Ballistic Atmospheric Entry

- Planar state equations (from Orbital Mechanics lecture)
- Standard atmospheres
- Orbital decay due to drag
- Straight-line (no gravity) ballistic entry based on density and altitude
- Planetary entries (at least a start)
- Basic equations of planar motion

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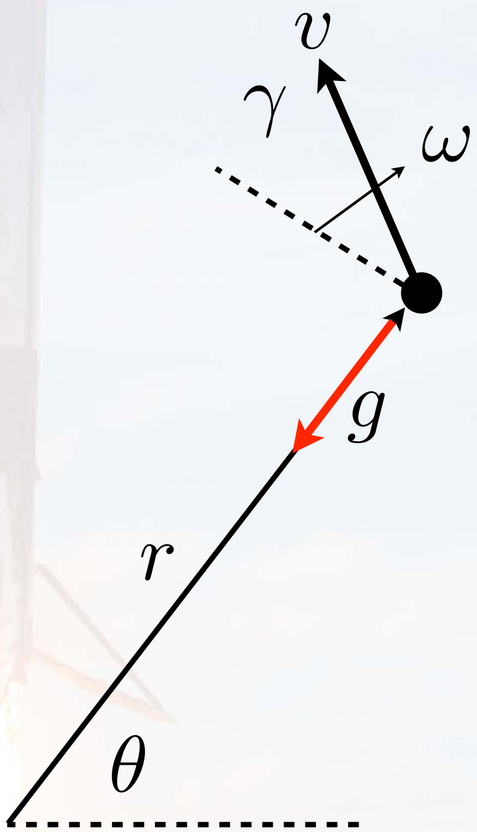
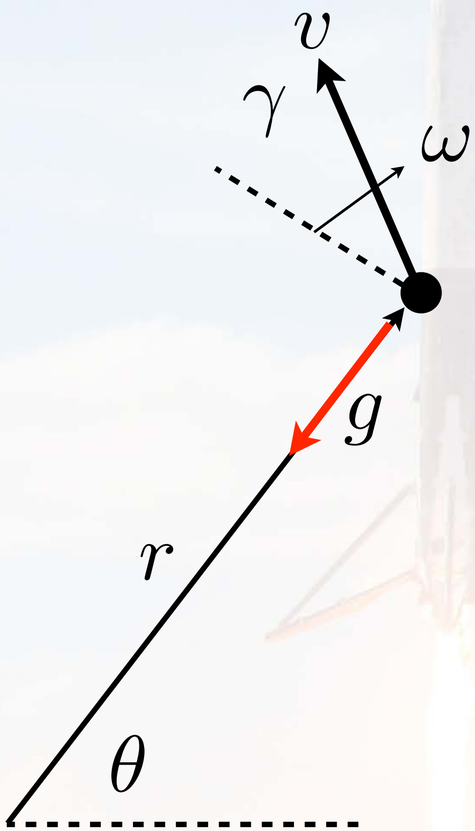
Free-Body Diagram with Spherical Planet



Orbital Planar State Equations



Inertial angular velocity

$$\omega = \dot{\gamma} - \dot{\theta}$$

Sum of accelerations
normal to velocity vector

$$-g\cos\gamma = \omega v$$

Sum of accelerations
perpendicular to velocity vector

$$-g\sin\gamma = \dot{v}$$

Orbital Planar State Equations (2)

$$\dot{r} = v \sin\gamma$$

$$r\dot{\theta} = v \cos\gamma$$

$$\omega = \dot{\gamma} - \dot{\theta} = \dot{\gamma} - \frac{v}{r}\cos\gamma$$

$$-g\cos\gamma = \left(\dot{\gamma} - \frac{v}{r}\cos\gamma\right) v$$

$$-\left(g - \frac{v^2}{r}\right)\cos\gamma = \dot{\gamma} v$$

$$-\left(1 - \frac{v^2}{rg}\right) g\cos\gamma = \dot{\gamma} v$$

Canonical Orbital Planar State Equations

$$\dot{\gamma} = -\frac{1}{v}\left(1 - \frac{v^2}{v_c^2}\right) g \cos\gamma$$

$$\dot{v} = -g \sin\gamma$$

$$\dot{r} = v \sin\gamma$$

$$\dot{\theta} = \frac{v}{r} \cos\gamma$$

Coupled first-order ODEs

$$g = g_o \left(\frac{r_o}{r}\right)^2$$

Numerical Integration - 4th Order R-K

Given a series of equations $\dot{\bar{y}} = \bar{f}(t, \bar{x})$

$$\bar{k_1} = \Delta t \ \bar{f}\left(t_n, \bar{y_n}\right)$$

$$\bar{k_2} = \Delta t \ \bar{f}\left(t_n + \frac{\Delta t}{2}, \bar{y_n} + \frac{\bar{k_1}}{2}\right)$$

$$\bar{k_3} = \Delta t \ \bar{f}\left(t_n + \frac{\Delta t}{2}, \bar{y_n} + \frac{\bar{k_2}}{2}\right)$$

$$\bar{k_4} = \Delta t \ \bar{f}\left(t_n + \Delta t, \bar{y_n} + \bar{k_3}\right)$$

$$\bar{y_{n+1}} = \bar{y_n} + \frac{\bar{k_1}}{6} + \frac{\bar{k_2}}{3} + \frac{\bar{k_3}}{3} + \frac{\bar{k_4}}{6} + O(\Delta t^5)$$

Atmospheric Density with Altitude

Pressure=the integral of the atmospheric density in the column above the reference area

$$P_o = \int_o^\infty \rho g dh = \rho_o g \int_o^\infty e^{-\frac{h}{h_s}} dh = -\rho_o g h_s \left[e^{-\frac{h}{h_s}} \right]_o^\infty$$

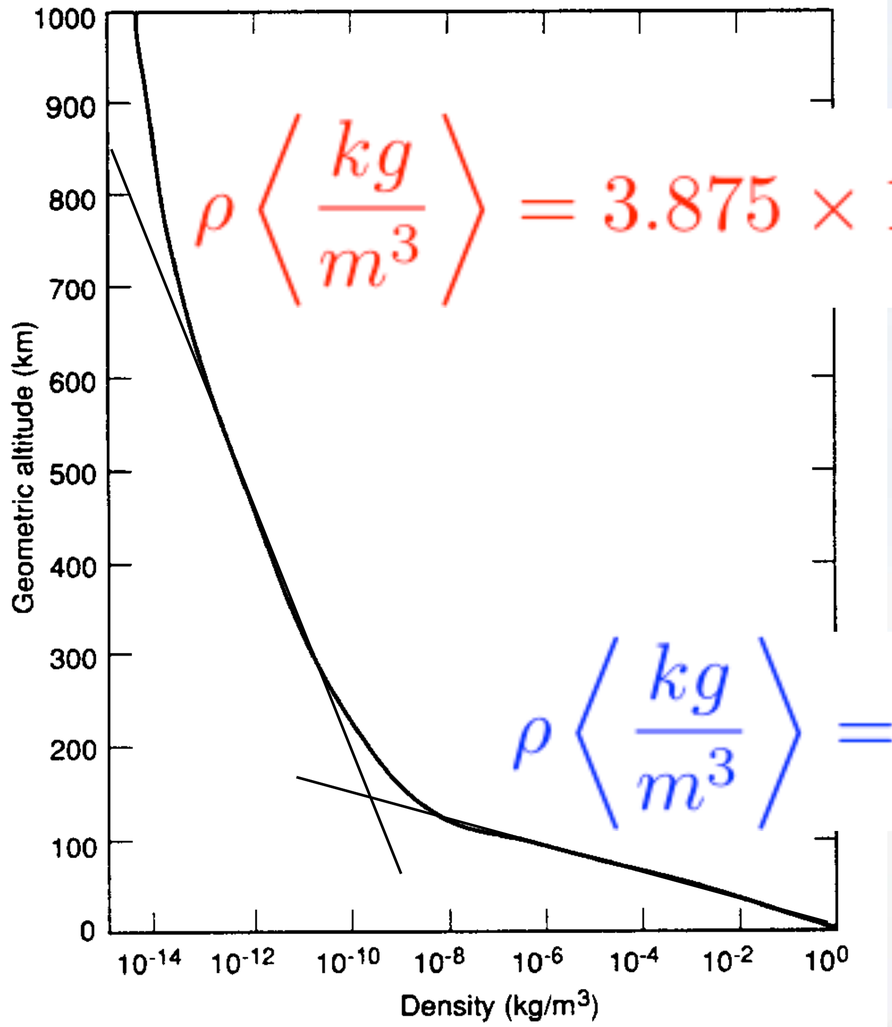
$$= -\rho_o g h_s \left[0 - 1 \right]$$

$$\boxed{P_o = \rho_o g h_s}$$

Earth: $\rho_o = 1.226 \frac{kg}{m^3}$; $h_s = 7524m$;

$P_o(calc) = 90,400\ Pa$; $P_o(act) = 101,300\ Pa$

Atmospheric Density with Altitude



$$\rho\left\langle \frac{kg}{m^3} \right\rangle = 3.875 \times 10^{-9} e^{-\frac{h\langle km \rangle}{59.06}}$$

$$\rho\left\langle \frac{kg}{m^3} \right\rangle = 1.226 e^{-\frac{h\langle km \rangle}{7.524}}$$

Ref: V. L. Pisacane and R. C. Moore, Fundamentals of Space Systems Oxford University Press, 1994

Energy Loss Due to Atmospheric Drag

$$\text{Drag } D \equiv \frac{1}{2}\rho v^2 A c_D$$

$$\text{Drag acceleration } a_d = \frac{D}{m} = \frac{\rho v^2}{2}\frac{A c_D}{m}$$

$$\boxed{\beta \equiv \frac{m}{c_D A}} \quad \text{<== Ballistic Coefficient}$$

$$a_d = \frac{\rho v^2}{2\beta}$$

$$\text{orbital energy} \equiv E = -\frac{\mu}{2a}$$

$$\frac{dE}{dt} = \frac{\mu}{2a^2}\frac{da}{dt}$$

Energy Loss Due to Atmospheric Drag

Since drag is highest at perigee, the first effect of atmospheric drag is to circularize the orbit (high perigee drag lowers apogee)

$$\frac{dE_{drag}}{dt} = a_d v$$

$$v_{circ}^2 = \frac{\mu}{a} \qquad \frac{dE_{drag}}{dt} = -\frac{\rho v^2}{2\beta}\sqrt{\frac{\mu}{a}}$$

$$\frac{dE_{drag}}{dt} = -\sqrt{\frac{\mu}{a}}\frac{\rho}{2\beta}\frac{\mu}{a} = -\left(\frac{\mu}{a}\right)^{\frac{3}{2}}\frac{\rho}{2\beta}$$

Derivation of Orbital Decay Due to Drag

Set orbital energy variation equal to energy lost by drag

$$\frac{\mu}{2a^2}\frac{da}{dt} = -\frac{\rho}{2\beta}\left(\frac{\mu}{a}\right)^{\frac{3}{2}}$$

$$\frac{da}{dt} = -\frac{\rho}{\beta}\sqrt{\mu a}$$

$$\rho = \rho_o e^{-\frac{h}{h_s}} \qquad a = h + r_E \Longrightarrow \frac{da}{dt} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{\sqrt{\mu\left(h + r_E\right)}}{\beta}\rho_o e^{-\frac{h}{h_s}}$$

Derivation of Orbital Decay (2)

This is a separable differential equation...

$$\frac{1}{\sqrt{r_E + h}} e^{\frac{h}{h_s}} dh = -\frac{\sqrt{\mu}}{\beta} \rho_o dt$$

$$\int_{h_o}^{h} \frac{1}{\sqrt{r_E + h}} e^{\frac{h}{h_s}} dh = -\frac{\sqrt{\mu}}{\beta} \rho_o \int_{t_o}^{t} dt$$

Assume $\sqrt{r_E + h} \sim \sqrt{r_E}$ for $r_E \gg h$

$$\frac{1}{\sqrt{r_E}} \int_{h_o}^{h} e^{\frac{h}{h_s}} dh = -\frac{\sqrt{\mu}}{\beta} \rho_o (t - t_o)$$

Derivation of Orbital Decay (3)

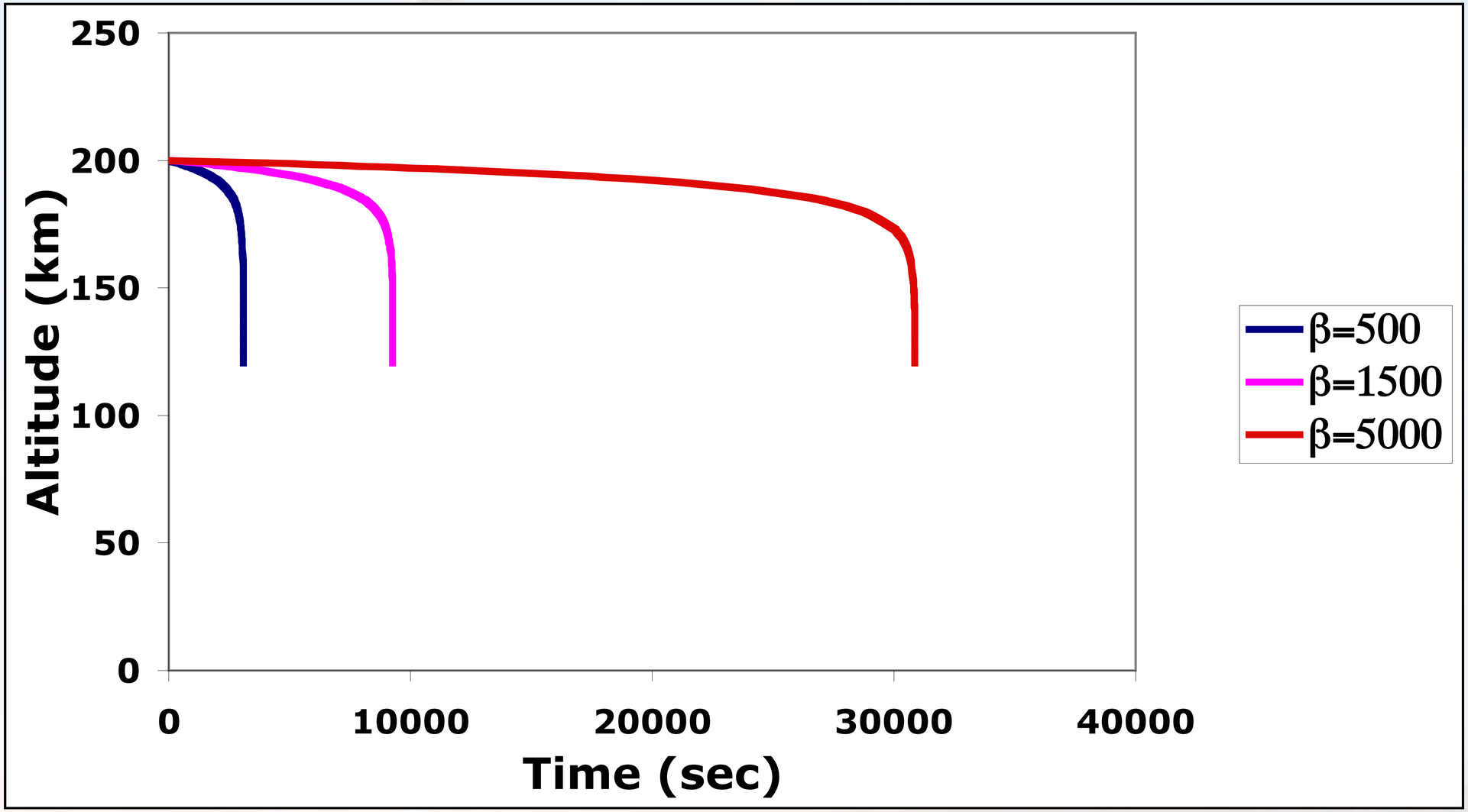
$$\frac{h_s}{\sqrt{r_E}}\left(e^{\frac{h}{h_s}} - e^{\frac{h_o}{h_s}}\right) = -\frac{\sqrt{\mu}}{\beta}\rho_o\left(t - t_o\right)$$

$$e^{\frac{h}{h_s}} - e^{\frac{h_o}{h_s}} = -\frac{\sqrt{\mu r_E}}{h_s\beta}\rho_o\left(t - t_o\right)$$

$$h(t) = h_s \ln\left[e^{\frac{h_o}{h_s}} - \frac{\sqrt{\mu r_E}}{h_s\beta}\rho_o\left(t - t_o\right)\right]$$

Note that some variables typically use km, and others are in meters - you have to make sure unit conversions are done properly to make this work out correctly!

Orbit Decay from Atmospheric Drag



Time Until Orbital Decay

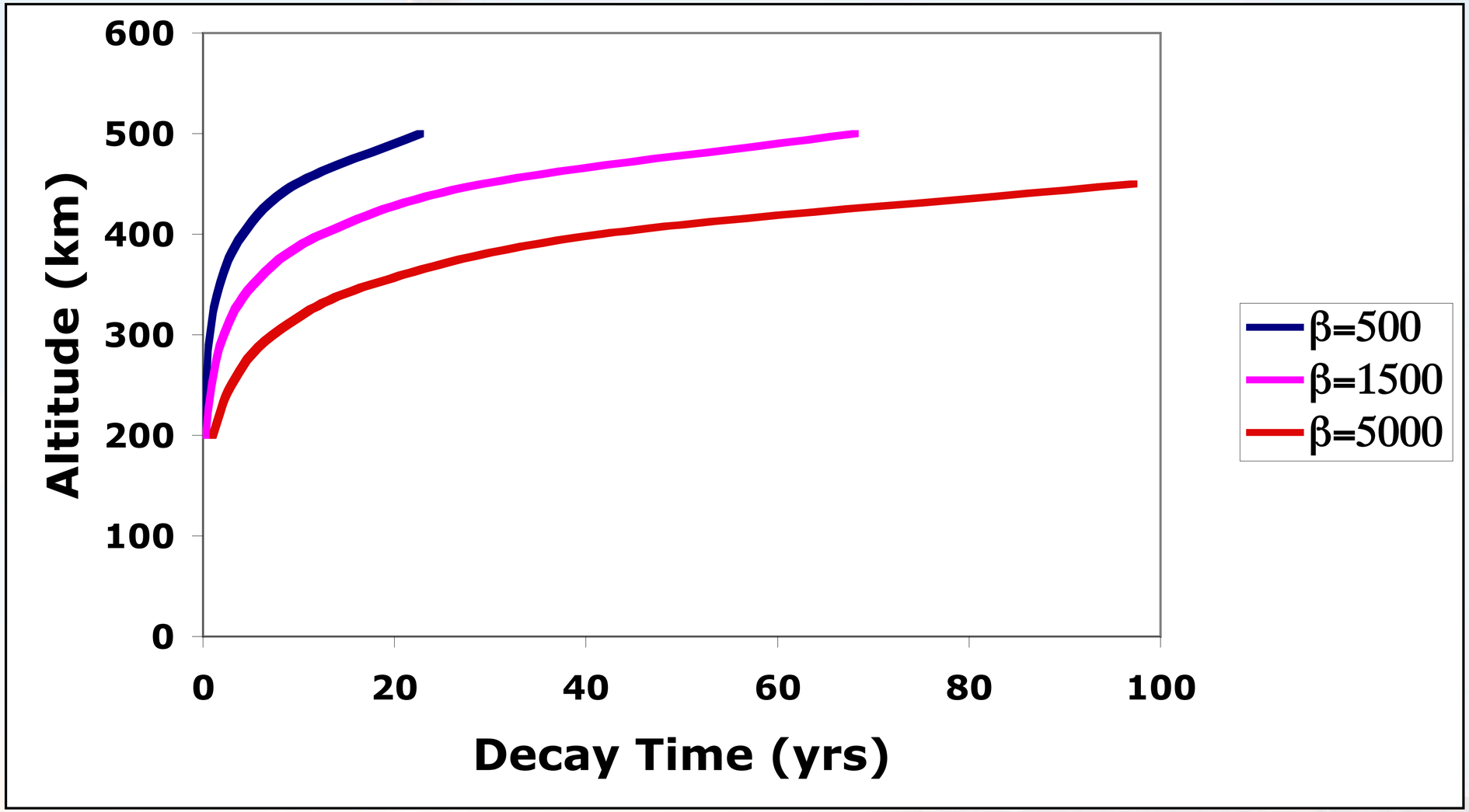
$$e^{\frac{h}{h_s}} - e^{\frac{h_o}{h_s}} = -\frac{\sqrt{\mu r_E}}{h_s \beta} \rho_o \left(t - t_o\right)$$

To find the time remaining (t_o=0) until the orbit reaches any given "critical" altitude, some algebra gives

$$t(h_{crit}) = \frac{h_s \beta}{\sqrt{\mu r_E} \rho_o} \left(e^{\frac{h_o}{h_s}} - e^{\frac{h_{crit}}{h_s}} \right)$$

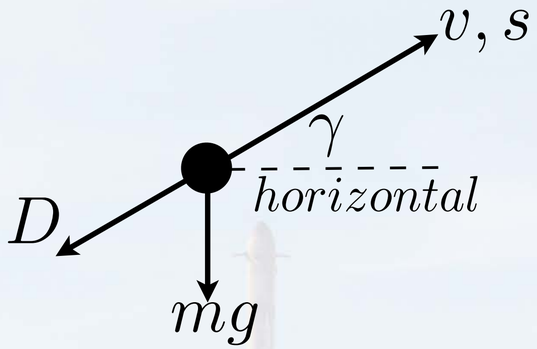
$$\boxed{t(h_{crit}) \propto \beta}$$

Decay Time to r=120 km



Ballistic Entry (no lift)

s = distance along the flight path



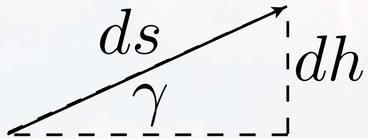
$$\frac{dv}{dt} = -g\sin\gamma - \frac{D}{m}$$

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = V\frac{dv}{ds} = \frac{1}{2}\frac{d(v^2)}{ds}$$

$$\frac{1}{2}\frac{d(v^2)}{ds} = -g\sin\gamma - \frac{D}{m}$$

$$\text{Drag } D \equiv \frac{1}{2}\rho v^2 A c_D$$

$$\frac{1}{2}\frac{d(v^2)}{ds} = -g\sin\gamma - \frac{\rho v^2}{2m}Ac_D$$



$$ds = \frac{dh}{\sin\gamma}$$

$$\frac{\sin\gamma}{2}\frac{d(v^2)}{dh} = -g\sin\gamma - \frac{\rho v^2}{2m}Ac_D$$

Ballistic Entry (2)

Exponential atmosphere $\Rightarrow \rho = \rho_o e^{-\frac{h}{h_s}}$

$$\frac{d\rho}{\rho_o} = e^{-\frac{h}{h_s}}\left(\frac{-dh}{h_s}\right) = \frac{\rho_o e^{-\frac{h}{h_s}}}{\rho_o}\left(\frac{-dh}{h_s}\right) = \frac{\rho}{\rho_o}\left(\frac{-dh}{h_s}\right)$$

$$dh = \frac{-h_s}{\rho} d\rho$$

$$\frac{\sin\gamma}{2}\frac{d(v^2)}{dh} = -g\sin\gamma - \frac{\rho v^2}{2m} A c_D$$

$$\frac{\sin\gamma}{2}\frac{d(v^2)}{d\rho}\left(\frac{-\rho}{h_s}\right) = -g\sin\gamma - \frac{\rho v^2}{2}\frac{A c_D}{m}$$

$$\frac{d(v^2)}{d\rho} = \frac{2gh_s}{\rho} + \frac{h_s v^2}{\sin\gamma}\frac{A c_D}{m}$$

Ballistic Entry (3)

Let $\beta \equiv \dfrac{m}{c_D A} \Rightarrow$ Ballistic Coefficient

$$\frac{d(v^2)}{d\rho} - \frac{h_s}{\beta \sin\gamma} v^2 = \frac{2gh_s}{\rho}$$

Assume $mg \ll D$ to get homogeneous ODE

$$\frac{d(v^2)}{d\rho} - \frac{h_s}{\beta \sin\gamma} v^2 = 0 \qquad \frac{d(v^2)}{v^2} = \frac{h_s}{\beta \sin\gamma} d\rho$$

Use $\left(v^2\right)$ as integration variable

$$\int_{v_e}^{v} \frac{d(v^2)}{v^2} = \frac{h_s}{\beta \sin\gamma} \int_0^{\rho} d\rho \qquad v_e = \text{velocity at entry}$$

Ballistic Entry (4)

Note that the effect of ignoring gravity is that there is no force perpendicular to velocity vector $\Rightarrow$ constant flight path angle $\gamma \Rightarrow$ straight line trajectories

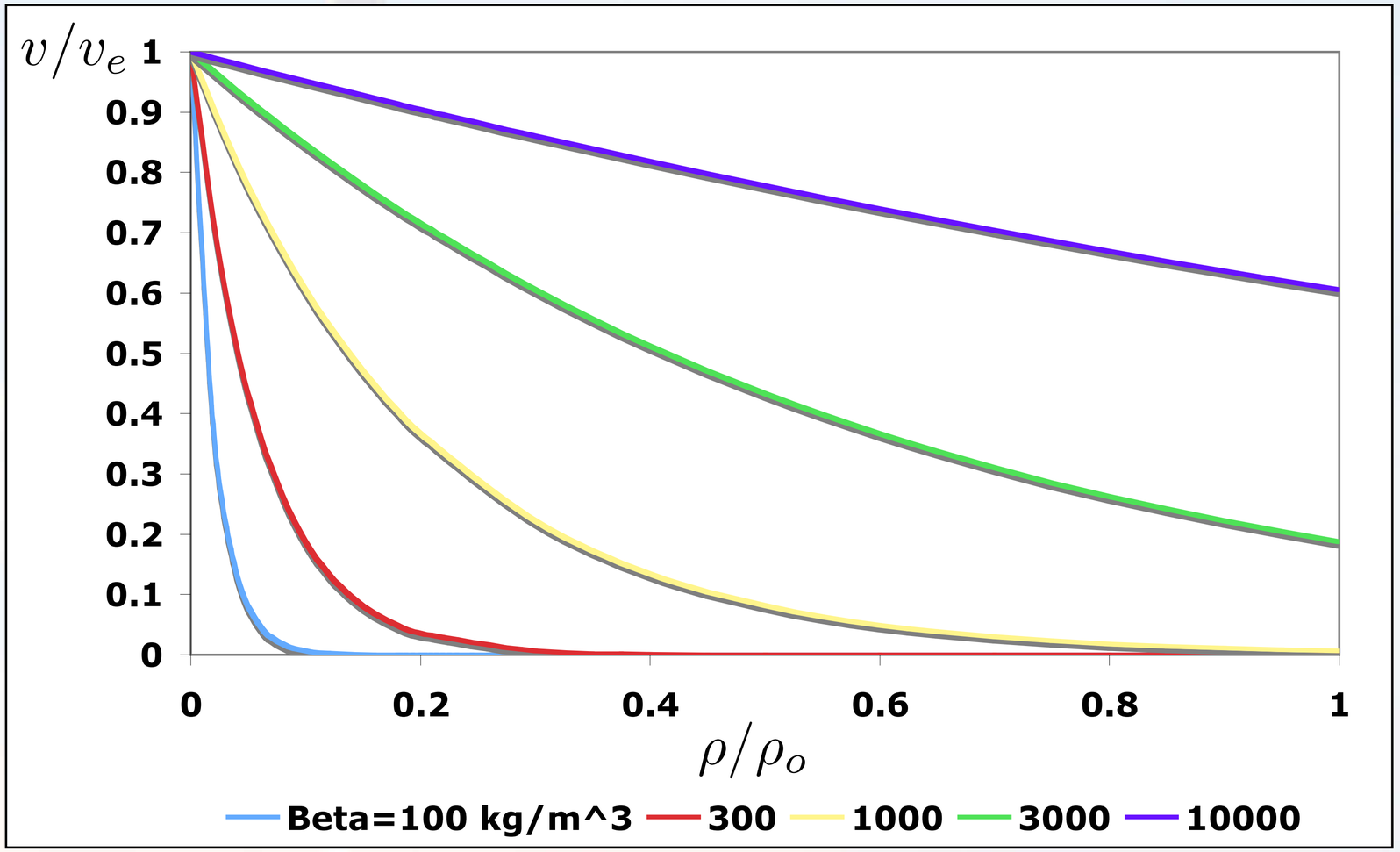
$$\ln \frac{v^2}{v_e^2} = 2 \ln \frac{v}{v_e} = \frac{h_s \rho}{\beta \sin \gamma}$$

$$\frac{v}{v_e} = \exp\left(\frac{h_s \rho}{2\beta \sin \gamma} \right)$$

$$\boxed{\frac{v}{v_e} = \exp\left(\frac{h_s \rho_o}{2\beta \sin \gamma} \frac{\rho}{\rho_o} \right)}$$

Check units: $\left(\frac{m \frac{kg}{m^3}}{\frac{kg}{m^2}} \right)$

Earth Entry, γ=-60°



What About Peak Deceleration?

$$n \equiv \frac{dv}{dt} = -\frac{\rho v^2}{2\beta}$$

To find n_{max}, set $\frac{d}{dt}\left(\frac{dv}{dt}\right) = \frac{d^2v}{dt^2} = 0$

$$\frac{d^2v}{dt^2} = -\frac{1}{2\beta}\left(2\rho v\frac{dv}{dt} + v^2\frac{d\rho}{dt}\right) = 0$$

$$\frac{d^2v}{dt^2} = -\frac{1}{2\beta}\left(-\frac{2\rho^2v^3}{2\beta} + v^2\frac{d\rho}{dt}\right) = 0$$

$$\frac{\rho^2v^3}{\beta} = v^2\frac{d\rho}{dt} \qquad \rho^2 v = \beta\frac{d\rho}{dt}$$

Peak Deceleration (2)

From exponential atmosphere,

$$\frac{d\rho}{dt} = -\frac{\rho_o}{h_s} e^{-\frac{h}{h_s}} \frac{dh}{dt} = -\frac{\rho}{h_s}\frac{dh}{dt}$$

From geometry, $\dfrac{dh}{dt} = v \sin\gamma$

$$\frac{d\rho}{dt} = -\frac{\rho v}{h_s}\sin\gamma \qquad \rho^2 v = \beta\frac{d\rho}{dt}$$

$$\rho^2 v = \beta\left(-\frac{\rho v}{h_s}\sin\gamma\right)$$

Remember that this refers to the conditions at max deceleration

$$\rho_{n_{max}} = -\frac{\beta}{h_s}\sin\gamma$$

Critical β for Deceleration Before Impact

At surface, $\rho = \rho_o$

$$\boxed{\beta_{crit} = -\frac{\rho_o h_s}{\sin\gamma}}$$ $\Leftarrow$ Value of β at which vehicle hits ground at point of maximum deceleration

How large is maximum deceleration?

$$\frac{dv}{dt} = \frac{\rho v^2}{2\beta} \qquad \Rightarrow \left|\frac{dv}{dt}\right|_{max} = \frac{\rho_{n_{max}} v^2}{2\beta}$$

$$\left|\frac{dv}{dt}\right|_{max} = \frac{v^2}{2\beta}\left(-\frac{\beta}{h_s}\sin\gamma\right) = -\frac{1}{2}\frac{v^2}{h_s}\sin\gamma$$

Note that this value of v is actually $v_{n_{max}}$

Peak Deceleration (3)

From page 14,

$$\frac{v}{v_e} = \exp\left(\frac{h_s \rho}{2\beta \sin\gamma}\right)$$

$$\frac{v_{n_{max}}}{v_e} = \exp\left[\frac{h_s}{2\beta \sin\gamma}\left(-\frac{\beta}{h_s}\sin\gamma\right)\right] = e^{-\frac{1}{2}}$$

$$\left|\frac{dv}{dt}\right|_{max} = -\frac{1}{2}\frac{\left(v_e e^{-\frac{1}{2}}\right)^2}{h_s}\sin\gamma = -\frac{v_e^2 \sin\gamma}{2h_s e}$$

Note that the velocity at which maximum deceleration occurs is always a fixed fraction of the entry velocity - it doesn't depend on ballistic coefficient, flight path angle, or anything else! Also, the magnitude of the maximum deceleration is not a function of ballistic coefficient - it is dependent on the entry trajectory (v_e and γ) but not spacecraft parameters (i.e., ballistic coefficient).

Terminal Velocity

Full form of ODE -

$$\frac{d\left(v^2\right)}{d\rho} - \frac{h_s}{\beta \sin\gamma} v^2 = \frac{2gh_s}{\rho}$$

At terminal velocity, $v = \text{constant} \equiv v_T$

$$-\frac{h_s}{\beta \sin\gamma} v_T^2 = \frac{2gh_s}{\rho}$$

$$\boxed{v_T^2 = \sqrt{-\frac{2g\beta \sin\gamma}{\rho}}}$$

"Cannon Ball" γ=-90° Ballistic Entry

6.75" diameter sphere, c_D=0.2, V_E=6000 m/sec

	Iron	Aluminum	Balsa Wood
Weight	**40 lb**	**15.6 lb**	**14.5 oz**
β (kg/m^2)	**3938**	**1532**	**89**
ρ_{md} (kg/m^3)	**0.555**	**0.216**	**0.0125**
h_{md} (m)	**5600**	**12,300**	**32,500**
V_{impact} (m/s)	**1998**	**355**	**0***
V_{term} (m/sec)	**251**	**156**	**38**

*Artifact of assumption that $D \gg mg$

Nondimensional Ballistic Coefficient

$$\frac{v}{v_e} = \exp\left(\frac{h_s \rho_o}{2\beta \sin\gamma}\frac{\rho}{\rho_o}\right) = \exp\left(\frac{P_o}{2\beta g \sin\gamma}\frac{\rho}{\rho_o}\right)$$

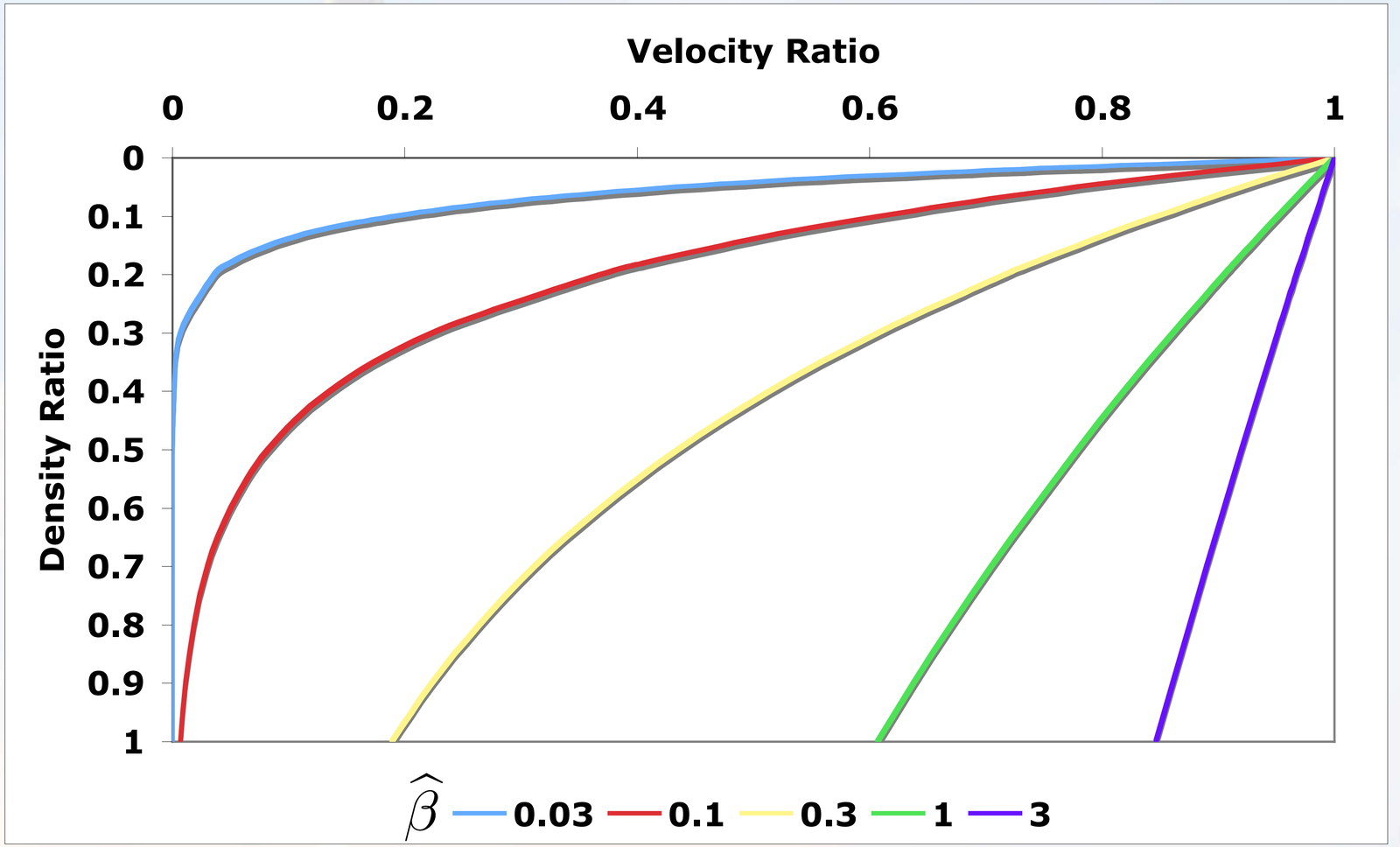
Let $\widehat{\beta} \equiv \dfrac{\beta}{\rho_o h_s} = \dfrac{\beta g}{P_o}$ (Nondimensional form of ballistic coefficient)

Note that we are using the estimated value of $P_o = \rho_o g h_s$, not the actual surface pressure.

$$\boxed{\frac{v}{v_e} = \exp\left(\frac{1}{2\widehat{\beta} \sin\gamma}\frac{\rho}{\rho_o}\right)}$$

$$\beta_{crit} = -\frac{\rho_o h_s}{\sin\gamma} \qquad \boxed{\widehat{\beta}_{crit} = -\frac{1}{\sin\gamma}}$$

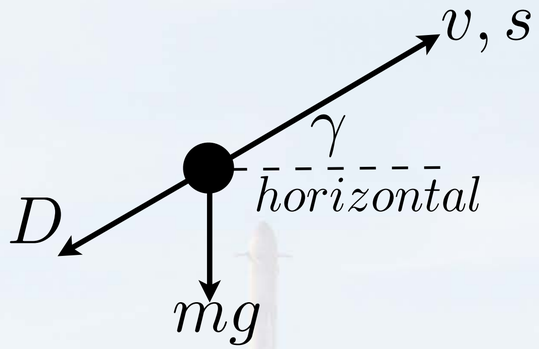
Entry Velocity Trends, γ=-90°



Ballistic Entry, Again (this time using altitude)

s = distance along the flight path

$$\frac{dv}{dt} = -g\sin\gamma - \frac{D}{m}$$



Again assuming $D \gg g$,

$$\frac{dv}{dt} = -\frac{D}{m}$$

$$\text{Drag } D \equiv \frac{1}{2}\rho v^2 A c_D$$

$$\frac{dv}{dt} = -\frac{\rho c_D A}{2m} v^2$$

Separating the variables,

$$\frac{dv}{v^2} = -\frac{\rho}{2\beta} dt$$

Calculating the Entry Velocity Profile

$$\frac{dh}{dt} = v\sin\gamma \Rightarrow dt = \frac{dh}{v\sin\gamma}$$

$$\frac{dv}{v^2} = -\frac{\rho}{2\beta v\sin\gamma}dh \quad \Rightarrow \frac{dv}{v} = -\frac{\rho}{2\beta\sin\gamma}dh$$

$$\frac{dv}{v} = -\frac{\rho_o}{2\beta\sin\gamma}e^{-\frac{h}{hs}}dh$$

$$\int_{v_e}^{v}\frac{dv}{v} = -\frac{\rho_o}{2\beta\sin\gamma}\int_{h_e}^{h}e^{-\frac{h}{hs}}dh$$

$$\ln\frac{v}{v_e} = \frac{\rho_o h_s}{2\beta\sin\gamma}\left[e^{-\frac{h}{hs}}\right]_{h_e}^{h} = \frac{1}{2\widehat{\beta}\sin\gamma}\left[e^{-\frac{h}{hs}} - e^{-\frac{h_e}{hs}}\right]$$

Deriving the Entry Velocity Function

Remember that $e^{-\frac{h_e}{h_s}} = \frac{\rho_e}{\rho_o} \approx 0$

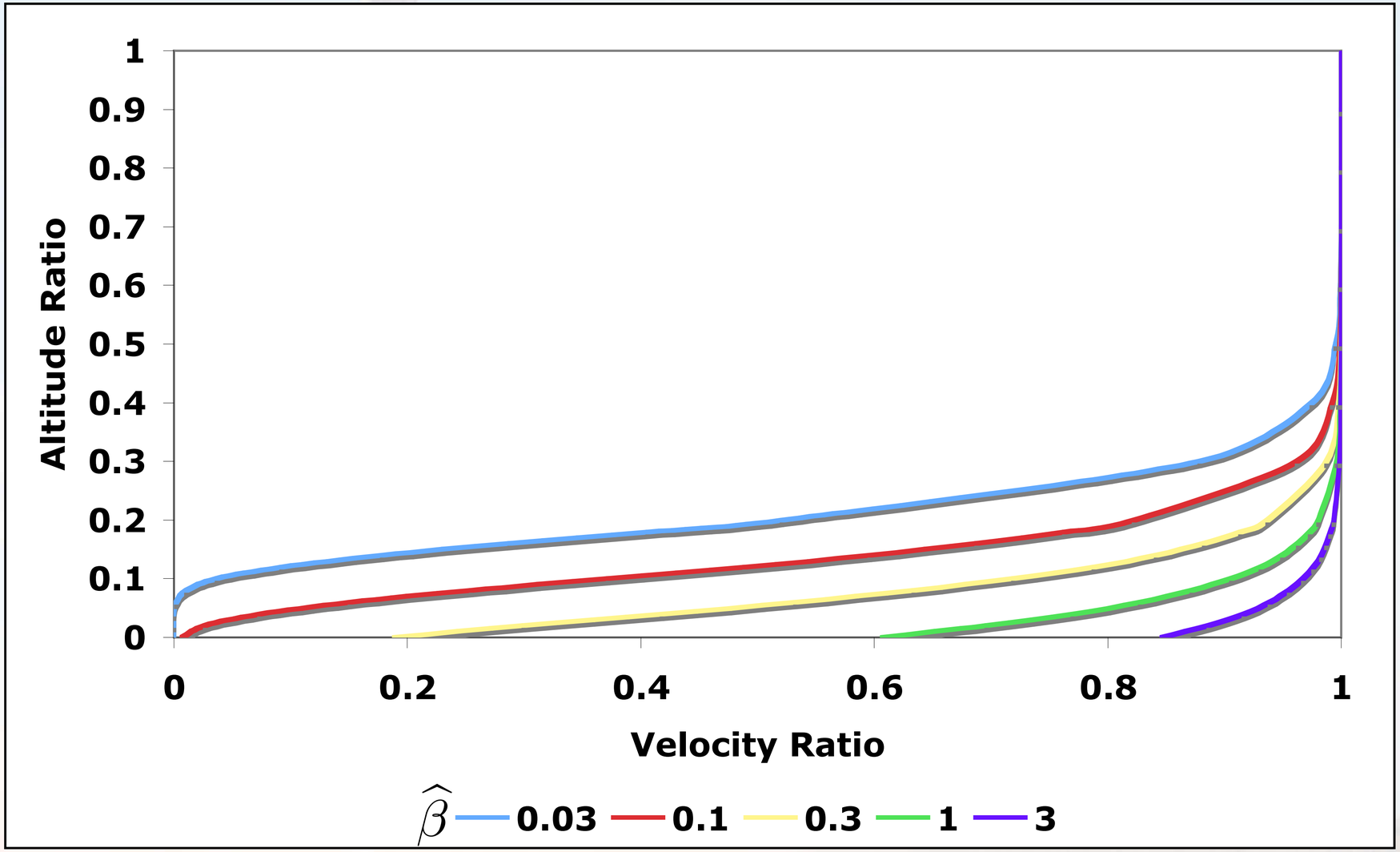
$$\frac{v}{v_e} = \exp\left[\frac{1}{2\widehat{\beta}\sin\gamma} e^{-\frac{h}{h_s}}\right]$$

We have a parametric entry equation in terms of nondimensional velocity ratios, ballistic coefficient, and altitude. To bound the nondimensional altitude variable between 0 and 1, rewrite as

$$\boxed{\frac{v}{v_e} = \exp\left[\frac{1}{2\widehat{\beta}\sin\gamma} e^{-\frac{h}{h_e}\frac{h_e}{h_s}}\right]}$$

$\frac{h_e}{h_s}$ and $\widehat{\beta}$ are the only variables that relate to a specific planet

Earth Entry, γ=-90°



Deceleration as a Function of Altitude

Start with

$$\frac{v}{v_e} = \exp\left[\frac{1}{2\widehat{\beta}\sin\gamma} e^{-\frac{h}{he}\frac{he}{hs}}\right]$$

Let $B \equiv \dfrac{1}{2\widehat{\beta}\sin\gamma}$

$$\frac{v}{v_e} = \exp\left(Be^{-\frac{h}{hs}}\right)$$

$$\frac{d}{dt}\left(\frac{v}{v_e}\right) = \exp\left(Be^{-\frac{h}{hs}}\right)\frac{d}{dt}\left(Be^{-\frac{h}{hs}}\right)$$

$$\frac{dv}{dt} = v_e \exp\left(Be^{-\frac{h}{hs}}\right)\frac{-B}{h_s}\left(e^{-\frac{h}{hs}}\right)\frac{dh}{dt}$$

$$\frac{dh}{dt} = v\sin\gamma \quad = v_e \sin\gamma \exp Be^{-\frac{h}{hs}}$$

Parametric Deceleration

$$\frac{dv}{dt} = v_e \exp\left(Be^{-\frac{h}{h_s}}\right) \frac{-B}{h_s} \left(e^{-\frac{h}{h_s}}\right) v_e \sin\gamma \exp\left(Be^{-\frac{h}{h_s}}\right)$$

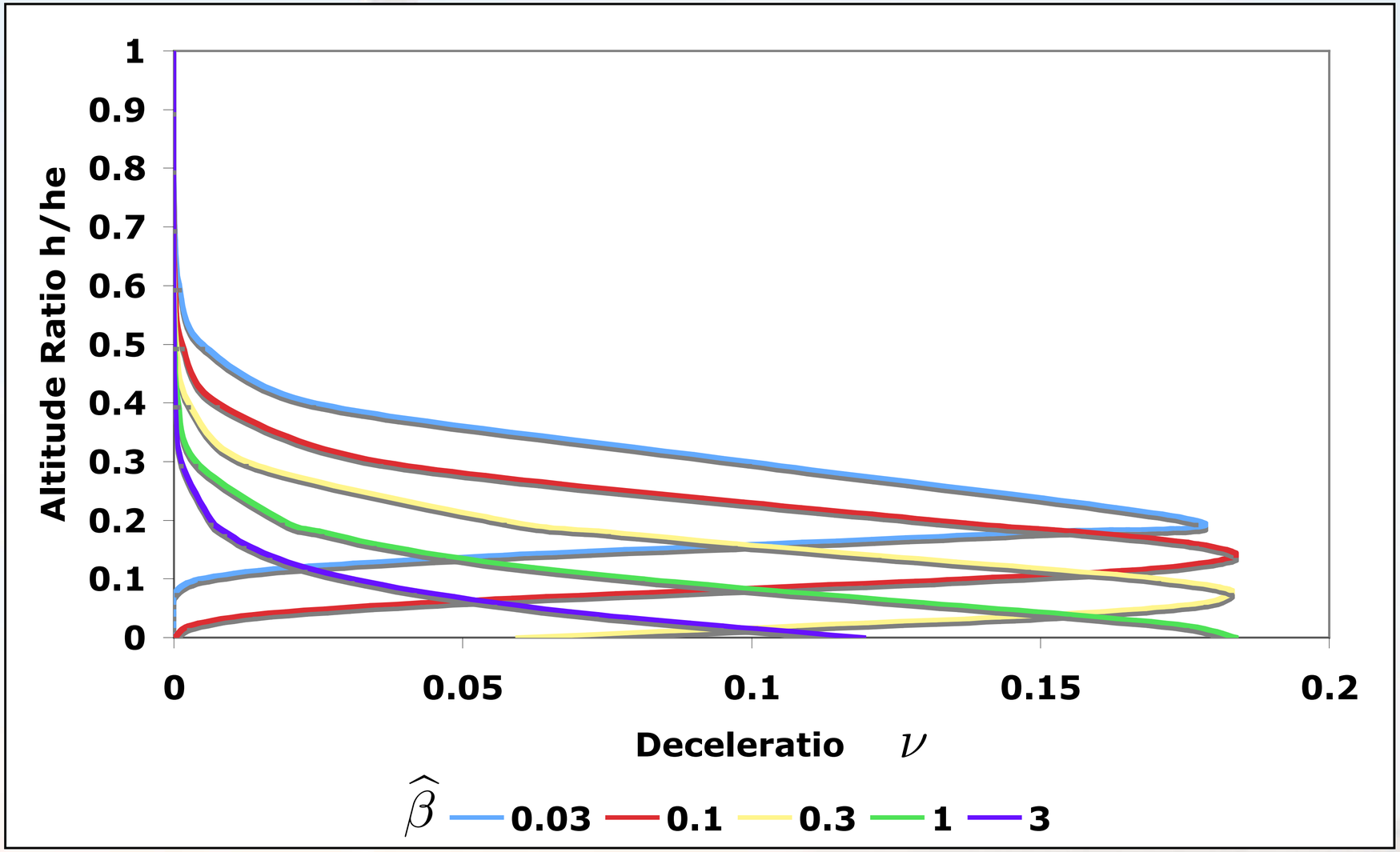
$$\frac{dv}{dt} = \frac{-Bv_e^2}{h_s} \sin\gamma \left(e^{-\frac{h}{h_s}}\right) \exp\left(2Be^{-\frac{h}{h_s}}\right)$$

$$\frac{dv}{dt} = \frac{-v_e^2}{2h_s\widehat{\beta}} \left(e^{-\frac{h}{h_s}}\right) \exp\left(\frac{1}{\widehat{\beta}\sin\gamma} e^{-\frac{h}{h_s}}\right)$$

$$\text{Let } n_{ref} \equiv \frac{v_e^2}{h_s},\ \nu \equiv \frac{dv/dt}{n_{ref}},\ \varphi \equiv \frac{h_e}{h_s}$$

$$\nu = \frac{-1}{2\widehat{\beta}} \left(e^{-\varphi\frac{h}{h_e}}\right) \exp\left(\frac{1}{\widehat{\beta}\sin\gamma} e^{-\varphi\frac{h}{h_e}}\right)$$

Nondimensional Deceleration, γ=-90°



Deceleration Equations

Nondimensional Form

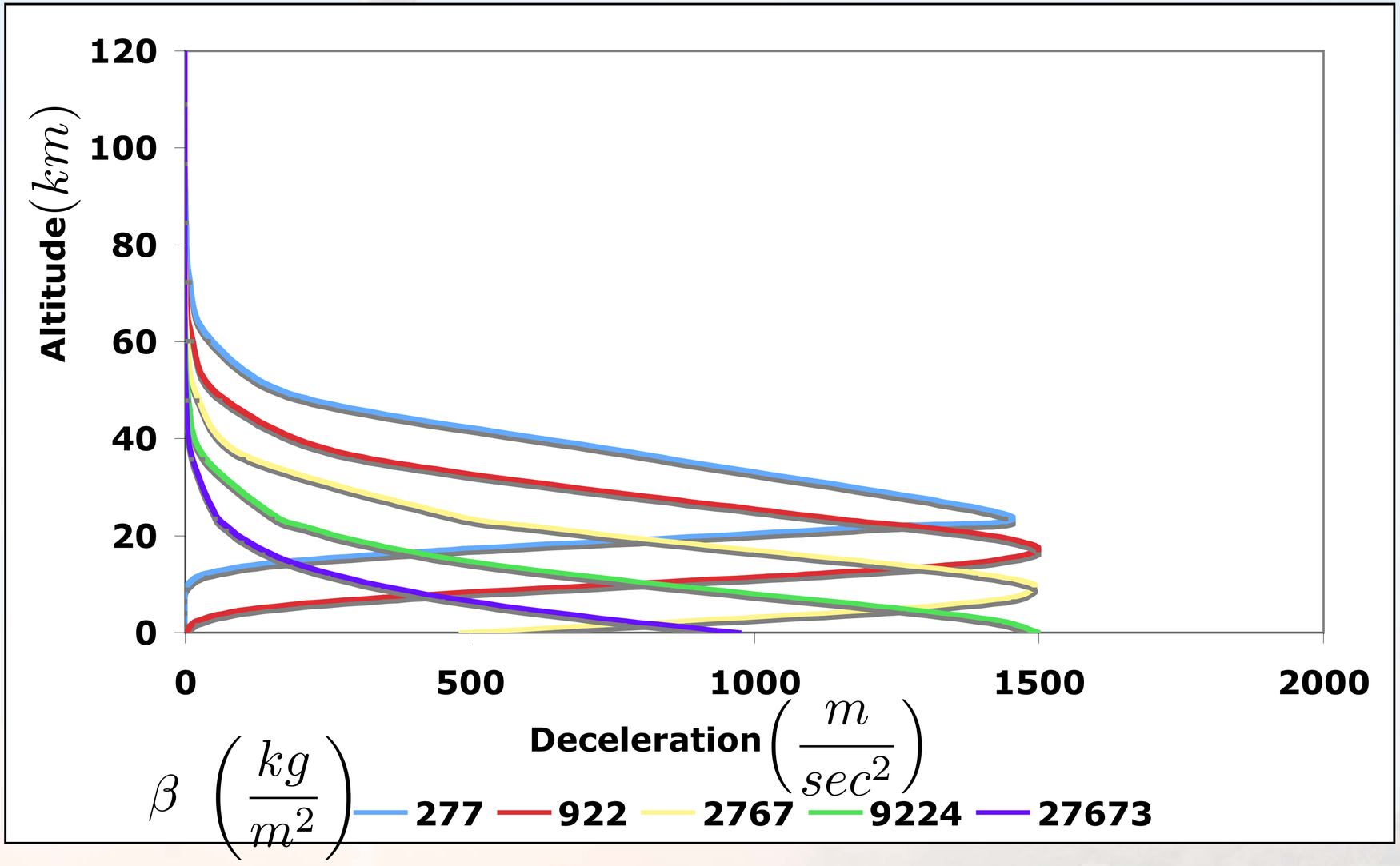
$$\nu = \frac{-1}{2\widehat{\beta}} \left(e^{-\varphi \frac{h}{h_e}} \right) \exp \left(\frac{1}{\widehat{\beta} \sin \gamma} e^{-\varphi \frac{h}{h_e}} \right)$$

Dimensional Form

$$n = \frac{-\rho_o V_e^2}{2\beta} \left(e^{-\frac{h}{h_s}} \right) \exp \left(\frac{\rho_o h_s}{\beta \sin \gamma} e^{-\frac{h}{h_s}} \right)$$

Note that these equations result in values <0 (reflecting deceleration) - graphs are absolute values of deceleration for clarity.

Dimensional Deceleration, γ=-90°



Altitude of Maximum Deceleration

Returning to shorthand notation for deceleration

$$\nu = -B \sin\gamma \left(e^{-\frac{h}{h_s}}\right) \exp\left(2Be^{-\frac{h}{h_s}}\right)$$

$$\text{Let } \eta \equiv \frac{h}{h_s}$$

$$\nu = -B \sin\gamma \left(e^{-\eta}\right) \exp\left(2Be^{-\eta}\right)$$

$$\frac{d\nu}{d\eta} = -B \sin\gamma \left[\frac{d}{d\eta}\left(e^{-\eta}\right) \exp\left(2Be^{-\eta}\right) + \left(e^{-\eta}\right) \frac{d}{d\eta} \exp\left(2Be^{-\eta}\right)\right]$$

$$\frac{d\nu}{d\eta} = -B \sin\gamma \left[-\left(e^{-\eta}\right) \exp\left(2Be^{-\eta}\right) + \left(e^{-\eta}\right) \left(-2Be^{-\eta}\right) \exp\left(2Be^{-\eta}\right)\right]$$

$$\frac{d\nu}{d\eta} = B \sin\gamma e^{-\eta} \exp\left(2Be^{-\eta}\right) \left[1 + \left(2Be^{-\eta}\right)\right] = 0$$

Altitude of Maximum Deceleration

$$1 + \left(2Be^{-\eta}\right) = 0 \quad \Rightarrow e^{\eta} = -2B$$

$$\eta_{n_{max}} = \ln\left(-2B\right)$$

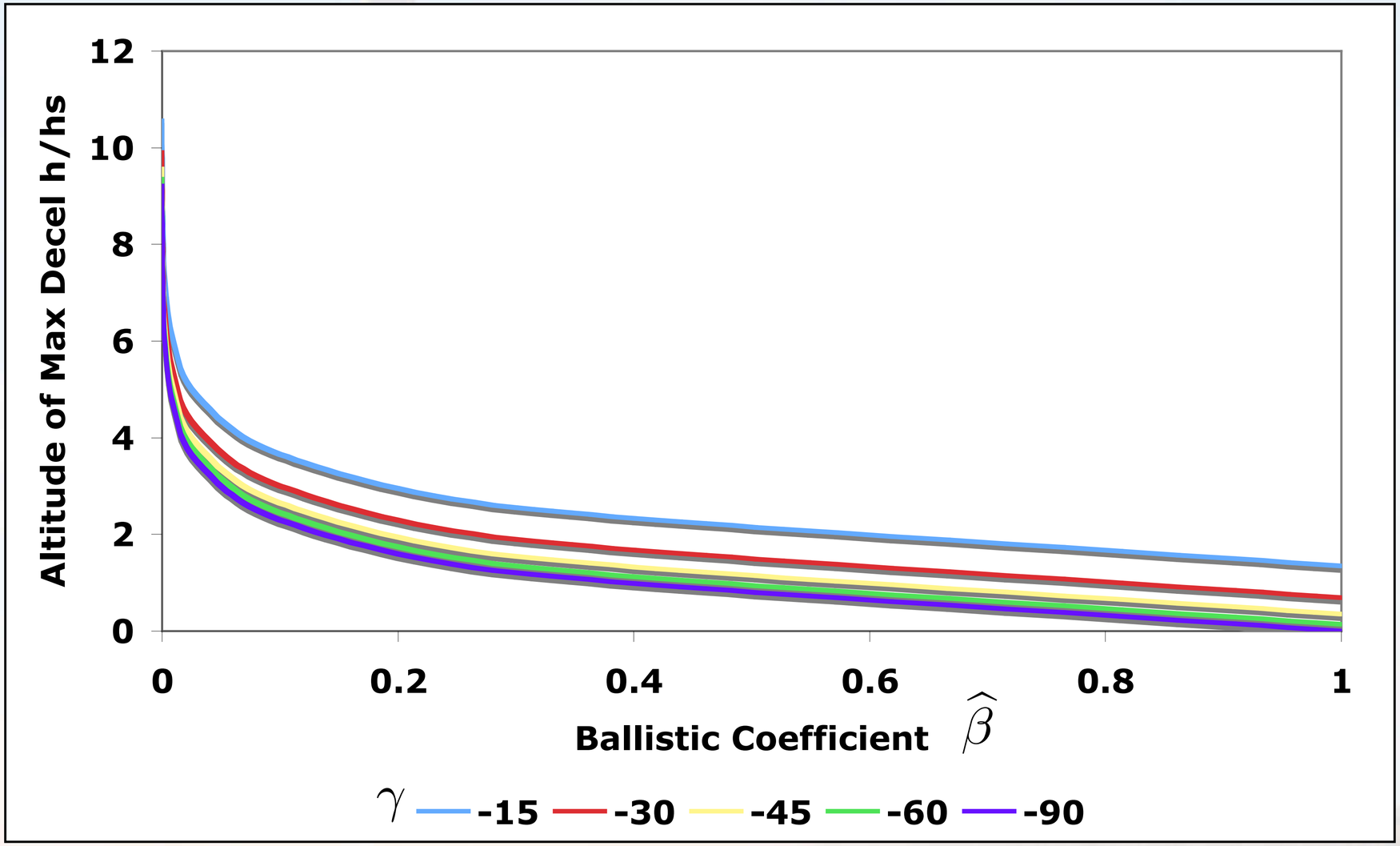
$$\boxed{\eta_{n_{max}} = \ln\left(\frac{-1}{\widehat{\beta}\sin\gamma}\right)}$$

Converting from parametric to dimensional form gives

$$\boxed{h_{n_{max}} = h_s \ln\left(\frac{-\rho_o h_s}{\beta\sin\gamma}\right)}$$

Altitude of maximum deceleration is independent of entry velocity!

Altitude of Maximum Deceleration



Magnitude of Maximum Deceleration

Start with the equation for acceleration -

$$\nu = \frac{-1}{2\widehat{\beta}} e^{-\eta} \exp\left(\frac{1}{\widehat{\beta}\sin\gamma} e^{-\eta}\right)$$

and insert the value of η at the point of maximum deceleration

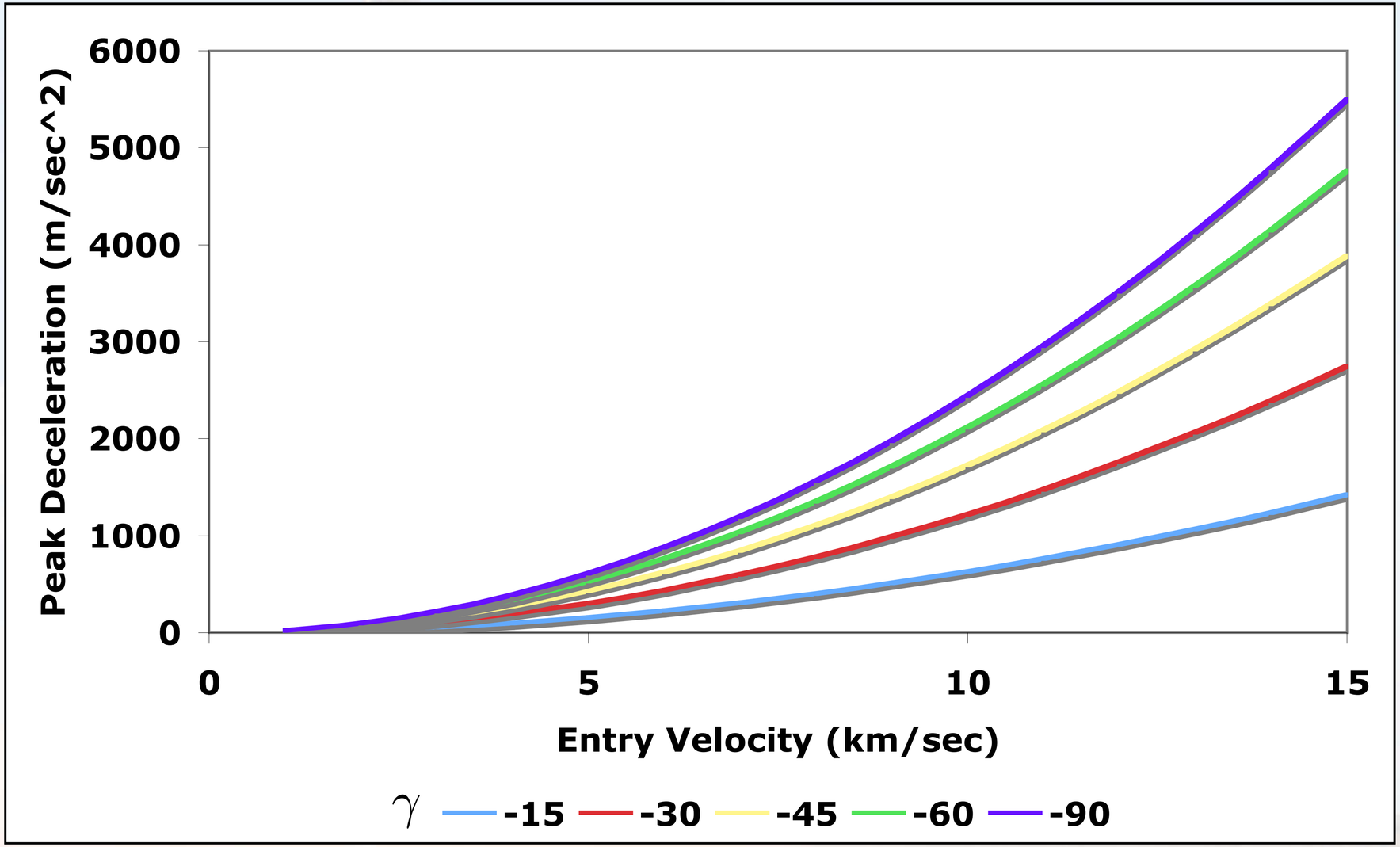
$$\eta_{n_{max}} = \ln\left(\frac{-1}{\widehat{\beta}\sin\gamma}\right) \quad \Rightarrow e^{-\eta} = -\widehat{\beta}\sin\gamma$$

$$\nu_{n_{max}} = \frac{-1}{2\widehat{\beta}}\left(-\widehat{\beta}\sin\gamma\right) \exp\left(\frac{-\widehat{\beta}\sin\gamma}{\widehat{\beta}\sin\gamma}\right) \quad \Rightarrow \nu_{n_{max}} = \frac{\sin\gamma}{2e}$$

$$\boxed{n_{max} = \frac{v_e^2}{h_s}\frac{\sin\gamma}{2e}}$$

Maximum deceleration is not a function of ballistic coefficient!

Peak Ballistic Deceleration for Earth Entry



Velocity at Maximum Deceleration

Start with the equation for velocity

$$\frac{v}{v_e} = \exp\left[\frac{1}{2\widehat{\beta}\sin\gamma}e^{-\eta}\right]$$

and insert the value of η at the point of maximum deceleration

$$\eta_{n_{max}} = \ln\left(\frac{-1}{\widehat{\beta}\sin\gamma}\right) \quad \Rightarrow e^{-\eta} = -\widehat{\beta}\sin\gamma$$

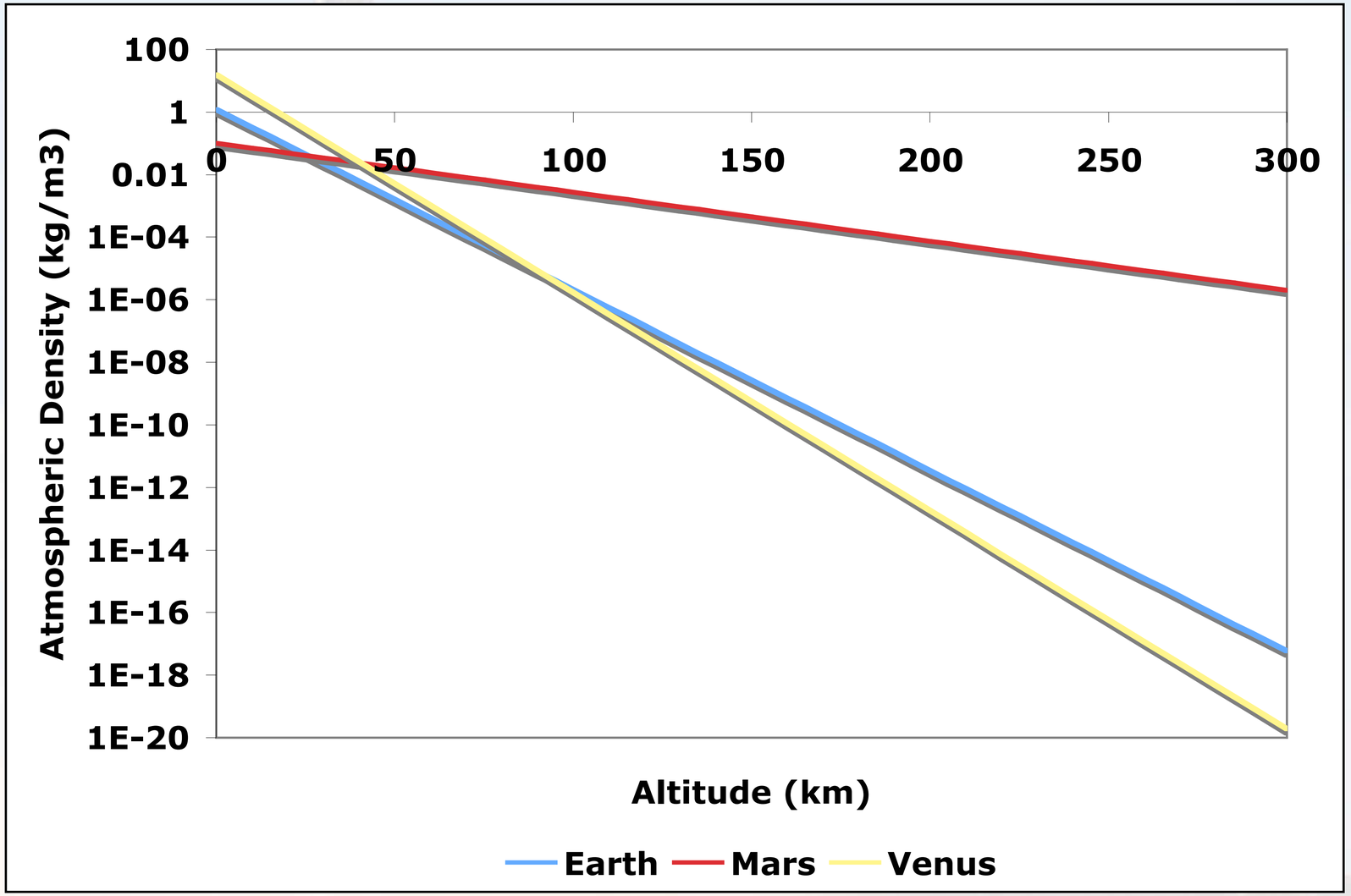
$$\frac{v}{v_e} = \exp\left[\frac{-\widehat{\beta}\sin\gamma}{2\widehat{\beta}\sin\gamma}\right] \quad \Rightarrow \boxed{v_{n_{max}} = \frac{v_e}{\sqrt{e}} \cong 0.606 v_e}$$

Velocity at maximum deceleration is independent of everything except v_e

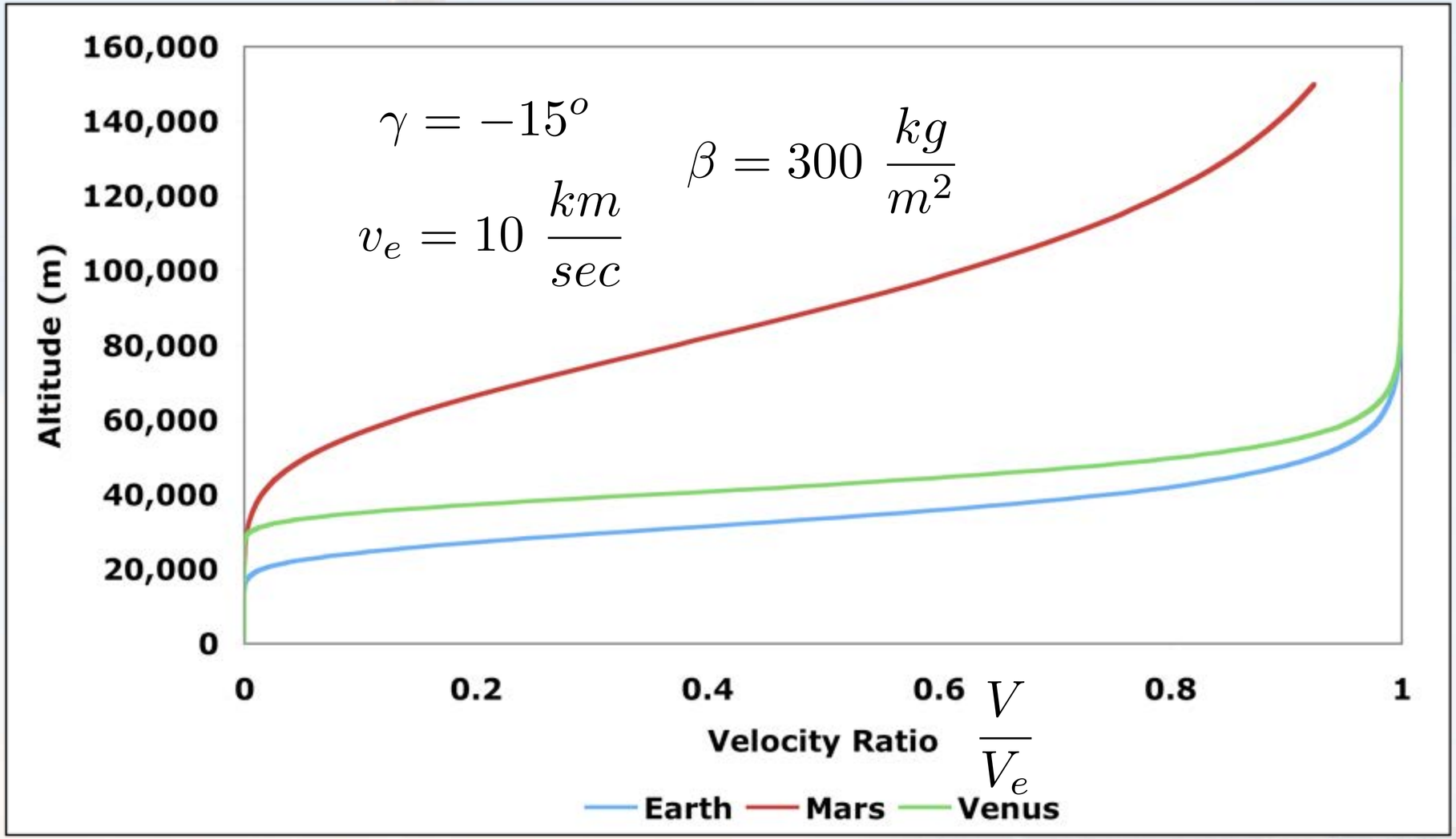
Planetary Entry - Physical Data

	Radius (km)	μ (km^3/sec^2)	ρ_o (kg/m^3)	h_s (km)	v_{esc} (km/sec)
Earth	6378	398,604	1.225	7.524	11.18
Mars	3393	42,840	0.0993	27.7	5.025
Venus	6052	325,600	16.02	6.227	10.37

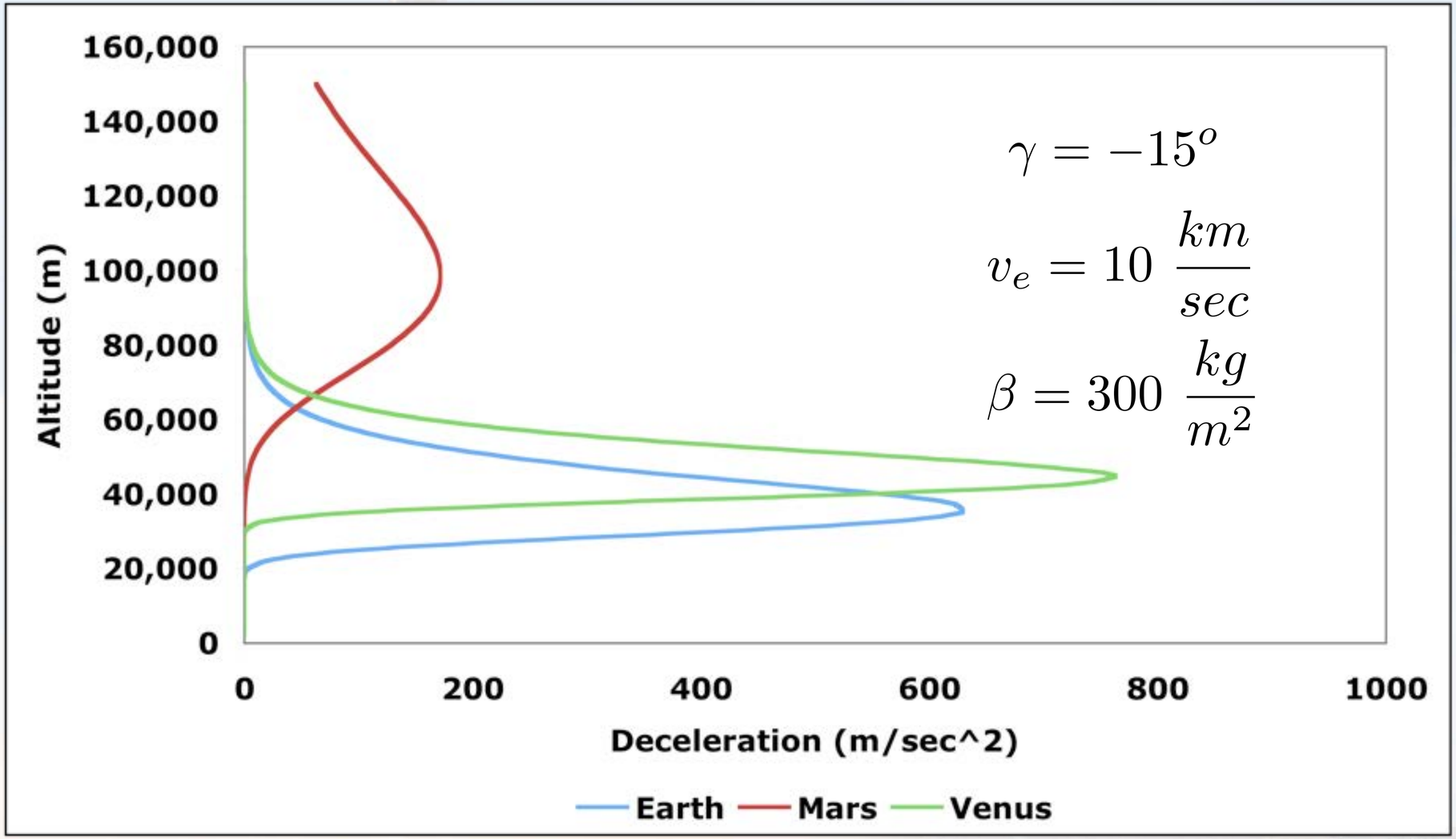
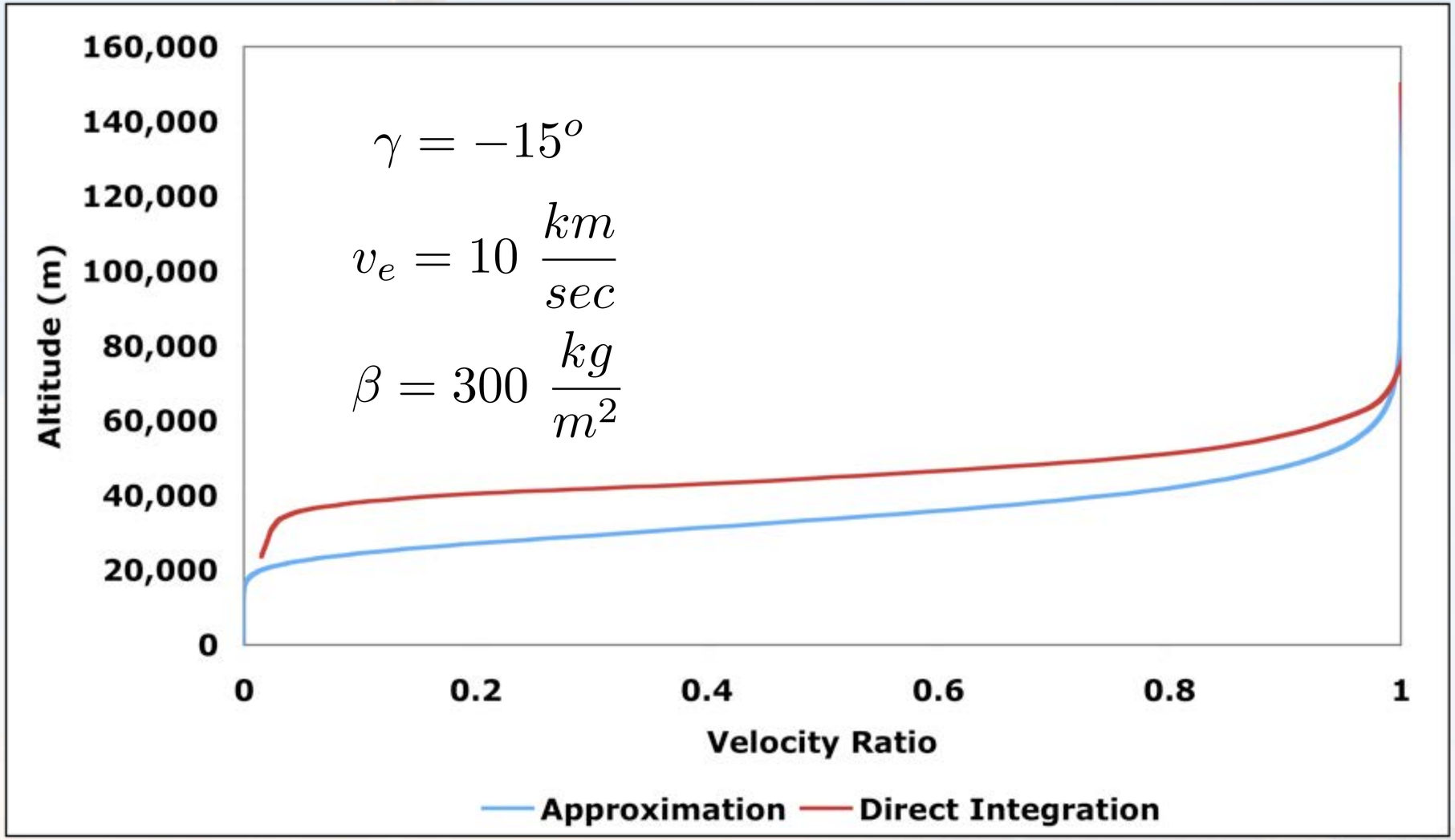
Comparison of Planetary Atmospheres



Planetary Entry Profiles



Planetary Entry Deceleration Comparison



Check on Approximation Formulas