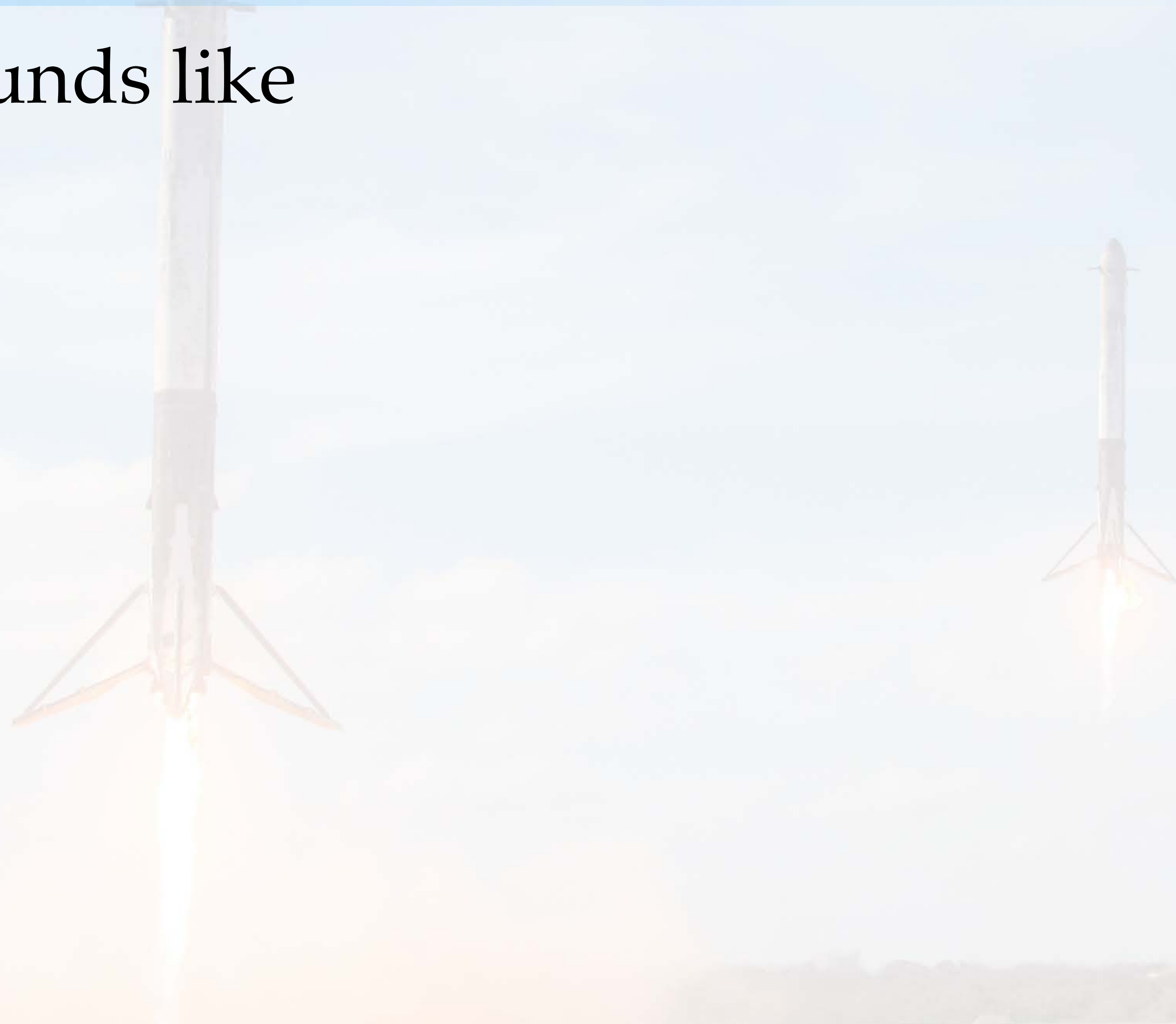


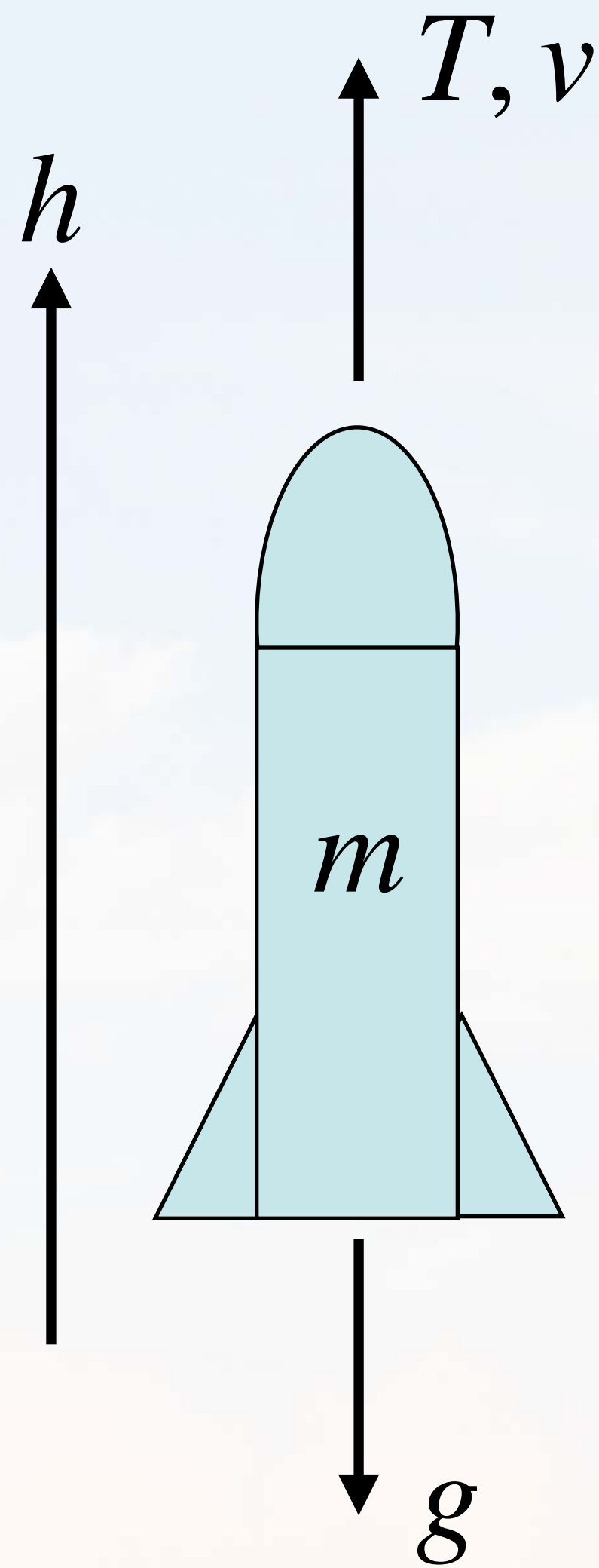
Sounding Rocket Trajectory Analysis

- It is what it sounds like



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Sounding Rocket Analysis



$$\frac{dh}{dt} = v$$

$$m \frac{dv}{dt} = T - mg$$

$$T = \dot{m} v_e, \quad \dot{m} = - \frac{dm}{dt}$$

$$m \frac{dv}{dt} = - \frac{dm}{dt} v_e - mg$$

$$dv = - v_e \frac{dm}{m} - g dt$$

Integrating Accelerations for Velocity

$$\int_0^v dv = -g \int_0^t dt - v_e \int_{m_0}^m \frac{dm}{m}$$

Assume that thrust is constant $\rightarrow \dot{m}$ is constant

$$m = m_0 - \dot{m}t \quad \Longrightarrow \quad t = \frac{m_0 - m}{\dot{m}}$$

$$v = -gt - v_e \ln \frac{m}{m_0} = \frac{dh}{dt}$$

Gravity loss



Integrating Velocity for Altitude

$$\frac{dh}{dt} = -gt - v_e \ln \frac{m_0 - \dot{m}t}{m_0}$$

$$\int_0^h dh = -g \int_0^t t dt - v_e \int_0^t \ln \frac{m_0 - \dot{m}t}{m_0} dt$$

$$\text{Let } u = \frac{m_0 - \dot{m}t}{m_0} \quad \implies \quad du = -\frac{\dot{m}}{m_0} dt$$

$$\text{Limits: } t = 0 \implies u = 1; \quad t = t_f \implies u_f = \frac{m_f}{m_0}$$



Solving for Altitude at Burnout

$$\int_0^{h_{bo}} dh = -g \int_0^{t_{bo}} t dt + \frac{v_e m_0}{\dot{m}} \int_1^{u_f} \ln u du$$

$$h_{bo} = -\frac{1}{2} g t_{bo}^2 + \frac{v_e m_0}{\dot{m}} \left[u \ln u - u \right]_1^{u_f}$$

$$h_{bo} = -\frac{1}{2} g t_{bo}^2 + \frac{v_e}{\dot{m}} \left[m_f \ln \frac{m_f}{m_0} - m_f + m_0 \right]$$

$$\dot{m} = \frac{m_0 - m_f}{t_{bo}}$$



Maximizing Burnout Altitude

$$v_{bo} = -gt_{bo} - v_e \ln \frac{m_f}{m_0}$$

$$h_{bo} = -\frac{1}{2}gt_{bo}^2 + \frac{v_e t_{bo}}{m_0 - m_f} \left[m_f \ln \frac{m_f}{m_0} - m_f + m_0 \right]$$

$$\frac{\partial h_{bo}}{\partial t_{bo}} = -gt_{bo} + \frac{v_e}{m_0 - m_f} \left[m_f \ln \frac{m_f}{m_0} - m_f + m_0 \right] = 0$$

For maximum burnout altitude, $t_{bo} \longrightarrow 0$



Solving for Height Attained After Burnout

$$\text{Energy } E = \frac{1}{2}mv^2 + mgh = \text{constant}$$

$$\frac{1}{2}mv_{bo}^2 + mgh_{bo} = mgh_{max}$$

$$\frac{1}{2g} \left(-gt_{bo} - v_e \ln \frac{m_f}{m_0} \right)^2 - \frac{1}{2}gt_{bo}^2 + \frac{v_e t_{bo}}{m_0 - m_f} \left(m_f \ln \frac{m_f}{m_0} - m_f + m_0 \right) = h_{max}$$

Maximum Altitude

Simplifying assumption: as $t_{bo} \rightarrow 0$, $h_{bo} \rightarrow 0$

$$\frac{1}{2g} v_{bo}^2 \approx h_{max}$$

$$\frac{v_e^2}{2g} \ln^2 \left(\frac{m_f}{m_0} \right) \approx h_{max}$$

Example: $\frac{m_f}{m_0} = 0.2$, $v_e = 3000 \frac{m}{sec} \implies h_{max} = 1189 \text{ km}$



Variation of Altitude with Burn Time

t_{bo} (sec)	v_{bo} (m/sec)	h_{bo} (km)	h_{max} (km)
0	4828	0	1189
10	4730	17	1142
20	4632	34	1095
30	4534	49	1049
40	4436	64	1004
50	4338	77	960
60	4240	90	917

