Mass Estimating Relationships

- Review of iterative design approach
- Mass Estimating Relationships (MERs)
- Sample vehicle design analysis

Overview of the Design Process

Program Objectives ⇔ System Requirements

Vehicle-level Estimation (based on a few parameters from prior art)

System-level Estimation (system parameters based on prior experience)

System-level Design (based on discipline-oriented analysis)

Vehicle-Level Preliminary Design - 1st Pass

- Single Stage to Orbit (SSTO) vehicle
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec
  - δ=0.08

$$r = e^{-\frac{\Delta V}{g I_{sp}}} = 0.1127$$

$$\lambda = r - \delta = 0.0327$$

$$M_0 = \frac{M_L}{\lambda} = 153{,}000\ kg$$

$$M_i = \delta M_0 = 12{,}240\ kg$$

$$M_p = M_0 (1 - r) = 135{,}800\ kg$$

System-Level Estimation

- Start with propellant tanks (biggest part)
- LOX/LH2 engines generally run at mixture ratio of 6:1 (by weight)
  - LH2: 19,390 kg
  - LOX: 116,400 kg
- Propellant densities

$$\rho_{LOX} = 1140 \frac{kg}{m^3} \qquad \rho_{LH_2} = 112 \frac{kg}{m^3}$$

LOX Tank MERs

- Mass of Tank

$$M_{LOX\ Tank}\left(kg\right) = 0.0152\,M_{LOX}\left(kg\right) + 318$$

- Mass of Insulation

$$M_{LOX\ Insulation}\left(kg\right) = 1.123\,\frac{kg}{m^2}$$

LOX Tank Design

- Mass of LOX=116,400 kg

$$M_{LOX\ Tank}(kg) = 0.0152(116,400) + 318 = 2087\ kg$$

- Need area to find LOX tank insulation mass
  - assume a sphere

$$V_{LOX\ Tank} = M_{LOX} / \rho_{LOX} = 102.1\ m^3$$

$$r_{LOX\ Tank} = \left(\frac{V_{LOX}}{4\pi/3}\right)^{1/3} = 2.90\ m$$

$$A_{LOX\ Tank} = 4\pi r_{LOX}^2 = 105.6\ m^2$$

$$M_{LOX\ Insulation}(kg) = 1.123\frac{kg}{m^2}\left(105.6\ m^2\right) = 119\ kg$$

LH2 Tank MERs

- Mass of Tank

$$M_{LH_2\ Tank}(kg) = 0.0694 M_{LH_2}(kg) + 363$$

- Mass of Insulation

$$M_{LH_2\ Insulation}(kg) = 2.88 \frac{kg}{m^2}$$

LH2 Tank Design

- Mass of LH2=19,390 kg

$$M_{LH_2\ Tank}(kg) = 0.0694(19,390) + 363 = 1709 kg$$

- Again, assume LH2 tank is spherical

$$V_{LH_2\ Tank} = M_{LH_2} / \rho_{LH_2} = 346.3\ m^3$$

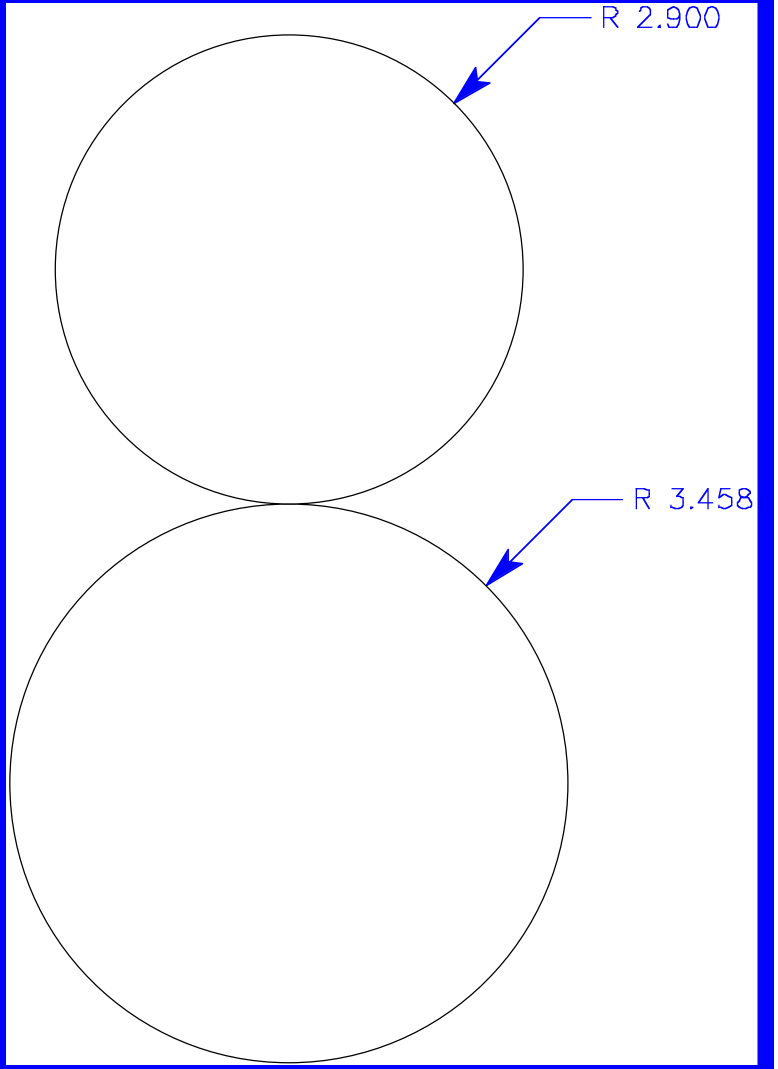
$$r_{LH_2\ Tank} = \left(\frac{V_{LH_2}}{4\pi / 3} \right)^{1/3} = 3.46\ m$$

$$A_{LH_2\ Tank} = 4\pi r_{LH_2}^2 = 150.2\ m^2$$

$$M_{LH_2\ Insulation}(kg) = 2.88 \frac{kg}{m^2} \left(150.2\ m^2 \right) = 433\ kg$$

Current Design Sketch

- Masses
  - LOX Tank 2089 kg
  - LOX Tank Insulation 1709 kg
  - LH2 Tank 119 kg
  - LH2 Tank Insulation 433 kg



Other Tankage MERs

- Storable Propellants (RP-1, N_2O_4, N_2H_4)

$$M_{Storables\ Tank}(kg) = 0.316\left[M_{Storables}(kg)\right]^{.6}$$

- Small tank (liquids)

$$M_{Small\ Liquid\ Tank}(kg) = 0.1 M_{contents}(kg)$$

- Small tank (pressurized gases)

$$M_{Small\ Gas\ Tank}(kg) = 2 M_{contents}(kg)$$

Boost Module Propellant Tanks

- Gross mass 23,000 kg
  - Inert mass 2300 kg
  - Propellant mass 20,700 kg
  - Mixture ratio N_2O_4/A50 = 1.8 (by mass)
- N_2O_4 tank
  - Mass = 13,310 kg
  - Density = 1450 kg/m^3
  - Volume = 9.177 m^3 --> r_{sphere}=1.299 m
- Aerozine 50 tank
  - Mass = 7390 kg
  - Density = 900 kg/m^3
  - Volume = 8.214 m^3 --> r_{sphere}=1.252 m

N_2O_4 Tank Sizing

- Need total N_2O_4 volume = 9.177 m^3
- Single tank
  - Radius = 1.299 m
  - Mass = 94.2 kg
- Dual tanks
  - Radius = 1.031 m
  - Mass = 62.2 kg (x2 = 124.3 kg)
- Triple tanks
  - Radius = 0.900 m
  - Mass = 48.7 kg (x3 = 146.2 kg)

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Other Structural MERs

- Fairings and shrouds

$$M_{Fairing}(kg) = 13.3\frac{kg}{m^2}$$

- Avionics

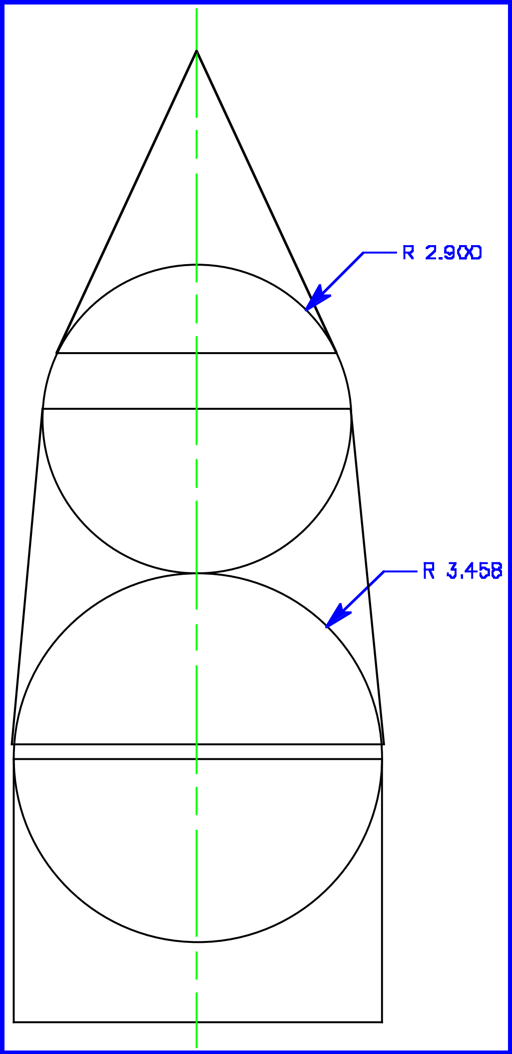
$$M_{Avionics}(kg) = 10\left[M_0(kg)\right]^{0.361}$$

- Wiring

$$M_{Wiring}(kg) = 1.058\sqrt{M_0(kg)}\ \ell^{0.25}$$

Fairing Analysis

- Payload Shroud
  - Area 51.74 m²
  - Mass 688 kg
- Intertank Fairing
  - Area 126.1 m²
  - Mass 1677 kg
- Aft Fairing
  - Area 107.7 m²
  - Mass 1433 kg



Avionics and Wiring Masses

- Avionics

$$M_{Avionics}(kg) = 10\left[153,000\ kg\right]^{0.361} = 744\ kg$$

- Wiring

$$M_{Wiring}(kg) = 1.058\sqrt{153,000}\,(16.95\ m)^{0.25} = 840\ kg$$

Propulsion MERs

- Liquid Pump-Fed Rocket Engine Mass

$$M_{Rocket\ Engine}(kg) = 7.81\times10^{-4}T(N) + 3.37\times10^{-5}T(N)\sqrt{\frac{A_e}{A_t}} + 59$$

- Solid Rocket Motor

$$M_{Motor\ Casing} = 0.135 M_{propellants}$$

- Thrust Structure Mass

$$M_{Thrust\ Structure}(kg) = 2.55\times10^{-4}T(N)$$

Propulsion MERs (continued)

- Gimbal Mass

$$M_{Gimbals}(kg) = 237.8\left[\frac{T(N)}{P_0(Pa)}\right]^{.9375}$$

- Gimbal Torque

$$\tau_{Gimbals}(N \cdot m) = 990,000\left[\frac{T(N)}{P_0(Pa)}\right]^{1.25}$$

Propulsion System Assumptions

- Initial T/mg ratio = 1.3
  - Keeps final acceleration low with reasonable throttling
- Number of engines = 6
  - Positive acceleration worst-case after engine out
- Chamber pressure = 1000 psi = 6897 kN
  - Typical for high-performance LOX/LH2 engines
- Expansion ratio A_e/A_t=30
  - Compromise ratio with good vacuum performance

Propulsion Mass Estimates

- Rocket Engine Thrust (each)

$$T(N) = \frac{m_0 g (T/W)_0}{n_{engines}} = 324,900 \ N$$

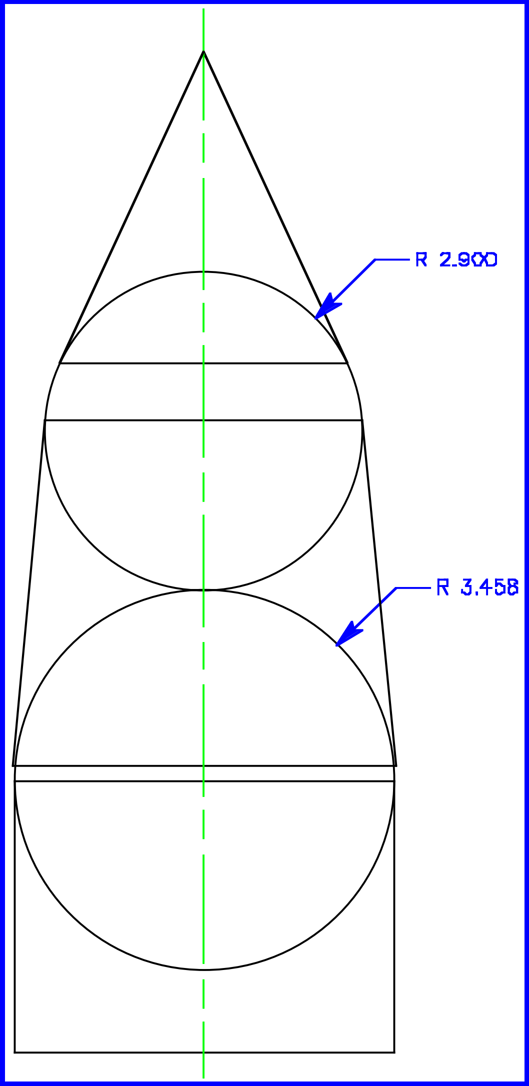
- Rocket Engine Mass (each)

$$M_{Rocket\ Engine}(kg) = 7.81 \times 10^{-4}(324,900) + 3.37 \times 10^{-5}(324,900)\sqrt{30} + 59 = 373 \ kg$$

- Thrust Structure Mass

$$M_{Thrust\ Structure}(kg) = 2.55 \times 10^{-4}(324,900) = 497 \ kg$$

First Pass Vehicle Configuration

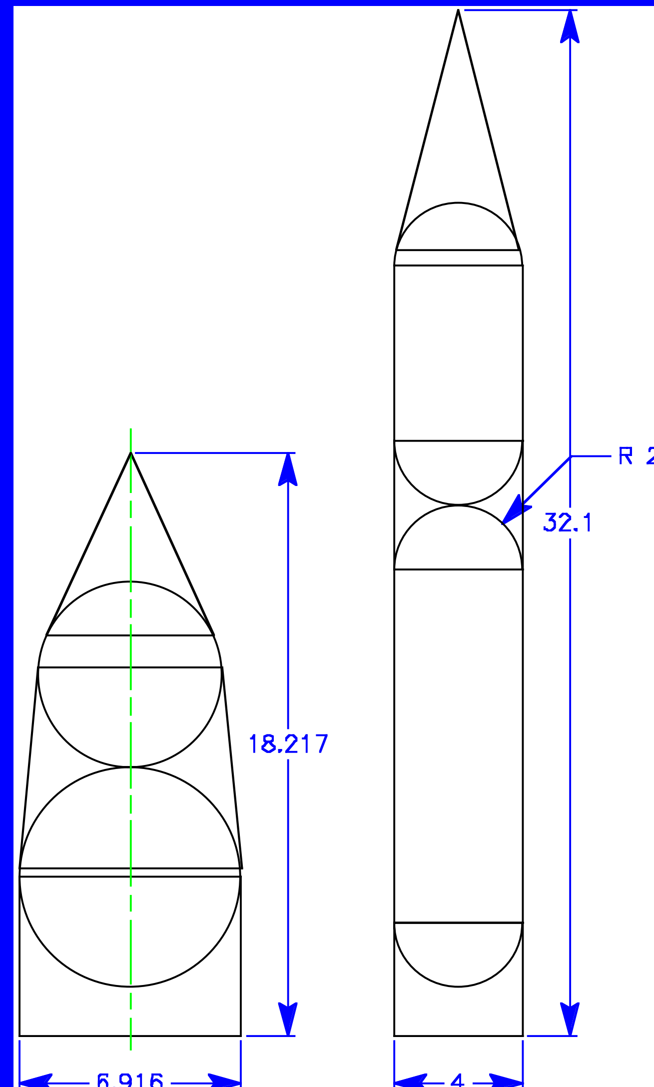


Mass Summary - First Pass

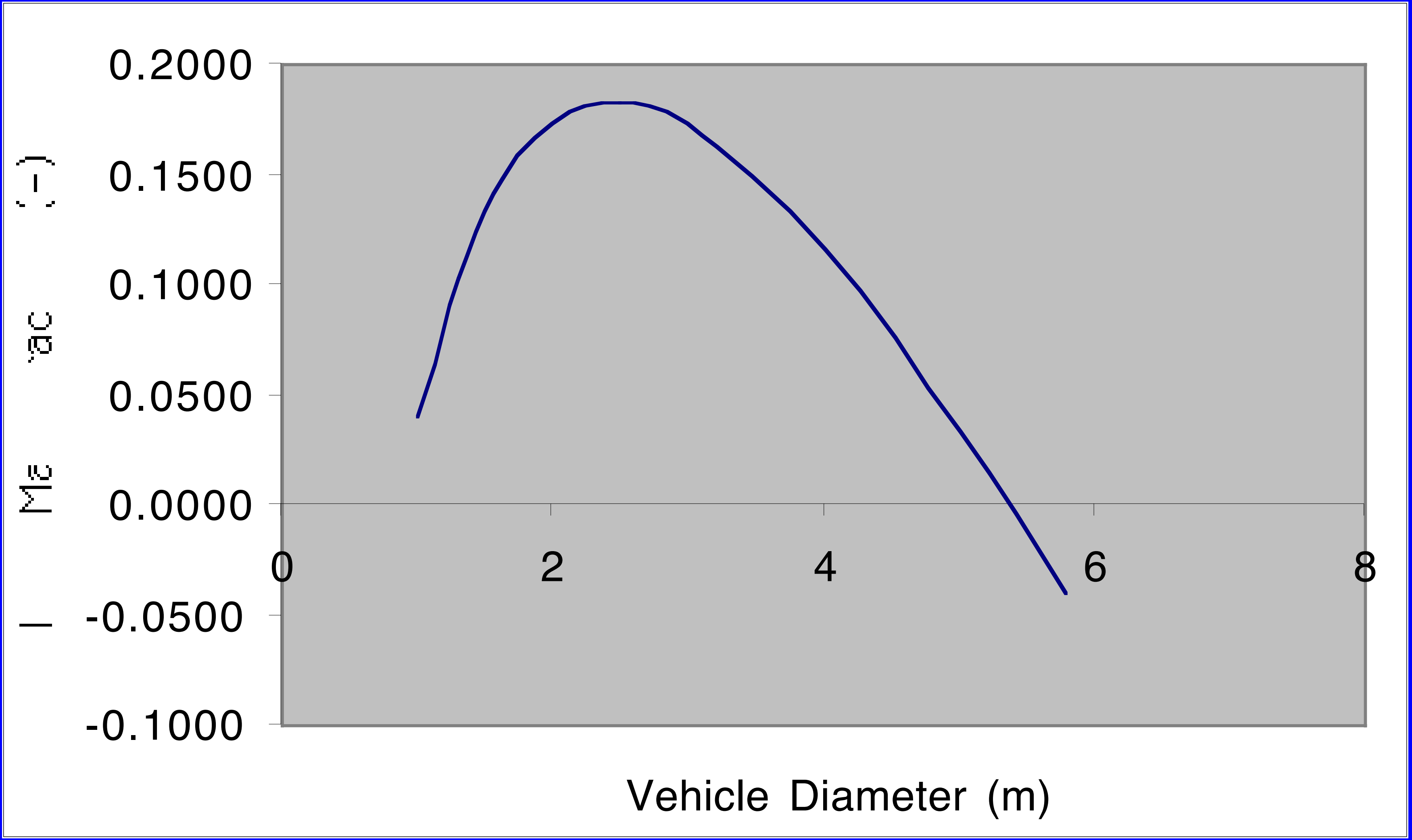
Initial Inert Mass Estimate	12,240 kg
LOX Tank	2087 kg
LH2 Tank	1709 kg
LOX Insulation	119 kg
LH2 Insulation	433 kg
Payload Fairing	688 kg
Intertank Fairing	1677 kg
Aft Fairing	1433 kg
Engines	2236 kg
Thrust Structure	497 kg
Gimbals	81 kg
Avionics	744 kg
Wiring	840 kg
Reserve	-
Total Inert Mass	12,543 kg
Design Margin	-2.4 %

Modifications for Second Pass

- Keep all initial vehicle sizing parameters constant
- Pick vehicle diameter and make tanks cylindrical to fit
- Redo MER analysis

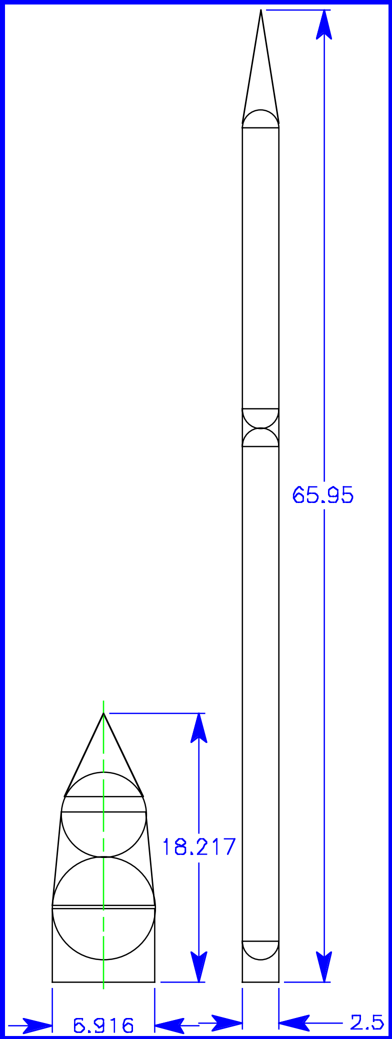


Effect of Vehicle Diameter on δ

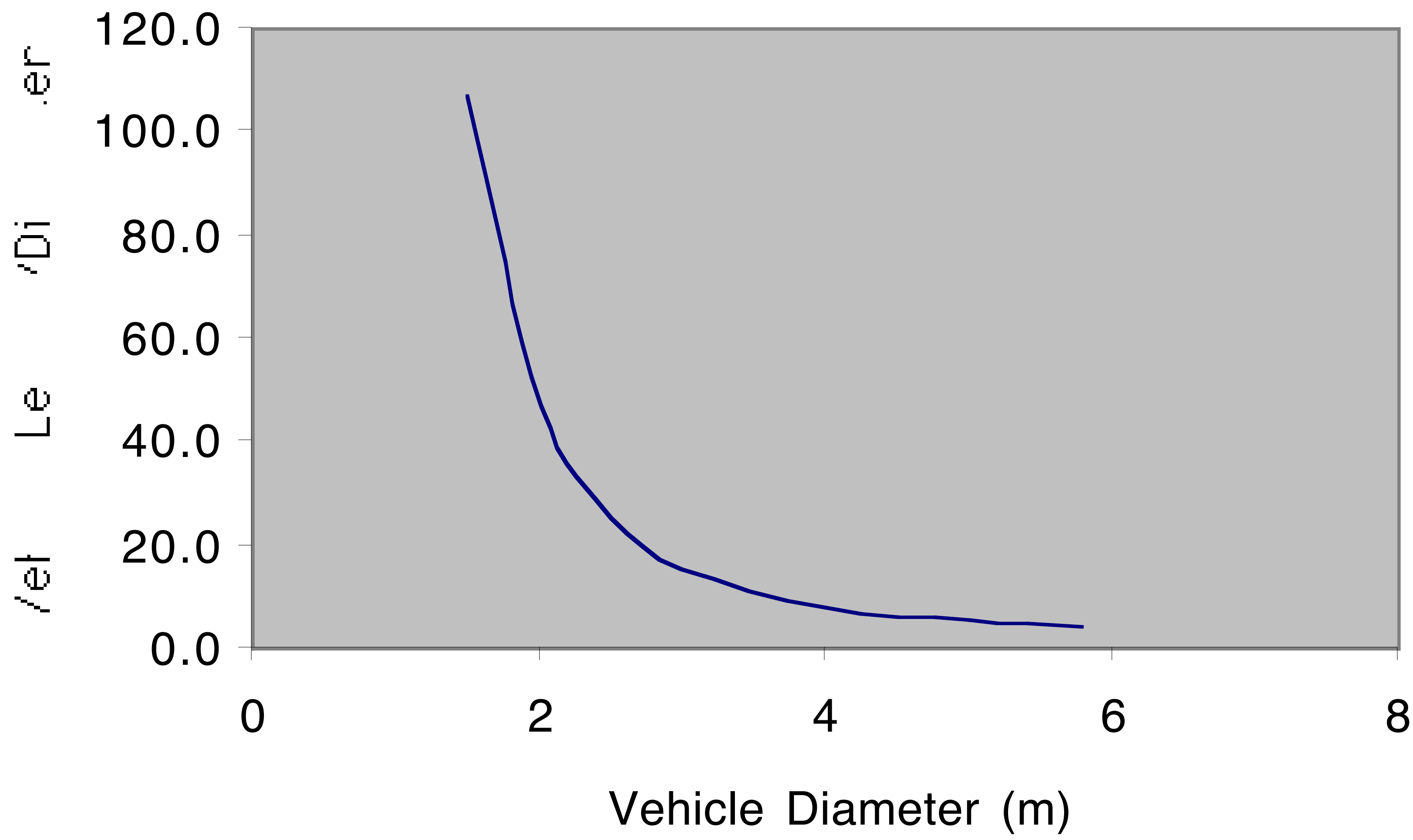


Effect of Mass-Optimal Diameter Choice

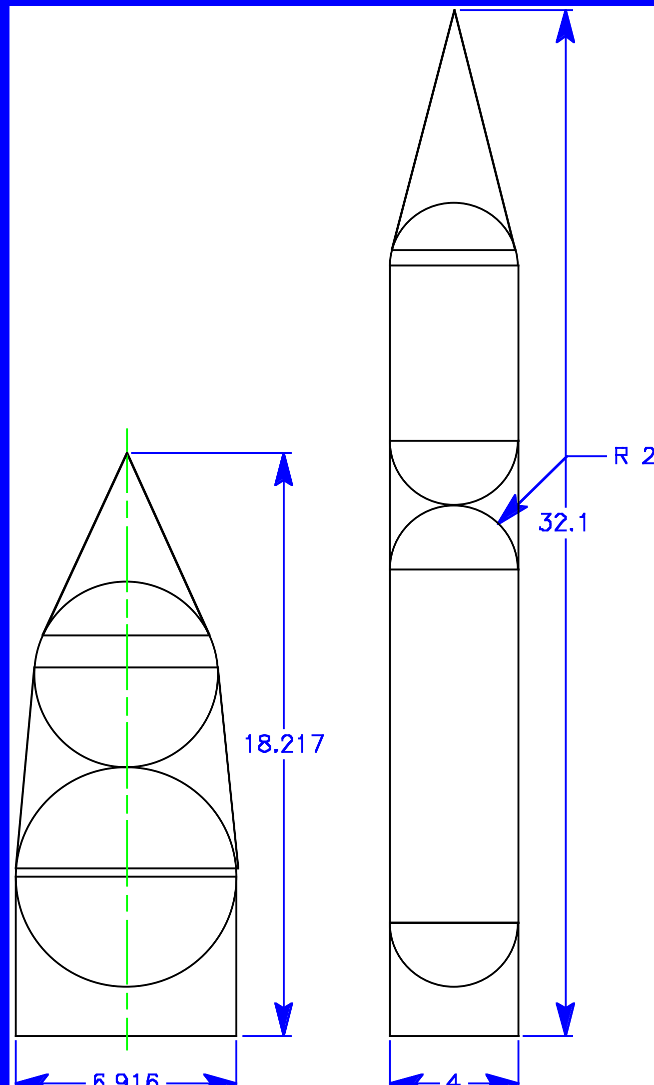
- Vehicle has L/D of 25.2 - severe complications from structural dynamics
- Mass margin goes from -2.4% to +18.3%
- Decreased volume for rocket engines in aft fairing
- Infeasible configuration



Effect of Diameter on Vehicle L/D



Second Pass Vehicle Configuration

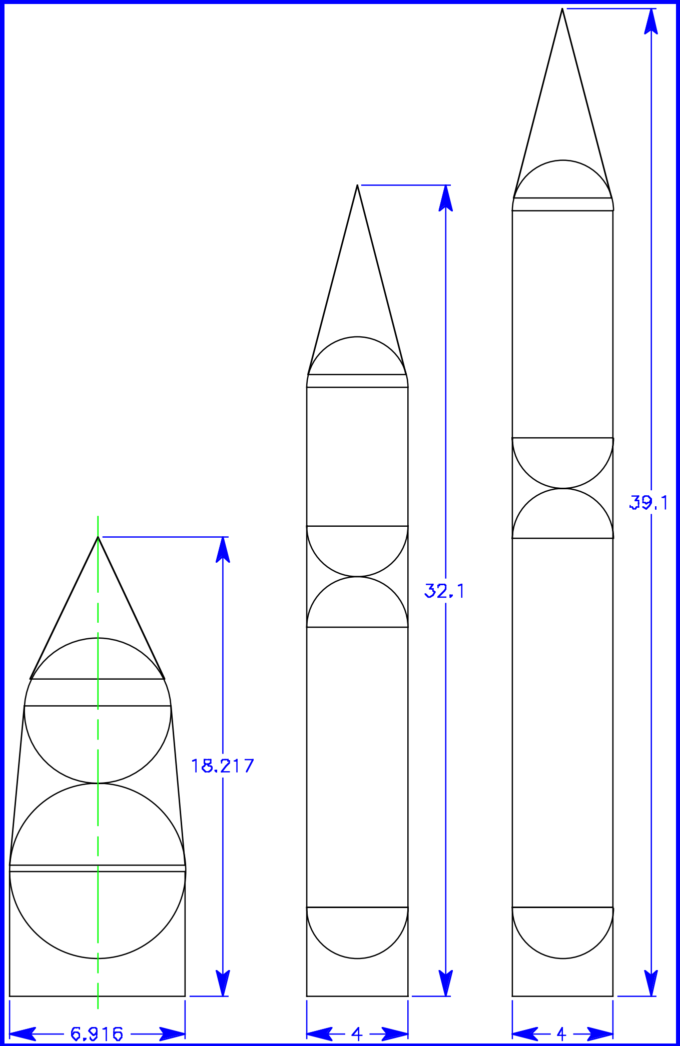


Mass Summary - Second Pass

Initial Inert Mass Estimate	12,240 kg	12,240 kg
LOX Tank	2087 kg	2087 kg
LH2 Tank	1709 kg	1709 kg
LOX Insulation	119 kg	133 kg
LH2 Insulation	433 kg	547 kg
Payload Fairing	688 kg	689 kg
Intertank Fairing	1677 kg	669 kg
Aft Fairing	1433 kg	585 kg
Engines	2236 kg	2236 kg
Thrust Structure	497 kg	497 kg
Gimbals	81 kg	81 kg
Avionics	744 kg	744 kg
Wiring	840 kg	985 kg
Reserve	-	-
Total Inert Mass	12,543 kg	10,960 kg
Design Margin	-2.4 %	+11.7 %

Modifications for Iteration 3

- Keep 4 m tank diameter
- Change initial assumption of δ iteratively, with resulting changes in m_0 and m_i, to reach 30% mass margin



Vehicle-Level Preliminary Design - 3rd Pass

- Single Stage to Orbit (SSTO) vehicle
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec
  - δ=0.08655

$$r = e^{-\frac{\Delta V}{g I_{sp}}} = 0.1127$$

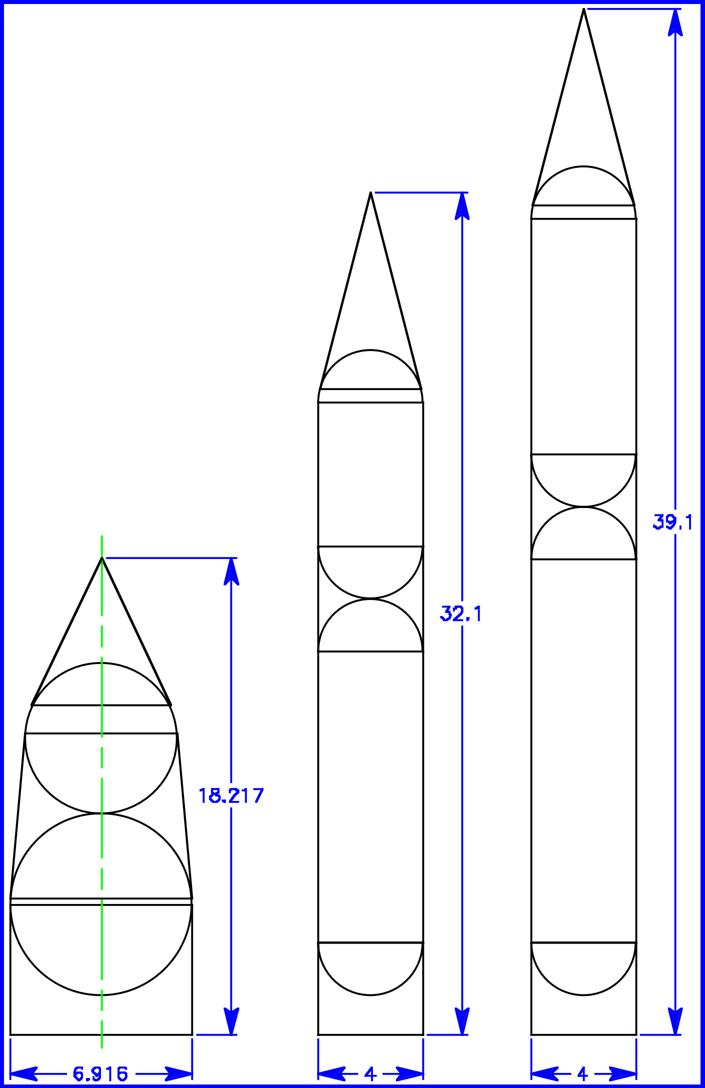
$$\lambda = r - \delta = 0.0261$$

$$M_0 = \frac{M_L}{\lambda} = 191,300\ kg$$

$$M_i = \delta M_0 = 16,560\ kg$$

$$M_p = M_0 (1 - r) = 169,800\ kg$$

Third Pass Vehicle Configuration



Mass Summary - Third Pass

Initial Inert Mass Estimate	12,240 kg	12,240 kg	16,560 kg
LOX Tank	2087 kg	2087 kg	2530 kg
LH2 Tank	1709 kg	1709 kg	2046 kg
LOX Insulation	119 kg	133 kg	162 kg
LH2 Insulation	433 kg	547 kg	672 kg
Payload Fairing	688 kg	689 kg	689 kg
Intertank Fairing	1677 kg	669 kg	669 kg
Aft Fairing	1433 kg	585 kg	585 kg
Engines	2236 kg	2236 kg	2708 kg
Thrust Structure	497 kg	497 kg	622 kg
Gimbals	81 kg	81 kg	100 kg
Avionics	744 kg	744 kg	807 kg
Wiring	840 kg	985 kg	1146 kg
Reserve	-	-	-
Total Inert Mass	12,543 kg	10,960 kg	12,740 kg
Design Margin	-2.4 %	+11.7 %	+30 %

Mass Budgeting

	Estimates	Budgeted	Margins
• Initial Inert Mass Estimate	16,560 kg	16,560 kg	3826 kg
• LOX Tank	2530 kg	2910 kg	380 kg
• LH2 Tank	2046 kg	2354 kg	307 kg
• LOX Insulation	162 kg	187 kg	24 kg
• LH2 Insulation	672 kg	773 kg	101 kg
• Payload Fairing	689 kg	793 kg	104 kg
• Intertank Fairing	669 kg	769 kg	100 kg
• Aft Fairing	585 kg	673 kg	88 kg
• Engines	2708 kg	3115 kg	407 kg
• Thrust Structure	622 kg	715 kg	93 kg
• Gimbals	100 kg	115 kg	15 kg
• Avionics	807 kg	928 kg	121 kg
• Wiring	1146 kg	1318 kg	172 kg
• Reserve	-	1913 kg	1913 kg
• Total Inert Mass	12,740 kg	16,560 kg	