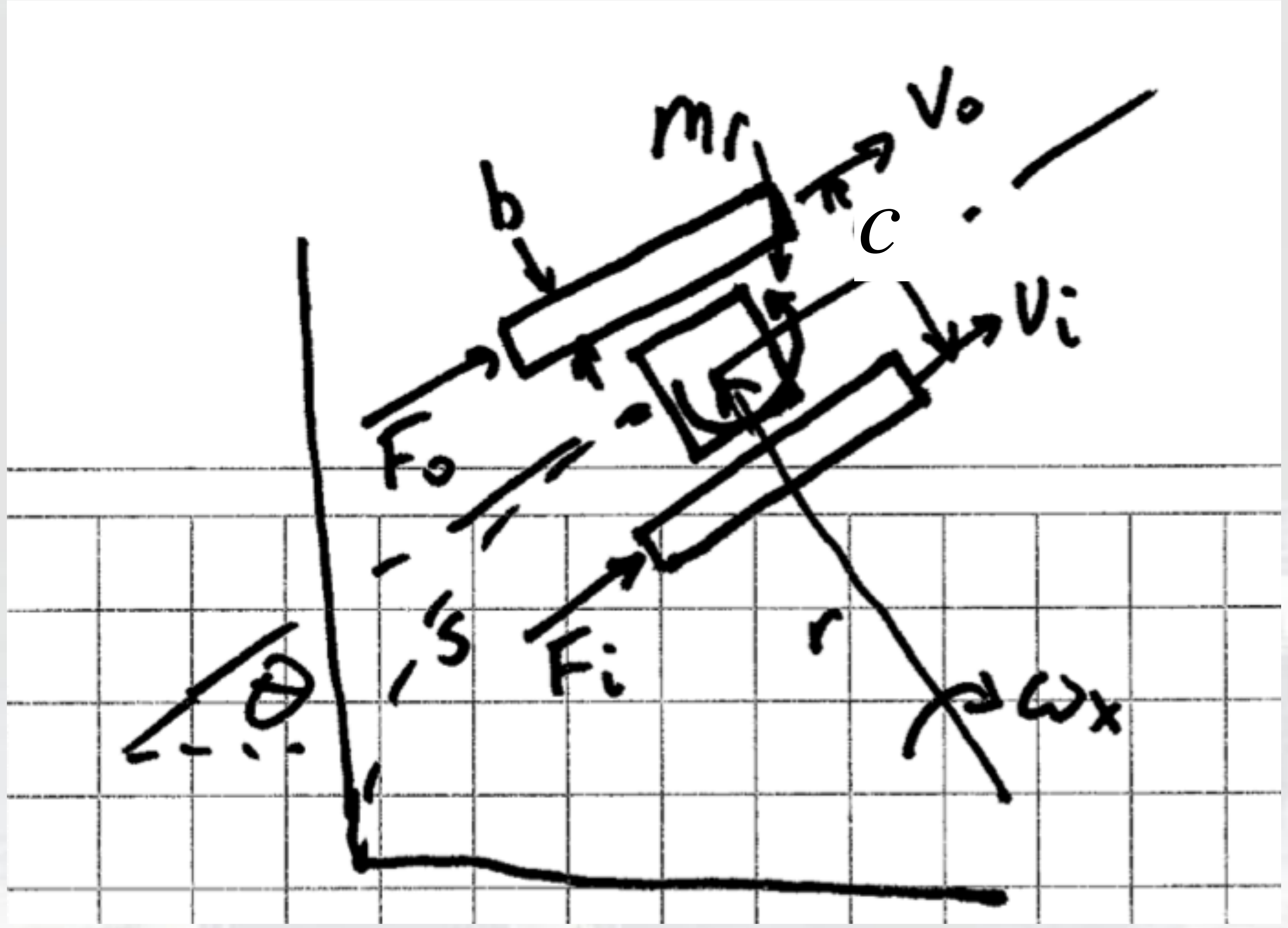
Effect of Track Loading Patterns (3)

Single sinusoid, 0 loading at front and back of track

$$P = \frac{W}{bl}\frac{\pi}{2}\sin\left(\frac{\pi x}{l}\right)$$

$$F = W\tan\phi\left[1 - \frac{e^{-sl/k}+1}{2\left(1 - i^2 l^2/\pi^2 k^2\right)}\right]$$

Steering of Tracked Vehicles



$$m\frac{d^2s}{dt^2} = F_o + F_i - R_{tot}$$

$$I_z\frac{d^2\theta}{dt^2} = \frac{c}{2}\left(F_o - F_i\right) - m_r$$

$M_r \equiv$ moment due to turning resistance

Under steady-state conditions (no acceleration),

$$F_o + F_i - R_{tot} = 0$$

$$\frac{c}{2}\left(F_o - F_1\right) - M_r = 0$$

Steering Forces (1)

$$F_o = \frac{R_{tot}}{2} + \frac{M_r}{2} = \frac{\mu_r W}{2} + \frac{m_c}{c}$$

$$F_i = \frac{R_{tot}}{2} - \frac{M_r}{2} = \frac{\mu_r W}{2} - \frac{m_c}{c}$$

For a normal pressure profile uniformly distributed,

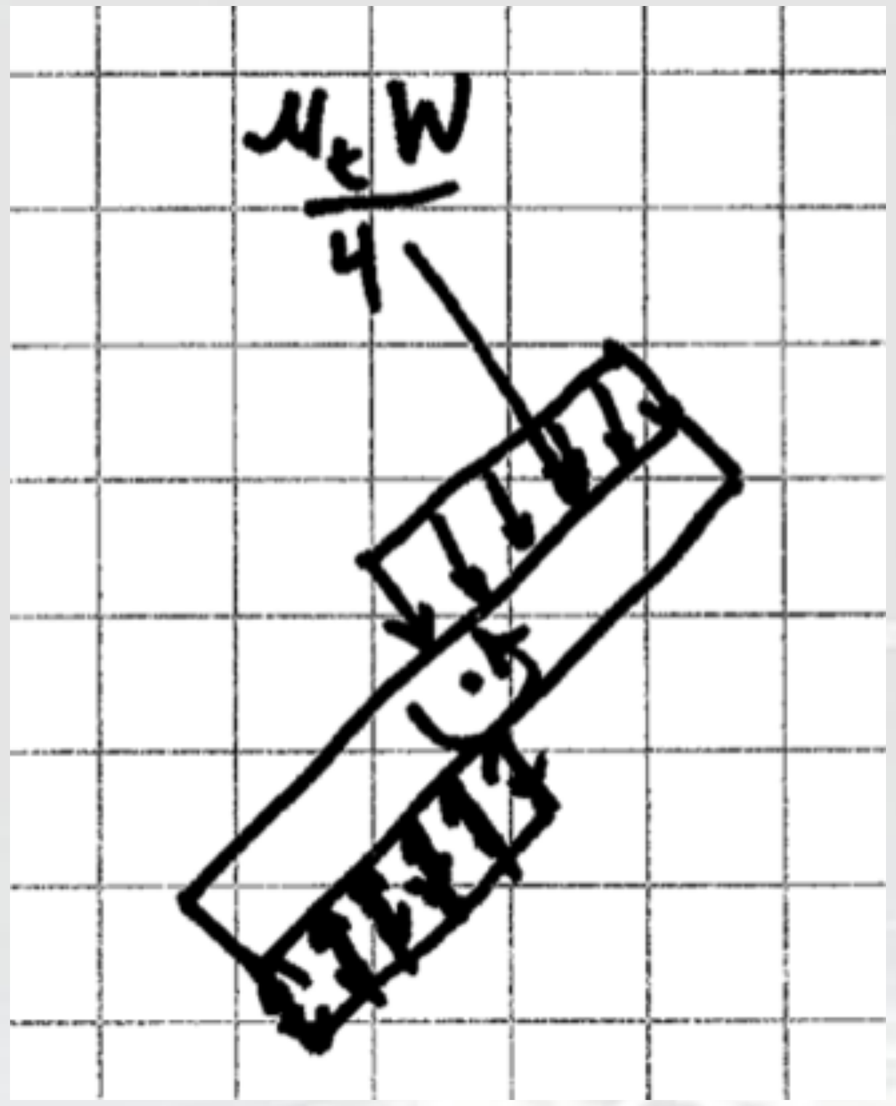
Lateral resistance: $R_l = \frac{\mu_t W}{2l}$

μ_t depends on soil and track design:

Steel track, hard ground: 0.55 – 0.58 grass: 0.87 – 1.11

Rubber tracks, hard ground: 0.65 – 0.66 grass: 0.67 – 1.14

Steering Forces (2)



$$M_r = 4\frac{W\mu_t}{2l}\int_0^{l/2} x dx = \frac{\mu_t W l}{4}$$

$$F_o = \frac{\mu_r W}{2} + \frac{\mu_t W l}{4c}$$

$$F_i = \frac{\mu_r w}{2} - \frac{\mu_t \omega l}{4c}$$

Vehicle with uniform normal pressure distribution turning at low speed on level ground

Maximum thrust of vehicle is limited by terrain and vehicle properties -

Outside track: $F_o \leqslant c_0 b l + \frac{W \tan \phi}{2}$

$$\frac{\mu_r W}{2} + \frac{\mu_t W l}{4c} \leqslant c_0 b l + \frac{W \tan \phi}{2}$$

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Steering Forces (3)

$$\frac{l}{c} \leqslant \frac{1}{\mu_t}\left(\frac{4c_0A}{W} + 2\tan\phi - 2\mu_r\right) \qquad A = bl$$

To steer without spinning outside track,

$$\frac{l}{c} \leqslant \frac{2}{\mu_t}\left(\frac{c_0}{P} + \tan\phi - \mu_r\right) = \frac{2}{\mu_t}\left(\frac{2c_0bl}{W} + \tan\phi - \mu_r\right)$$

Examples:

Sandy terrain: $c_0 = 0 \quad \phi = 30° \quad \mu_t = 0.5 \quad y = 0.1 \Rightarrow \quad \frac{l}{c} \leqslant 1.9$

Clay soil: $c_0 = 3.45\ kPa \quad \phi = 10° \quad P = 6.9\ kPa\ (= 1psi) \quad \mu_t = 0.4 \quad \mu_r = 0.1 \Rightarrow$

$$\frac{l}{c} \leqslant 2.88$$

$$F_i = \frac{\mu_r W}{2} - \frac{\mu_t W l}{4c} \Rightarrow \frac{\mu_t l}{2c} < \mu_r \text{ for } F_i > 0$$

$$\mu_t = 0.5 \quad \mu_r = 0.1 \quad \frac{l}{c} = 1.5 \quad \frac{0.5}{2}(1.5) = 0.375 \nless 0.1$$

$\Rightarrow$ inside track has to brake to turn

Max thrust of outside track is limited by terrain

$\Rightarrow$ braking inner track diminishes steering, drawbar pull

Tracked Vehicle Steering Example

$$W = 155.7 \text{ kN } (75{,}000 \text{ lb}) \quad c = 203.2 \text{ cm}$$

$$l = 304.8 \text{ cm} \quad b = 76.2 \text{ cm} \quad c_0 = 3.45 \text{ kPa} \quad \phi = 25^\circ \quad \mu_r = 0.15 \quad \mu_t = 0.5$$

1) Determine steerability: $\dfrac{l}{c} \leq \dfrac{2}{\mu_t}\left(\dfrac{c_0}{P} + \tan\phi - \mu_r\right) = 1.67$

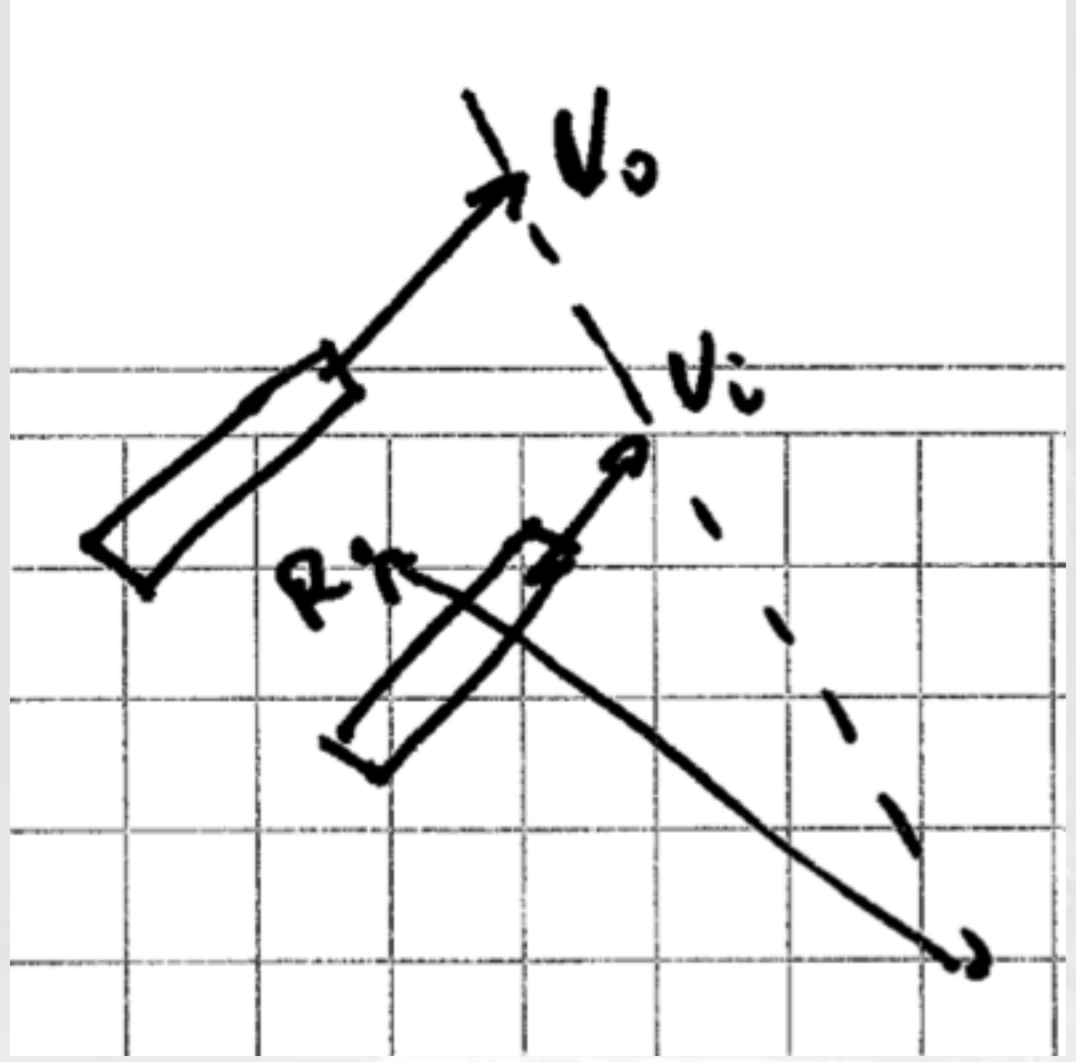
Actual $\dfrac{l}{b} = 1.5$ so it is steerable

2) Analyze thrust during turn

$$F_0 = \frac{\mu_r W}{2} + \frac{\mu_t W l}{4c} = 40.87 \text{ kN}$$

$$F_i = \frac{\mu_r W}{2} - \frac{\mu_t W l}{4c} = -17.52 \text{ kN}$$

Kinematics of Tracked Skid Steering (1)



Outside drive wheel turns at ω_o

Inside drive wheel turns at ω_i

Assume tracks do not skid or slip

Turn radius R yaw velocity Ω_z

By similar triangles, $\dfrac{v_o}{R + c/2} = \dfrac{v_i}{R - c/2} \Rightarrow v_o R - v_o \dfrac{c}{2} = v_i R + v_i \dfrac{c}{2}$

$$R = \frac{c}{2} \frac{v_o + v_i}{v_o - v_i}$$

drive sprocket radius $\equiv r$ $v = \omega r$

Kinematics of Tracked Skid Steering (2)

$$R = \frac{c}{2}\frac{\omega_o r + \omega_i r}{\omega_o r - \omega_i r} \qquad K_s \equiv \frac{\omega_o}{\omega_i}$$

$$R = \frac{c}{2}\frac{K_s + 1}{K_s - 1}$$

$$\Omega_z = \frac{r\omega_o + r\omega_i}{2R} = \frac{r\omega_i\left(K_s - 1\right)}{c}$$

Outside track always thrusts $\Rightarrow$ always slips

$$R' = \frac{c\left[r\omega_o\left(1 - s_0\right) + r\omega_i\left(1 - s_i\right)\right]}{2\left[r\omega_o\left(1 - s_0\right) - r\omega_i\left(1 - s_i\right)\right]}$$

Kinematics of Tracked Skid Steering (3)

$$R' = \frac{c\left[K_s\left(1 - s_0\right) + \left(1 - s_i\right)\right]}{2\left[K_s\left(1 - s_0\right) - \left(1 - s_i\right)\right]}$$

$$\Omega'_z = \frac{r\omega_0(1 - s_o) + r\omega_i\left(1 - s_i\right)}{2R'}$$

$$\Omega'_z = \frac{r\omega_i\left[K_s\left(1 - s_o\right) - \left(1 - s_i\right)\right]}{c}$$

$\frac{R'}{R}$ always > 1 (slip causes larger turning radius)

$\frac{\Omega'_z}{\Omega_z}$ always < 1 (slip causes slower turn)