ENAE 791 PROBLEM SET 1 – SPRING, 2020

DUE 2/25/20

(1) A spacecraft is in orbit around the Earth. Its position and velocity can be expressed in an inertial Cartesian frame centered on the gravitational center of the Earth with the Z axis oriented through the north pole as $\bar{X}$={0, 11681, 0} (km) and $\bar{V}$={5.134, 4.226, 2.787} (km/sec). Calculate

(a) Current scalar quantities for radius r and velocity v
(b) Semimajor axis a
(c) Eccentricity e
(d) Perigee altitude h_p
(e) Apogee radius r_a
(f) True anomaly θ
(g) Orbital period P
(h) Time since perigee passage
(i) Orbital inclination i

(2) Write a computer routine (program, MatLab script, or Excel spreadsheet, whatever works for you) to numerically integrate the planar orbital motion state equations derived in class. Starting with the conditions in a the previous problem, propagate the orbit forward through one orbital period. What are the position and velocity errors in your numerical prediction as compared to the calculated orbital state? (Note: we're going to be adding on to this program throughout the term to incorporate atmospheric drag, lift, and launch thrust, as well as out-of-plane motions. It's in your enlightened self-interest to write the code cleanly enough you can continue to modify and reuse it throughout the term.)

(3) We are going to "reverse engineer" the SpaceX Falcon 9 launch vehicle family. Some critical parameters: (note that throughout this course, "MT" refers to metric tons, or thousands of kg).

First stage:
m_{prop} **:** 433.1 MT
m_{inert} **:** 22.2 MT
I_{sp} **:** 310 sec

Second stage:
m_{prop} **:** 111.5 MT
m_{inert} **:** 4 MT
I_{sp} **:** 348 sec
Payload to LEO: 22.8 MT

(a) What is the vehicle gross mass?
(b) Calculate the Δv's for the first and second stages, and the total Δv for this launch vehicle
(c) Calculate the inert mass fraction δ and stage inert mass fraction ϵ for each stage

(d) Use the mass estimating relations from the Rocket Performance lecture notes to calculate predicted ϵ for each stage. How is SpaceX doing compared to the predicted values?
(e) Using the mass fractions obtained in (c), find the Δv distribution which would theoretically maximize the payload mass fraction
(f) Calculate the trade-off ratios for each stage
    (i) Effect of inert mass change on payload
    (ii) Effect of marginal propellent mass on payload

(4) SpaceX recovers and reuses the first stage by reserving propellent to decelerate and propulsively land the empty stage back at the launch site. Assume this requires a Δv of 500 m/sec.
(a) How much propellant would be required for this return and landing maneuver?
(b) Since that amount of propellant is no longer available for launching the payload, how much payload can the vehicle carry to orbit (the same Δv as you calculated in 2b) on a mission which recovers the first stage?

(5) The Falcon Heavy uses two additional Falcon 9 first stages as "strap-on boosters" for the core vehicle, which is (for the purposes of this problem) otherwise a standard Falcon 9. Assume the two first stages used as boosters have an additional 3MT of inert mass due to the need for aerodynamic fairings and other modifications. Assume that the Falcon Heavy must achieve the same Δv as you calculated in 2b.
(a) If all three first stages modules burn together, what is the payload to orbit?
(b) For the first flight, the center core operated at 85% thrust while the boosters are burning, then burned to completion and was jettisoned. What payload does this approach provide?
(c) For high-performance missions, the two outer boosters could be "cross-strapped": their propellant tanks will also supply the engines in the center core until booster depletion. At that point, the boosters will be jettisoned and the fully fueled center module will continue to burn. What payload to orbit would this provide?
(d) SpaceX is quoting a 63.8 MT payload to LEO. Based on your calculations, is this optimistic, pessimistic, or just about right?